CHARGE CONSERVATION AND EMERGENT GRAVITY

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Higgs couplings follow the SM:



There exist no new colored particles in the TeV domain:



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But, the SM still needs be extended for various physical reasons:







Reconciling SM with GR is an intricate problem.





(Poincare breaking scale)







All loop momenta are cut off at Λ_{\wp} . Then all the bosons, including the gauge bosons, acquire $\mathcal{O}(\Lambda_{\wp}^2)$ masses:



(D'Attanasio & Morris, hep-ph/<u>9602156</u>, '96)

(Peskin & Schroeder, QFT book, '95)







(Haque, 2014)

The SM effective action contains then a «hard mass term» for each gauge boson:

$$\delta S_V(\eta, \Lambda_{\wp}) = \int d^4x \sqrt{-\eta} c_V \Lambda_{\wp}^2 \operatorname{tr}[V_{\mu} V^{\mu}] + 0 \log \Lambda_{\wp} + 0 \, (\text{finite terms})$$

| Gauge Boson (V^{μ}) | Loop Factor (c_V) | Broken Gauge Symmetry |
|---------------------------|------------------------------------|-----------------------|
| $g_{\mu}^{a=1,,8}$ | $c_g = \frac{21}{16 \pi^2} g_s^2$ | $SU(3)_C$ |
| $W^{i=1,,3}_{\mu}$ | $c_W = \frac{21}{16 \pi^2} g_2^2$ | $SU(2)_L$ |
| B_{μ} | $c_B = \frac{39}{32\pi^2} g_Y^2$ | $U(1)_Y$ |

Color breaking demolishes confinement and destructs therefore all the hadronic structures.



Isospin is broken explicitly and spontaneously (by $\langle H \rangle \neq 0$). Electromagnetism is broken by $c_W \neq 2 c_B$.

$$\tan 2\tilde{\theta}_W = \frac{(g_2^2 - g_Y^2) \langle H \rangle^2}{(g_2^2 - g_Y^2) \langle H \rangle^2 + 2(c_W - 2c_B)\Lambda_{\wp}^2} \tan 2\theta_W \implies \partial_\mu J^\mu \neq 0$$



(Okun&Voloshin, '77; Ignatiev&Joshi, '96)

How to prevent charge and color breaking (CCB)? How to sweep away $\delta S_V(\eta, \Lambda_{\wp})$?

Start with the rather trivial identity:

$$\delta S_V \equiv -I_V + \delta S_V + I_V$$

in which

$$I_V(\eta) = \int d^4x \, \sqrt{-\eta} \, \frac{c_V}{2} \operatorname{tr} \left[V_{\mu\nu} \, V^{\mu\nu} \right]$$

is a kinetic structure involving the loop factor c_V .

(DD, <u>arXiv:1901.07244</u>, '19; <u>arXiv:1605.00377</u>, '16)

Now, expand the second I_V via by-parts integration and combine the result with $\delta S_V(\eta, \Lambda_{\wp})$. This leads to the renewed $\delta S_V(\eta, \Lambda_{\wp})$:

$$\delta S_V(\eta, \Lambda_{\wp}) \equiv -I_V(\eta) + \int d^4x \sqrt{-\eta} c_V \operatorname{tr}[V^{\mu}(-D^2_{\mu\nu} + \Lambda^2_{\wp} \eta_{\mu\nu})V^{\nu} + \partial_{\mu}(V_{\nu}V^{\mu\nu})]$$

in which D_{μ} is gauge-covariant derivative, and

$$D_{\mu\nu}^2 = D^2 \eta_{\mu\nu} - D_\mu D_\nu - V_{\mu\nu}$$

is the usual inverse V_{μ} propagator.

