Quantum Travel Time and Tunnel Ionization of Atoms

DURMUŞ DEMİR

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Annals of Physics, 386 (2017) 291

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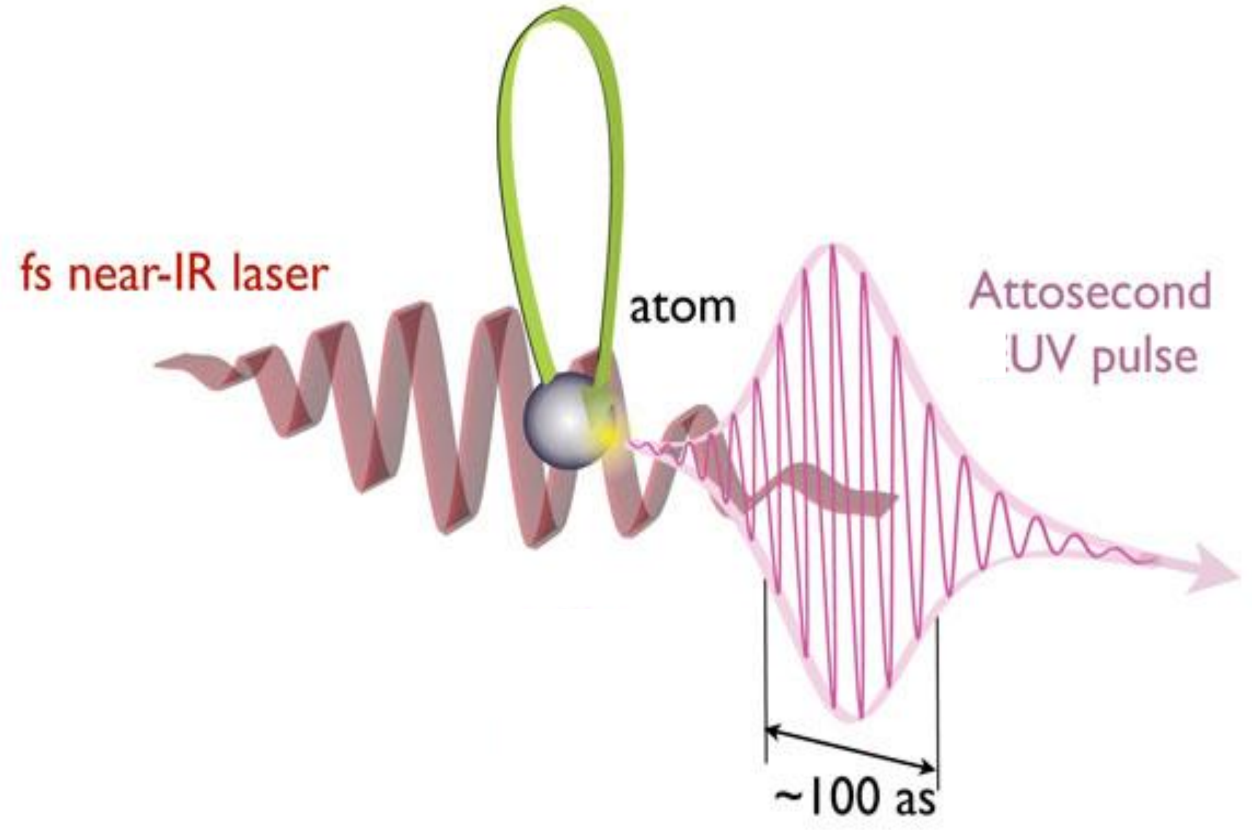
arXiv:2001.06071 [quant-ph]

Sabancı Üniversitesi 20 years

Quantum Optics and Information Meeting 4, 14-15 May 2020, IZTECH-İZMİR

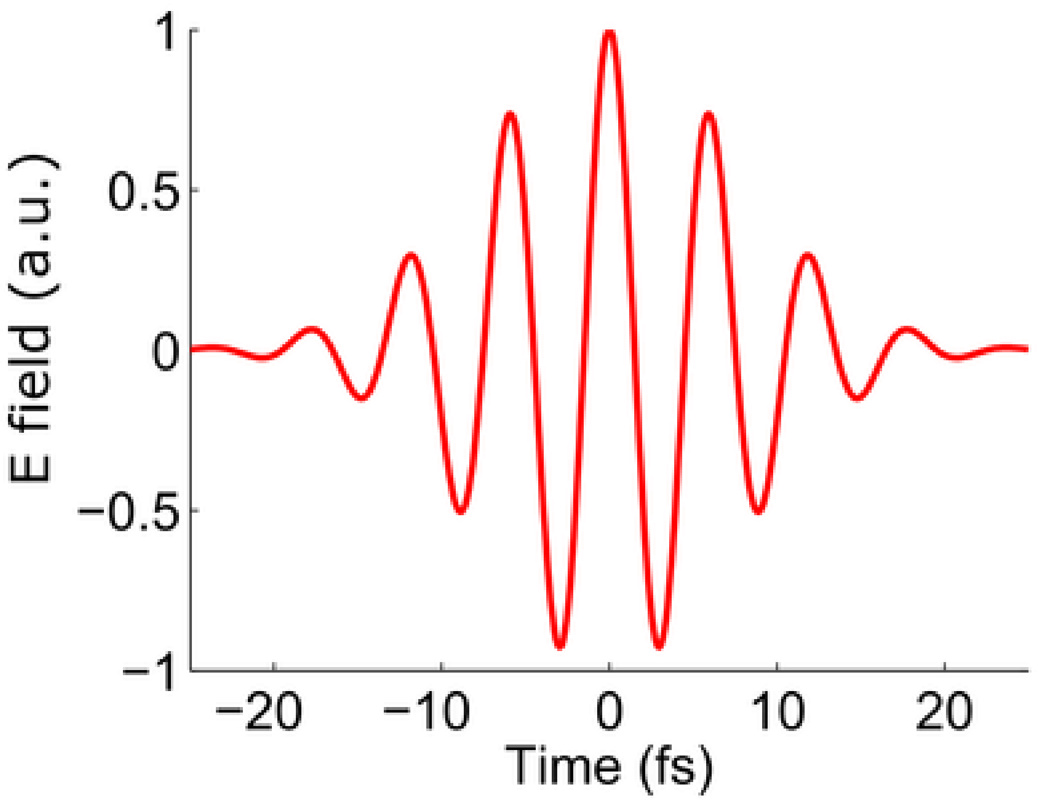
Atomic phenomena lie at the *attosecond* scales:

