ENS 526 - Spring 2012

Mathematical Methods for Scientists and Engineers II

(May 22, 2012)

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Lectures: W 11:40-13:30 FASS 1096
Th 10:40-11:30 FENS L065

Recitation: Th 9:40-10:30 FENS L065

Course Description: This is the second in a series of two courses that aim to equip students in engineering and natural sciences with abstract concepts for approaching as well as practical tools for solving a diverse range of mathematical problems that they will encounter throughout their post-graduate education. The presentation starts with a discussion of ordinary differential equations, where the focus is on linear, second order DEs. Obtaining the power series solutions of such DEs will lead us to a variety of special functions, including Legendre polynomials and Bessel functions. Viewing differentiation as a linear operator in the vector space of differentiable functions will give us a fresh look at the Fourier transform as a mere change of basis. Equipped with this abstract understanding we turn to the systematic analysis of second order linear differential operators by studying their eigenvalue problems. We will see that the eigenfunctions of second order linear differential operators can be chosen such that they form complete orthonormal bases. Any other function can be written as a linear combination of these basis functions. Then, we will look at partial differential operators. In particular, we will examine the representation of the Laplacian in various curvilinear coordinates. The method of separation of variables will be employed to solve the Helmholtz equation in different curvilinear coordinate systems. At this point, the knowledge about ordinary DEs acquired in the first part of the course will prove useful. The course will end with a discussion of the calculus of variations. Coming full circle, we will derive the differential equations considered throughout the course as the solutions of variational problems.

Evaluation:

Homework and quizzes	15 %
Exam I (Mar 29)	25 %
Exam II (May 17)	25 %
Comprehensive final exam	35 %

Detailed Course Content:

I. Ordinary differential equations

   A. Classification of second order, linear DEs
   B. Series solutions of homogeneous, linear DEs
   C. Solutions of hypergeometric and confluent hypergeometric DEs
   D. Invariant form of second order LDEs

II. Some special functions

A. Gamma and beta functions
B. Bessel and modified Bessel functions

III. Introduction to abstract linear vector spaces

A. Algebraic foundations (group, field, linear vector space, algebra)
B. Hilbert space (basis, inner product, norm, metric)
C. Linear transformations and operators
D. Translation operator and Fourier transform

IV. Sturm-Liouville problems

A. Green's identity and self-adjoint form of linear differential operators
B. Weighted Green's identity and Hermitization procedures
C. Expansions in orthogonal functions

V. Partial differential equations

A. Differential operators in curvilinear coordinates
B. Solution of Laplace, wave and diffusion equations through separation of variables
C. Green's functions of partial differential equations through expansion in orthogonal functions

VI. Calculus of variations

A. Variation and the Euler-Lagrange equation
B. Lagrangian multipliers and variation with constraints
C. Calculus of variation and Sturm-Liouville problems

Reference Books:

- Haluk Beker, *Fen ve Mühendislikte Matematiksel Metotlar I*, Boğaziçi Üniversitesi Yayınevi, Istanbul, 2006.
- George B. Arfken and Hans J. Weber, *Mathematical Methods for Physicists*, 6th edition, Academic Press, 2005.
- John D. Jackson, *Classical Electrodynamics*, 3rd edition, Wiley, 1998.
- Philippe Dennery and André Krzywicki, *Mathematics for Physicists*, Harper & Row, New York, 1967. (Dover edition, 1996.)
- Cornelius Lanczos, *Linear Differential Operators*, D. Van Nostrand Co., Princeton, N.J., 1961. (Dover edition, 1997.)
- Cornelius Lanczos, *The Variational Principles of Mechanics*, 4th edition, University of Toronto Press, Toronto, 1970. (Dover edition, 1986.)