The first step is to go to curved spacetime of a putative metric $g_{\mu\nu}$. Thus, in view of the general covariance, let

 $\eta_{\mu
u} \hookrightarrow g_{\mu
u}$

under which

$$\partial_{\mu} \rightarrow V_{\mu}, \ D_{\mu} \rightarrow D_{\mu}, \ D_{\mu\nu}^2 \rightarrow D_{\mu\nu}^2 = D^2 g_{\mu\nu} - D_{\mu} D_{\nu} - V_{\mu\nu}$$

so that $\delta S_V(\eta, \Lambda_{\wp})$ changes to

$$\delta S_V(g,\Lambda_{\wp}) \equiv -I_V(g) + \int d^4x \sqrt{-g} c_V \operatorname{tr}[V^{\mu}(-\mathcal{D}^2_{\mu\nu} + \Lambda^2_{\wp} g_{\mu\nu})V^{\nu} + \nabla_{\mu}(V_{\nu}V^{\mu\nu})]$$

in which the covariant derivative V_{μ} , satisfying $V_{\mu} g_{\alpha\beta} = 0$, is that of the Levi-Civita connection

$${}^{g}\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} (\partial_{\mu}g_{\nu\rho} + \partial_{\nu}g_{\rho\mu} - \partial_{\rho}g_{\nu\mu})$$

It is natural to associate the Poincare breaking scale Λ_{\wp} to spacetime curvature. It is thus conceivable to extend the metrical map

 $\eta_{\mu
u} \hookrightarrow g_{\mu
u}$

by the curvature mape

 $\Lambda^2_{\wp} g_{\mu\nu} \hookrightarrow R_{\mu\nu}({}^g\Gamma)$

where $R_{\mu\nu}(\Gamma(g))$ is the Ricci curvature of the Levi-Civita connection. These two maps do indeed nullify the problematic gauge boson mass action

$$\delta S_V(g,R) \equiv -I_V(g) + \int d^4x \sqrt{-g} \operatorname{c_V tr} \left[V^{\mu} \left(-\mathcal{D}^2_{\mu\nu} + R_{\mu\nu} ({}^g\Gamma) \right) V^{\nu} + \nabla_{\!\mu} (V_{\nu} V^{\mu\nu}) \right]$$
$$\equiv -I_V(g) + I_V(g) = 0 \,!$$

if c_V is held unchanged under curvature map! This seems to yield precisely what is sought! The CCB seems over !

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if c_V is held unchanged under curvature map! This seems to yield precisely what is sought! The CCB seems over !

How to prevent contradiction ? How to make curvature approach work? One possibility is to replace the Levi-Civita connection ${}^{g}\Gamma^{\lambda}_{\mu\nu}$ by an "affine connection" $\Gamma^{\lambda}_{\mu\nu}$. Namely, assume now that the metrical map

 $\eta_{\mu
u} \hookrightarrow g_{\mu
u}$

is followed by an «affine curvature map» of the form

 $\Lambda^2_{\wp} g_{\mu\nu} \hookrightarrow \mathbb{R}_{\mu\nu}(\Gamma)$

where $\mathbb{R}_{\mu\nu}(\Gamma)$ is the Ricci curvature of the affine connection $\Gamma^{\lambda}_{\mu\nu}$. These two maps now lead to

$$\delta S_V(g,\mathbb{R}) \equiv -I_V(g) + \int d^4x \sqrt{-g} \operatorname{c}_V \operatorname{tr} \left[V^{\mu} \left(-\mathcal{D}^2_{\mu\nu} + \mathbb{R}_{\mu\nu}(\Gamma) \right) V^{\nu} + \nabla_{\!\!\mu} (V_{\nu} V^{\mu\nu}) \right]$$

$$\equiv \int d^4x \sqrt{-g} c_V \operatorname{tr} \left[V^{\mu} \left(\mathbb{R}_{\mu\nu}(\Gamma) - R_{\mu\nu}({}^g\Gamma) \right) V^{\nu} \right]$$

if c_V is held unchanged while $\Lambda^2_{\wp} g_{\mu\nu} \hookrightarrow \mathbb{R}_{\mu\nu}(\Gamma)$.