- Length scale: $a_0 \approx 0.05\ \mathrm{nm}$
- Speed scale: $v_0 \approx \alpha\ c$
- Time scale: $\frac{a_0}{v_0} \approx 24\ \mathrm{as}$

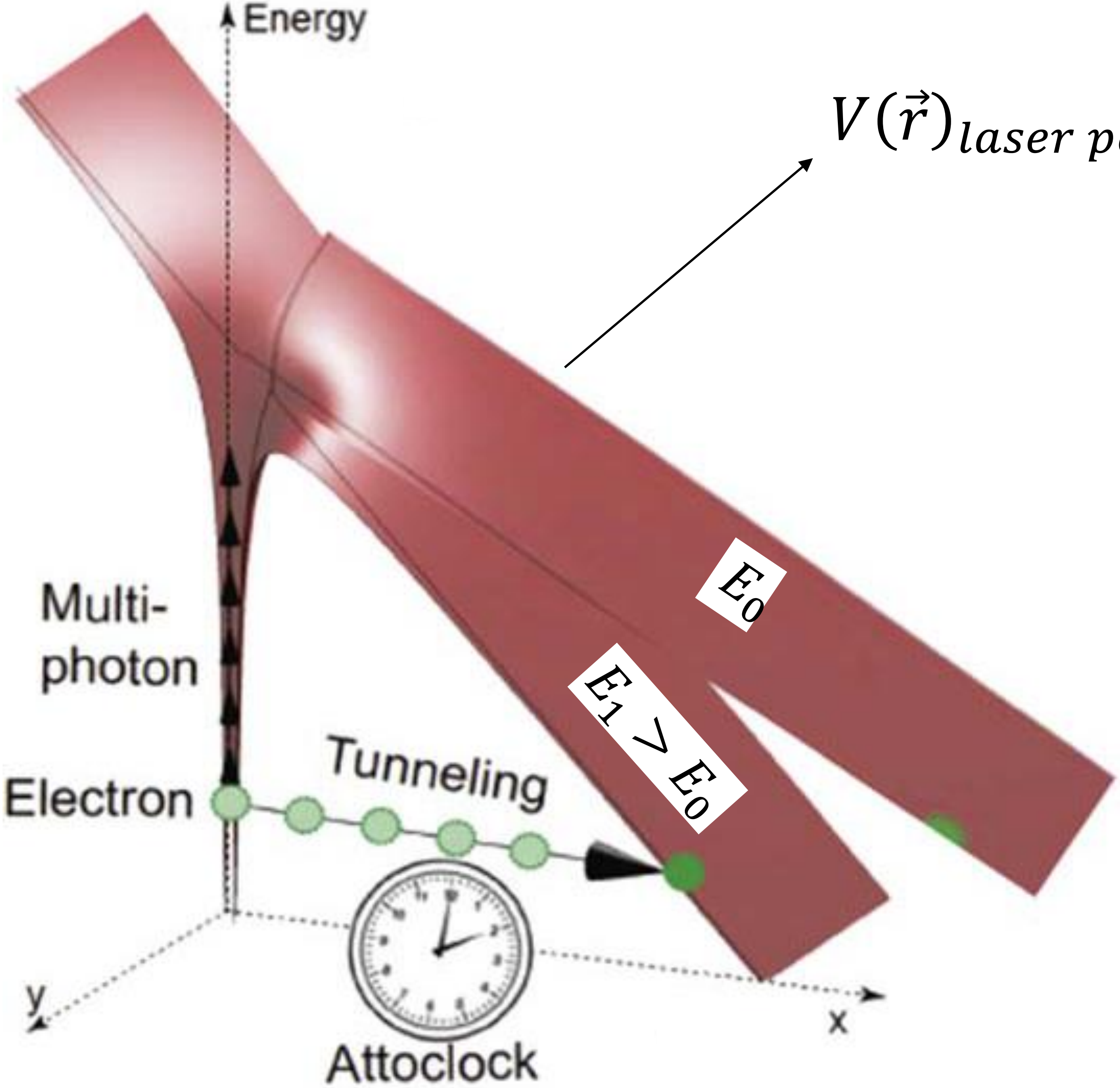


(C. Hernández-García *et al*., PRL, 2013)

$$\vec{E} = \frac{E_0}{\sqrt{1+\epsilon^2}} f(t)\ (\hat{x}\cos(\omega t + \phi_{CEP}) + \hat{y}\,\epsilon\sin(\omega t + \phi_{CEP}))$$



- $f(t) = \cos^4\left(\frac{\omega t}{N}\right)$ (for $-N\pi \le 2\omega t \le N\pi$)
- $E_0 = (0.8 - 1.5) \times 10^{14}\ \mathrm{W/cm^2}$
- $\epsilon = 0.8 - 1$
- $\mathrm{FWHM} = (1.6 - 6.0)\ \mathrm{fs}$
- $\omega^{-1} = (424 - 409)\ \mathrm{as}$



