ENS 525 - Fall 2010

Mathematical Methods for Scientist and Engineers I

(December 27, 2010)

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Lectures: M 11:40-13:30 FENS L063
W 12:40-13:30 FENS L056

Recitation: W 17:40-18:30 FENS G021

Course Description: This is the first in a series of two courses that aim to equip students in engineering and natural sciences with abstract concepts for approaching as well as practical tools for solving a diverse range of mathematical problems that they will encounter throughout their post-graduate education. The current course is concerned with complex calculus and linear vector spaces. Ordinary and partial differential equations are left for the second course. The presentation will start with a review of real-number calculus in one and two dimensions. Then, we will turn to the complex plane, where our main goal will be to introduce the calculus of residues and use it to evaluate integrals that are difficult to approach by other means. The skills developed in the process will prove useful in the discussion of Fourier and Laplace transforms, widely used in engineering applications. Our subsequent review of matrix algebra aims to highlight the role of eigenvalues and eigenvectors as means for characterizing and solving linear problems. Some of the concepts encountered in the presentation of matrix algebra and Fourier transforms will be unified in the discussion of abstract linear vector spaces. This will allow us to view Fourier transforms from an abstract perspective amenable to generalizations—wavelets and multiresolution analysis exemplifying one such possibility.

Who can take this course: The course is catered to starting graduate students in engineering and natural sciences who feel the need to refresh their knowledge of complex calculus, vector calculus, and linear algebra. Alternatively, the course can serve as a streamlined overview of these fields for students who have taken only two semesters of undergraduate Calculus. Therefore, it should be attractive to motivated, advanced undergraduate students as well. Since the mathematical concepts developed in this course are intended as tools for solving practical problems, most of the time the lack of mathematical rigor in their presentation will surely upset a mathematician.

Evaluation:

Homework and quizzes	15 %
Exam I (Nov 3)	25 %
Exam II (Dec 22)	25 %
Comprehensive final exam	35 %

Detailed Course Content:

I. Real-number calculus in one and two dimensions

A. One dimension

1. Elementary functions
2. Differentiation and integration
3. Power series and convergence
4. The Dirac delta function

B. Calculus in the plane

1. Plane curves
2. Differential calculus in two variables
3. Integral calculus in the plane and Green's theorem

II. Complex variables and transforms

A. Complex-number calculus

1. Complex numbers and complex functions
2. Differentiation and the Cauchy-Riemann conditions
3. Contour integration and the Cauchy formula
4. Analytic functions and power series
5. Calculus of residues

B. Transforms

1. Fourier transform I
2. Laplace transform

III. Linear vector spaces

A. Review of matrix algebra

1. Matrix operations and functions of matrices
2. The eigenvalue problem and (bi)orthogonal systems

B. Introduction to abstract linear vector spaces

1. Vector spaces and dual spaces
2. Linear transformations (or operators)
3. Eigenvalue problem and resolution of identity
4. Translation operator and Fourier transform II
5. Dilation operator, wavelets, and multiresolution analysis

Reference Books:

- Haluk Beker, *Fen ve Mühendislikte Matematiksel Metotlar*, Boğaziçi Üniversitesi Yayınevi, Istanbul, 2006.
- Bruce R. Kusse and Eric A. Westwig, *Mathematical Physics: Applied Mathematics for Scientists and Engineers*, 2nd edition, Wiley-VCH, Weinheim, 2006.
- Roger Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe*, Alfred A. Knopf, New York, 2005.
- Francis J. Flanigan, *Complex Variables: Harmonic and Analytic Functions*, Allyn and Bacon Inc., Boston, 1972. (Dover edition, 1983.)

- Philippe Dennery and André Krzywicki, *Mathematics for Physicists*, Harper & Row, New York, 1967. (Dover edition, 1996.)
- Cornelius Lanczos, *Applied Analysis*, Prentice-Hall Inc., Englewood Cliffs, N.J., 1956. (Dover edition, 1988.)
- Cornelius Lanczos, *Linear Differential Operators*, D. Van Nostrand Co., Princeton, N.J., 1961. (Dover edition, 1997.)