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How to prevent contradiction ? How to make curvature approach work? One possibility is to replace the Levi-Civita connection ${}^{g}\Gamma^{\lambda}_{\mu\nu}$ by an "affine connection" $\Gamma^{\lambda}_{\mu\nu}$. Namely, assume no $\sum_{\mu\nu}$ metrical map where $\mathbb{R}_{\mu\nu}(\Gamma)$ where $\mathbb{R}_{\mu\nu}(\Gamma)$ is the Ricci curver esseed affine connection $\Gamma^{\lambda}_{\mu\nu}$. These two maps now lead to $\delta S_{V}(g,\mathbb{R}) \equiv -I_{P} \left(be supplies curver x \sqrt{-g} c_{V} \operatorname{tr} \left[V^{\mu} \left(-D_{\mu\nu}^{2} + \mathbb{R}_{\mu\nu}(\Gamma) \right) V^{\nu} + \nabla_{\mu}(V_{\nu}V^{\mu\nu}) \right] \right]$ $CCB \stackrel{can be}{can}_{\mu\nu} \left(x \sqrt{-g} c_{V} \operatorname{tr} \left[V^{\mu} \left(\mathbb{R}_{\mu\nu}(\Gamma) - R_{\mu\nu}(^{g}\Gamma) \right) V^{\nu} \right] \right]$ s held unchanged while $\Lambda^{2}_{g\rho} g_{\mu\nu} \hookrightarrow \mathbb{R}_{+} \left(\Gamma^{n} \right)$ $\eta_{\mu\nu} \hookrightarrow g_{\mu\nu}$

 $\Gamma_{\mu\nu}^{\lambda}$ dynamics is set by curvature sector, and curvature sector stems from corrections to the vacuum and Higgs sectors:

 $\delta S_{OH}(\eta, \Lambda_{\wp}) = -\int d^4x \sqrt{-\eta} \left\{ c_4 \operatorname{str}[1] \Lambda_{\wp}^4 + c_m \operatorname{str}[m^2] \Lambda_{\wp}^2 + c_h \Lambda_{\wp}^2 h^2 \right\}$



Employing the metrical and curvature maps, the vacuum and Higgs sectors lead to the curvature sector:

$$\delta S_{OH}(g,\mathbb{R}) = -\int d^4x \sqrt{-g} \left\{ \frac{c_4}{16} \operatorname{str}[1](\mathbb{R}(g,\Gamma))^2 + \frac{c_m}{4} \operatorname{str}[m^2]\mathbb{R}(g,\Gamma) + \frac{c_h}{4} \mathbb{R}(g,\Gamma)h^2 \right\}$$

$$\mathbb{R}(g,\Gamma) = g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma)$$
Higgs-curvature coupling $\zeta = \frac{c_h}{2}$
would be M_{Pl}^2 if it were not wrong in size $(\operatorname{str}[m^2] \sim \Lambda_W^2)$ and sign $(\operatorname{str}[m^2] < 0)$!



The BSM sector must have requisite degrees of freedom to generate M_{Pl} correctly.

- > BSM sector is spanned by scalars h', gauge fields V'_{μ} , fermions f', ...
- SM+BSM is spanned by scalars $\mathcal{H} = \{h, h'\}$, gauge fields $\mathcal{V}_{\mu} = \{V_{\mu}, V_{\mu}'\}, \cdots$

▶ BSM mass spectrum is
$$m' = \{m_{h'}, m_{V'}, m_{f'}, \dots\}$$

- > SM+BSM mass spectrum is $\mathcal{M} = \{m_h, m_V, m_f, m_{h'}, m_{V'}, m_{f'}, \cdots\}$
- > Then, fundamental scale of gravity takes the form

$$M_{Pl}^{2} = \frac{1}{2} (c_{m} \operatorname{str}[m^{2}] + c_{m'} \operatorname{str}[m'^{2}]) \rightarrow \frac{1}{64\pi^{2}} \operatorname{str}[\mathcal{M}^{2}]$$
one-loop
scale or stack of m' sets M_{Pl}