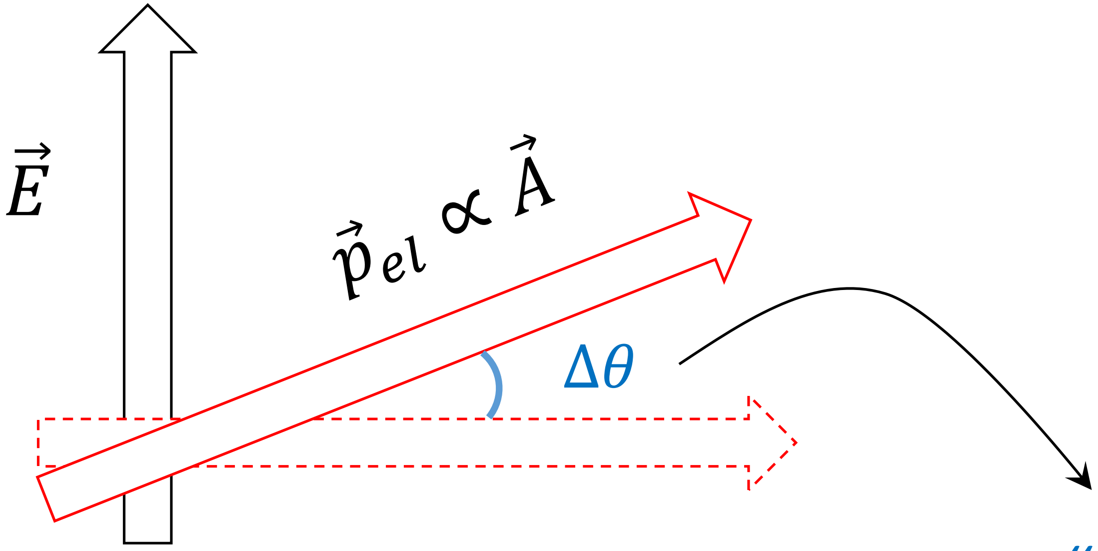
(A. Kheifets, JPhysB-Optical, 2020)

$$V(\vec{r})_{laser\ peak} = -\frac{1}{r} - \frac{xE_0}{\sqrt{1+\epsilon^2}}$$

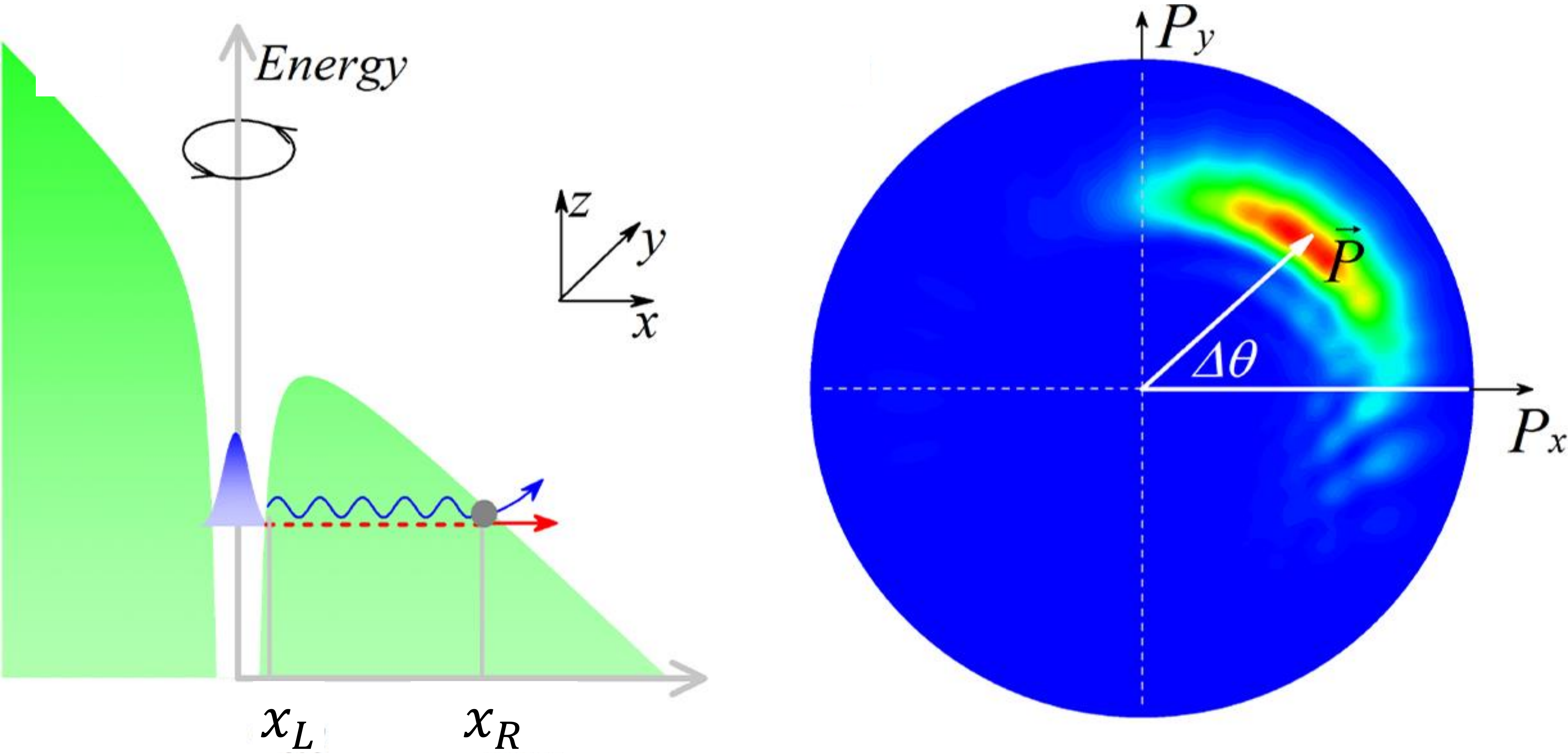
Strong laser fields (large E_0) result in thin potential barriers through which valence electron (with ionization energy $-I_p$) can tunnel within a finite time !