The complete curvature sector, reinstated with BSM effects, can be put into the form:

$$\delta S(g,\mathbb{R}) = \int d^4x \sqrt{-g} \left\{ -Q^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) + \frac{c_4}{16} \operatorname{str}[1] \big(\mathbb{R}(g,\Gamma) \big)^2 - c_V R_{\mu\nu}({}^g\Gamma) \operatorname{tr}[\mathcal{V}^{\mu}\mathcal{V}^{\nu}] \right\}$$
$$Q^{\mu\nu} = \left(\frac{M_{Pl}^2}{2} + \frac{c_4}{8} \operatorname{str}[1] \mathbb{R}(g,\Gamma) \right) g^{\mu\nu} + \mathcal{K}^{\mu\nu}$$
$$\mathcal{K}^{\mu\nu} = \frac{c_{\mathcal{H}}}{4} \mathcal{H}^2 g^{\mu\nu} - c_{\mathcal{V}} \operatorname{tr}[\mathcal{V}^{\mu}\mathcal{V}^{\nu}]$$

At last the $\Gamma^{\lambda}_{\mu\nu}$ dynamics! $\Gamma^{\lambda}_{\mu\nu}$ obeys the equation of motion:

$${}^{\Gamma}\nabla_{\alpha} Q_{\mu\nu} = 0$$

Its solution is:

$$\Gamma^{\lambda}_{\mu\nu} = {}^{g}\Gamma^{\lambda}_{\mu\nu} + \frac{1}{2} \left(Q^{-1} \right)^{\lambda\rho} \left(\nabla_{\mu} Q_{\nu\rho} + \nabla_{\nu} Q_{\rho\mu} - \nabla_{\rho} Q_{\mu\nu} \right)$$

this solution is actually a non-linear PDE for $\Gamma^{\lambda}_{\mu\nu}$ because $Q_{\mu\nu}$ involves the affine curvature $\mathbb{R}(g,\Gamma) \sim \partial\Gamma + \Gamma\Gamma$

Dropping ${\cal H}$ and ${\cal V}_{\mu}$ for simplicity and clarity, the curvature is found to satisfy the equation



At high curvatures gravity may deviate from EH form!

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At last the $\Gamma^{\lambda}_{\mu\nu}$ dynamics! It will now be possible to determ $\Gamma^{\lambda}_{\mu\nu}$ obeys the equation of motion: ¹, does indeed approach to ${}^{g}\Gamma^{\lambda}_{\mu\nu}$. In this regard,

- u - and this happens if - and this happens if - str[1] = 0 - str[1 ${}^{\Gamma}\nabla_{\alpha} Q_{\mu\nu} = 0$ Its solution is $\Gamma^{\lambda}_{\mu\nu} =$ γ is actually a partial γ Jecause $Q_{\mu
u}$ involves the affine curvature $\mathbb{R}(q, \Gamma) \sim \partial \Gamma + \Gamma \Gamma$

 $\Gamma_{\mu\nu}^{\lambda}$ does indeed approach to ${}^{g}\Gamma_{\mu\nu}^{\lambda}$! Indeed, with str[1] = 0, the affine connection takes an algebraic form

$$\Gamma_{\mu\nu}^{\lambda} = {}^{g}\Gamma_{\mu\nu}^{\lambda} + \frac{1}{2}\left(\left(\frac{M_{Pl}^{2}}{2}g + \mathcal{K}\right)^{-1}\right)^{\lambda\rho}\left(\nabla_{\!\!\mu}\mathcal{K}_{\nu\rho} + \nabla_{\!\!\nu}\mathcal{K}_{\rho\mu} - \nabla_{\!\!\rho}\mathcal{K}_{\mu\nu}\right)$$

$$={}^{g}\Gamma^{\lambda}_{\mu\nu} + \frac{1}{M_{Pl}^{2}} \left(\nabla_{\mu}\mathcal{K}_{\nu\rho} + \nabla_{\nu}\mathcal{K}_{\rho\mu} - \nabla_{\rho}\mathcal{K}_{\mu\nu} \right) + \mathcal{O}\left(\frac{\nabla\mathcal{K}^{2}}{M_{Pl}^{4}}\right)$$

involves only the scalars ${\mathcal H}$ and gauge bosons ${\mathcal V}_\mu$ in SM+BSM!