$$\vec{E} = \frac{E_0}{\sqrt{1+\epsilon^2}} \cos^4\left(\frac{\omega t}{N}\right) \left(\hat{x}\cos(\omega t + \delta) + \hat{y}\,\epsilon\sin(\omega t + \delta)\right)$$

$$\vec{p}_{el} \propto \vec{A} \approx \frac{E_0}{\omega\sqrt{1+\epsilon^2}} \cos^4\left(\frac{\omega t}{N}\right) \left(\hat{x}\sin(\omega t + \delta) - \hat{y}\,\epsilon\cos(\omega t + \delta)\right)$$

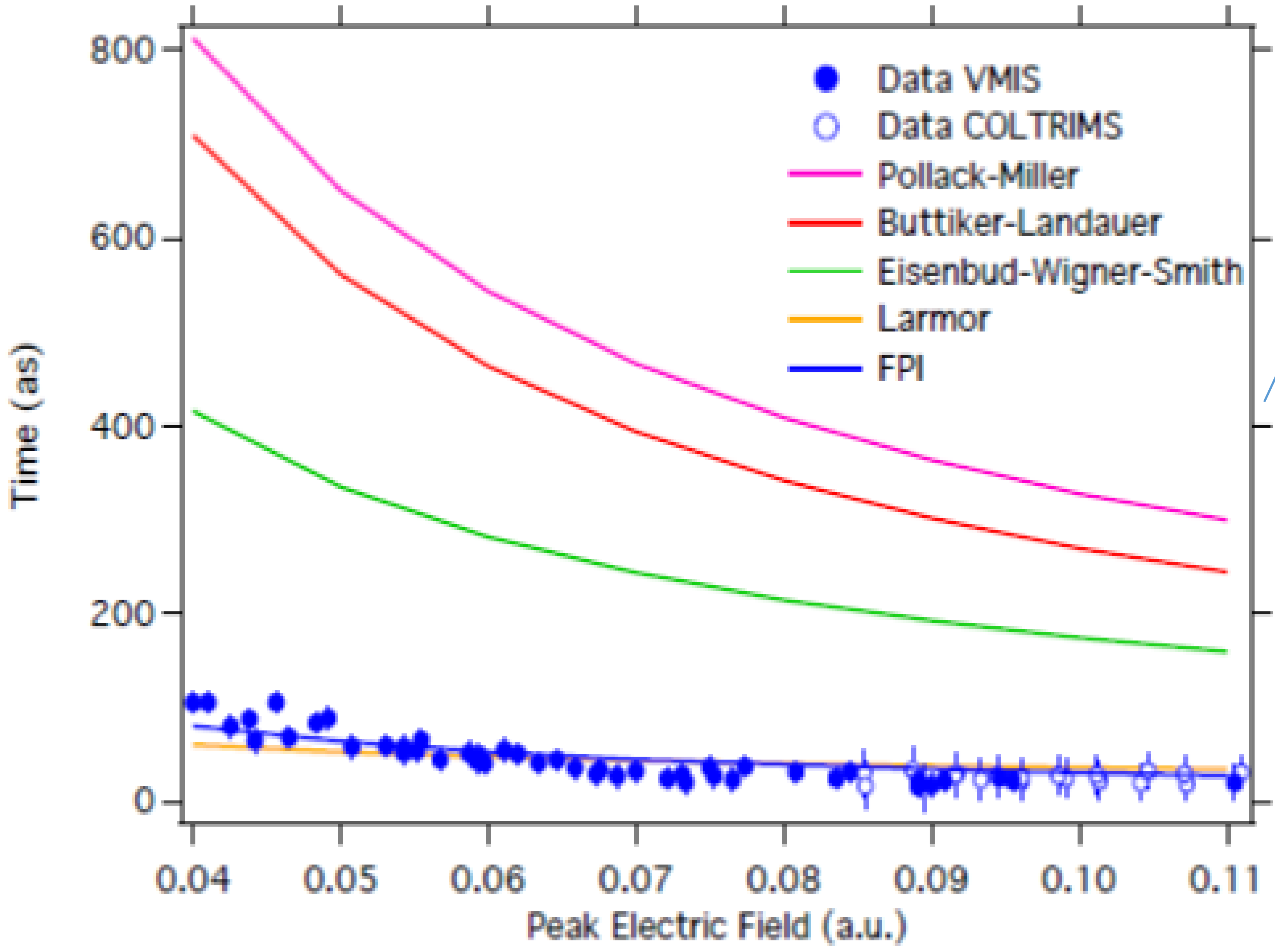


"delay" $= \Delta t = \frac{\Delta\theta}{\omega}$



(M. Yuan, Optics Express, 2019)

He ionization time:



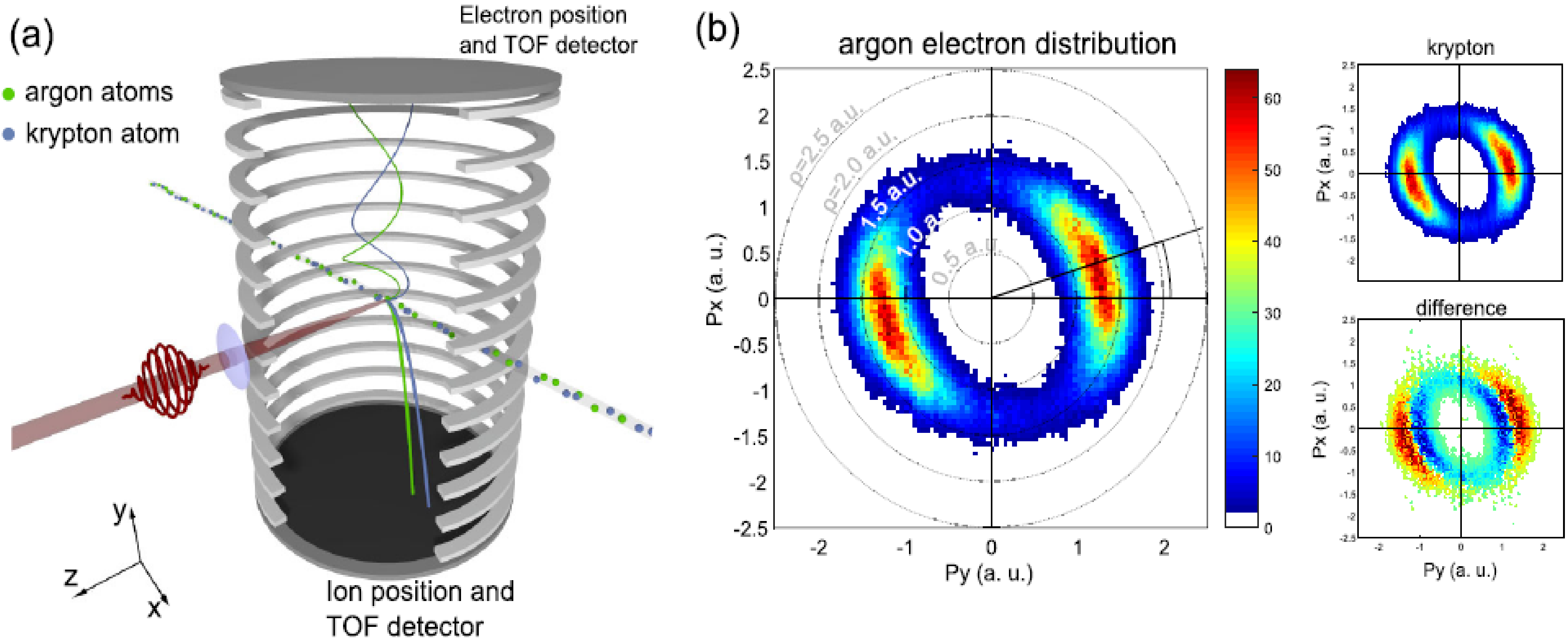
all known time models seem excluded

(**We need a working model**)

$(\Delta t)_{LM} = \mathrm{Re}\{(\Delta t)_{FPI}\}$ ve $|(\Delta t)_{FPI}|$ agrees with experiments because paths are coarse-grained according to the experimental resolution

(A. Landsman *et al.*, Optica, 2016)

Ar vs. Kr ionization times:



(E. Yakaboylu *et al.*, PRL, 2017)