Corresponding to the affine connection, the affine curvature takes the form

$$\mathbb{R}_{\mu\nu}(\Gamma) = R_{\mu\nu}({}^{g}\Gamma) + \mathcal{O}(\frac{\nabla^{2}\mathcal{K}}{M_{Pl}^{2}})$$

so that the notorious CCB gauge-boson mass action becomes

$$\delta S_V(g,\mathbb{R}) \equiv \int d^4x \sqrt{-g} c_V \operatorname{tr} \left[V^{\mu} \left(\mathbb{R}_{\mu\nu}(\Gamma) - R_{\mu\nu}({}^g\Gamma) \right) V^{\nu} \right] = 0 + \int d^4x \sqrt{-g} c_V \mathcal{O}(\frac{\nabla^2 \mathcal{K}}{M_{Pl}^2})$$

no contribution to scalar and gauge boson masses!

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With the affine curvature

$$\mathbb{R}_{\mu\nu}(\Gamma) = R_{\mu\nu}({}^{g}\Gamma) + \mathcal{O}(\frac{\nabla^{2}\mathcal{K}}{M_{Pl}^{2}})$$

the complete curvature sector takes the form

$$\delta S(g,\mathbb{R}) = \int d^4x \sqrt{-g} \left\{ -Q^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) + \frac{c_4}{16} \operatorname{str}[1] \big(\mathbb{R}(g,\Gamma) \big)^2 - c_V R_{\mu\nu}({}^g\Gamma) \operatorname{tr}[\mathcal{V}^{\mu}\mathcal{V}^{\nu}] \right\}$$
$$= \int d^4x \sqrt{-g} \left\{ -\frac{M_{Pl}^2}{2} R(g) - \frac{c_{\mathcal{H}}}{4} R(g) \mathcal{H}^2 + \mathcal{O}\left(\frac{\mathcal{K}\nabla^2 \mathcal{K}}{M_{Pl}^2}\right) \right\}$$

With the affine curvature

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complete curvature sector takes the form

$$\delta S(g,\mathbb{R}) = \int d^4x \sqrt{-g} \left\{ -Q^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) + \frac{c}{9} \right\} \left[\mathbb{R}(g,\Gamma) \right]^2 - c_V R_{\mu\nu}(^g\Gamma) \operatorname{tr}[\mathcal{V}^{\mu}\mathcal{V}^{\nu}] \right\}$$
$$= \int d^4x \sqrt{-g} \left\{ \mathcal{O}[\mathcal{V}^2\mathcal{H}]_2 - \frac{c_{\mathcal{H}}}{4} R(g) \mathcal{H}^2 + \mathcal{O}\left(\frac{\mathcal{K}\nabla^2\mathcal{H}}{M_{Pl}^2}\right) \right\}$$



no higher-curvature terms !

Symmergence left behind only logarithmic UV-sensitivities. Nevertheless, the equivalence relation

$$\log \frac{\Lambda_{\wp}}{\Lambda_W} = \frac{1}{2\epsilon} + \log \frac{\mu}{\Lambda_W}$$

enables passage to dimensional regularization! Independence from μ leads to RGE's.

| Flat Spacetime | Curved Spacetime |
|---|---------------------------------------|
| $SM(\psi,\eta,\Lambda_{\wp}^2,\log\Lambda_{\wp})$ | $SM(\psi, g, \mathbb{R}, \log \mu)$ |
| \oplus | \oplus |
| $BSM(\psi',\eta,\Lambda_{\&}^{2},\log\Lambda_{\bigotimes})$ | $BSM(\psi', g, \mathbb{R}, \log \mu)$ |

Gravity is incorporated into the SM in such a way that:

CCB is suppressed; GHP is neutralized; BSM is specified; Dim. Reg. is recovered.

There exist numerous problems to be investigated:

- What is the high curvature limit? Are there BH solutions?
- ➢ Is there an underlying SUSY? Can it have a say on the CCP?
- > Is inflation a scalar field? Or, is it a vector?
- Can Poincare breaking be made dynamical?
- How to characterize various BSM phenomena?

Thank You