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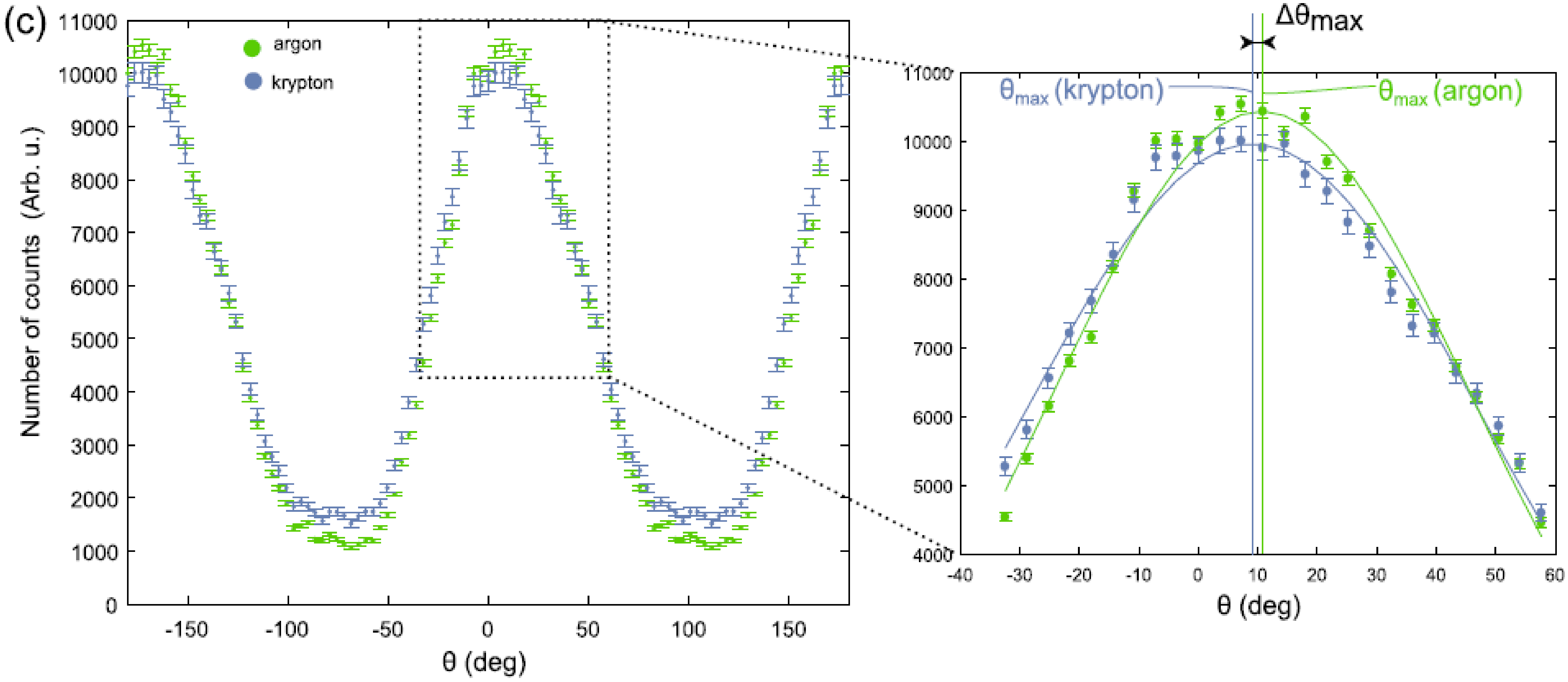
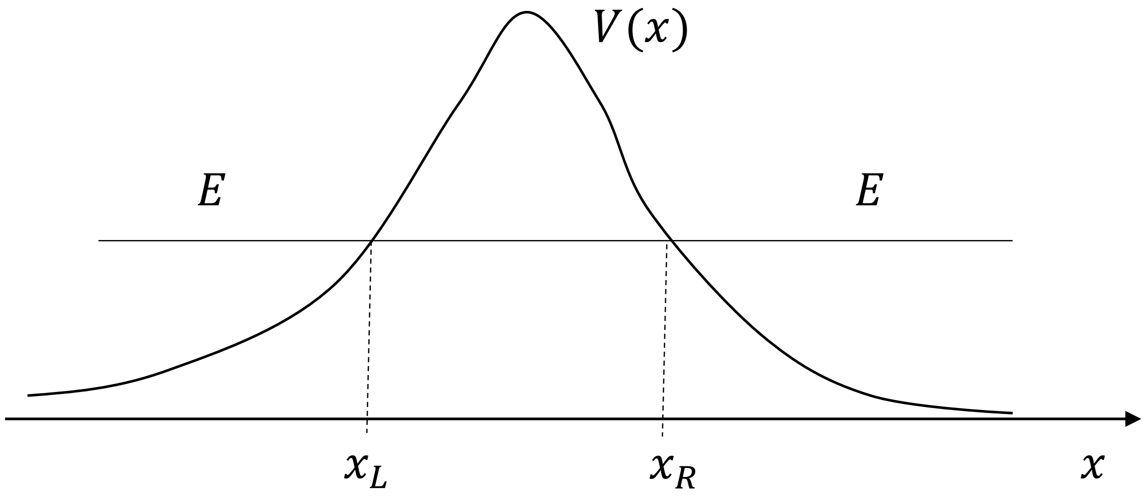


(E. Yakaboylu *et al.*, PRL, 2017)

Tunneling Time



Tunneling time = time it takesfortheparticletogetfromx_L to x_R

Experiment: Tunneling time exists and is finite!

Theory: There no method to compute tunneling time from first principles!

→ **We need a working model !**

a working «tunneling time» model

(D. D, T. Guner, Annals of Physics, **386** (2017) 291)

- Under the barrier ($x_L < x < x_R$) time is imaginary for the classical path:

$$\tau_c = \int_{x_L}^{x_R} \frac{m\,dx}{\sqrt{2m(V(x) - E)}}$$

- Imaginary time $(it) \equiv$ Temperature $\left(\frac{\hbar}{k_B T}\right)$

$$\sum_n e^{-\frac{i}{\hbar} E_n t} \Longleftrightarrow \sum_n e^{-\frac{E_n}{k_B T}}$$

- The number of microstates for a single particle can be defined as (phase space volume)/$\hbar$:

$$V_{\mathrm{PS}} = \int_{x_L}^{x_R} \sqrt{2m(V(x) - E)}\,dx \Longrightarrow \text{\# of microstates} = \Phi = \frac{V_{PS}}{\hbar}$$

- Entropy: $S = k_B\, p \log(1 - \log p)$

with $p = e^{-2\Phi} =$ «probability that particle goes directly to x_R from x_L»

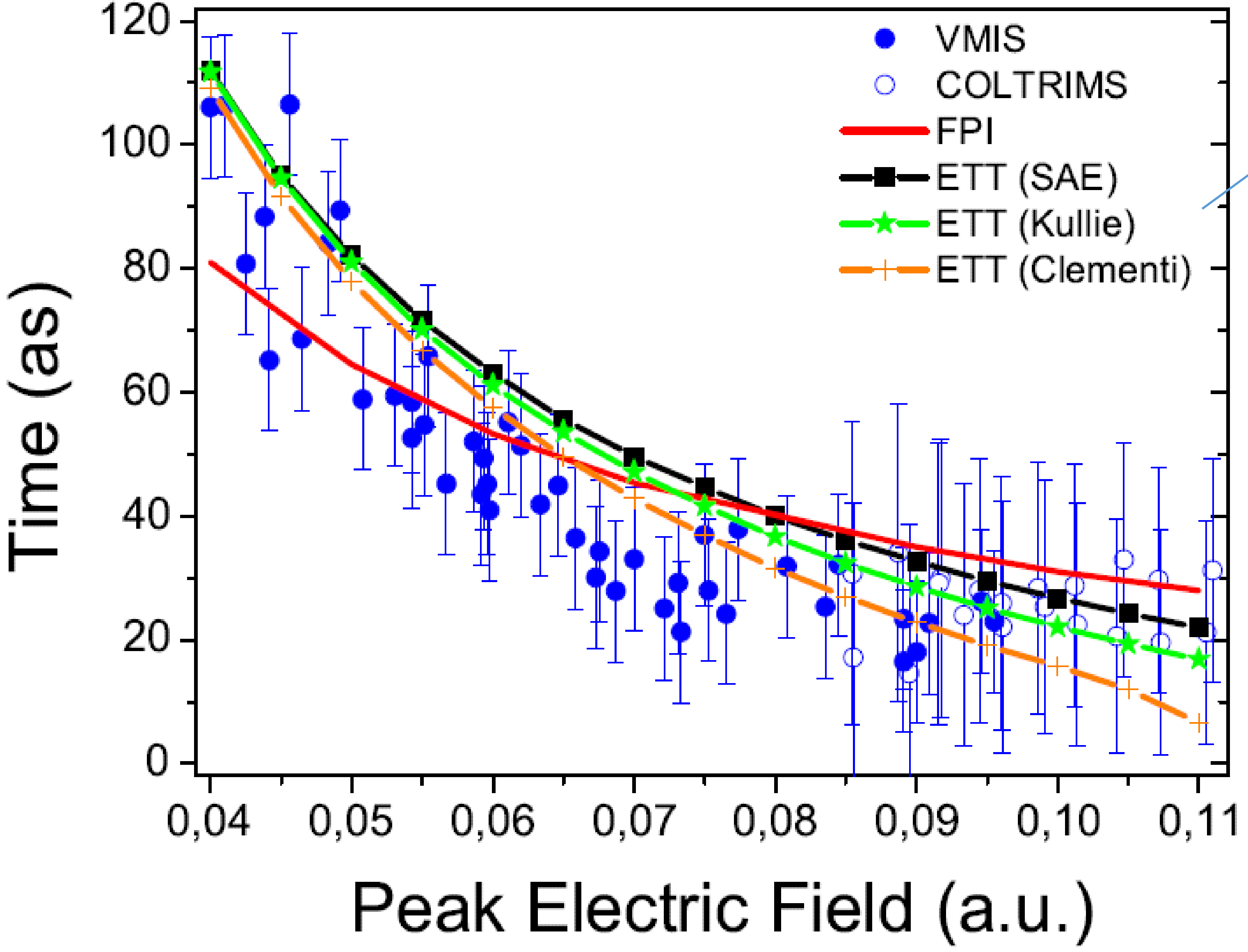
- Temperature: $$\frac{1}{T} = \frac{\partial S}{\partial E} = -\frac{2k_B\tau_c}{\hbar}\, e^{-2\,\Phi}\left(\frac{1}{1+2\,\Phi} + \log\frac{1}{1+2\Phi}\right)$$

- "Thermal Energy – Time" Uncertainty Product: $(k_B T) \times (\Delta\mathrm{t})_{\mathrm{ETT}} = \frac{\hbar}{2}$

- Entropic Tunneling Time (subluminal, physical, purely quantum):

$$(\Delta t)_{ETT} = -\frac{\tau_c}{8\pi}\left(1 + 2e^{-2\,\Phi} + e^{-4\Phi}\right)\left(\frac{1}{1+2\,\Phi} + \log\frac{1}{1+2\Phi}\right)$$

ETT vs He Ionization:

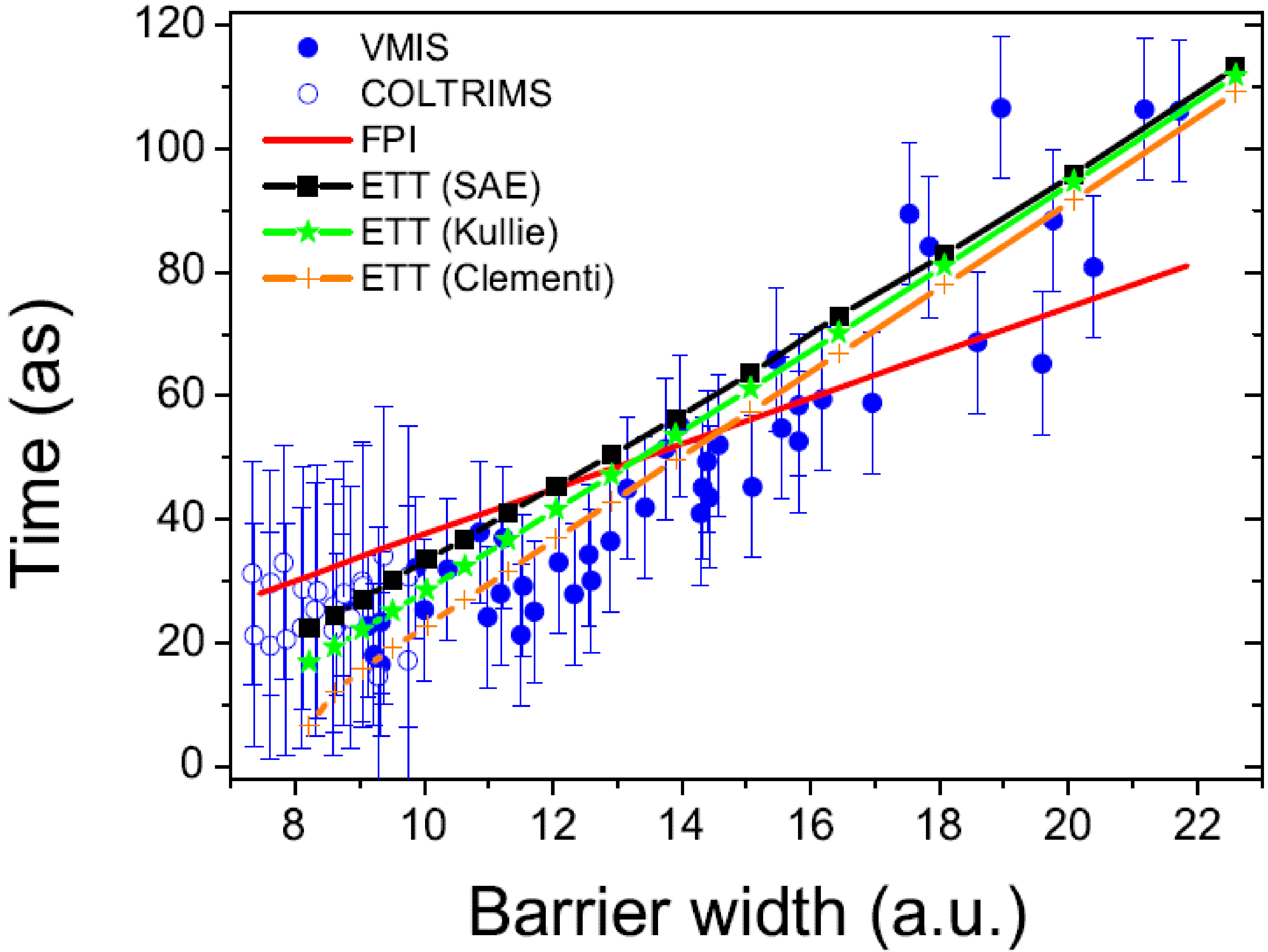


different $V(x)$ models

(D. D, T. Guner, Annals of Physics, 2017)

(A. Landsman *et al*, Optica 1 (2016) 343)

ETT vs He Ionization:



(D. D, T. Guner, Annals of Physics, 2017)

(A. Landsman *et a*l, Optica 1 (2016) 343)

… but we have two crucial problems:

- ETT holds only in the tunneling region. (It does not extend to outside.)
- ETT ignores interference effects. (It does not involve reflected waves.)

a working «time» model

(D. D, S. Paçal, arXiv: 2001.06071 [quant-ph])

- Tunneling is a stationary process (making sense of "time" is thus crux of the problem!)

- In stationary processes, time is trivialized as $\psi(x,t) = \phi(x)e^{-\frac{i}{\hbar}Et}$ so that

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} + V(x)\phi(x) = E\phi(x)$$

- For such processes one can introduce a "guiding equation"

"time-guiding equation": $\frac{d}{dx}t(x) = \frac{\rho(x)}{J(x)}$

which is nothing but the inverse of David Bohm's

"position-guiding equation": $\frac{d}{dt}x(t) = \frac{J(x)}{\rho(x)}$ (D. Bohm, Physical Review, 1953)

Time $t(x)$ and wavefunction $\phi(x)$ arise together (reminiscent of time in quantum gravity):

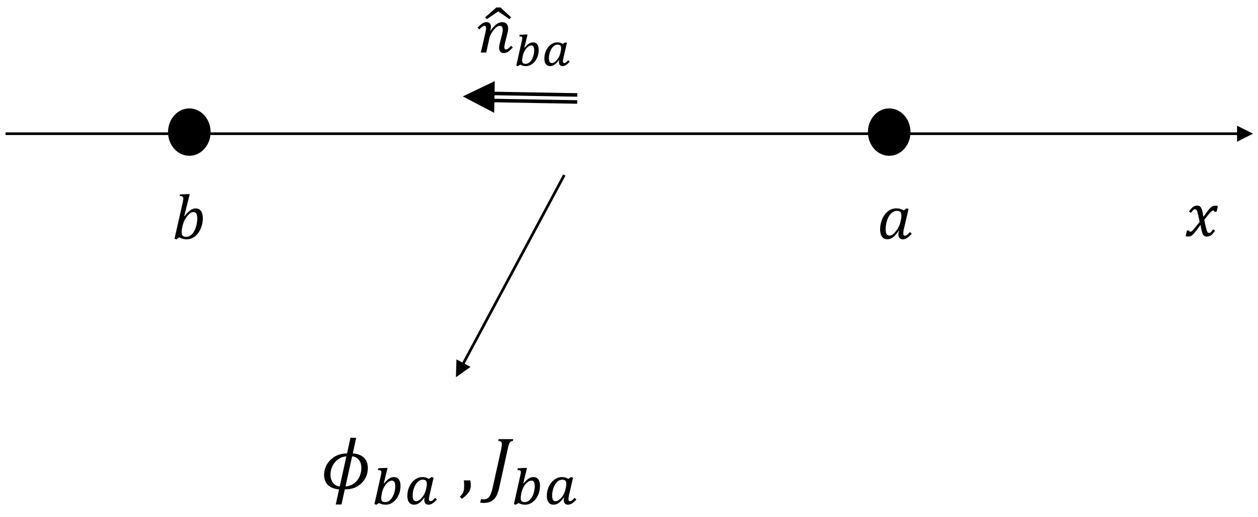
Quantum Travel Time: $(\Delta t)_{ba} = \int_a^b dx \, \frac{(\rho - \rho_{ab})}{J_{ba}}$

$$\rho = |\phi_{ba} + \phi_{ab}|^2$$

$$\rho_{ab} = |\phi_{ab}|^2$$

$$\phi_{ba} \sim e^{-\frac{i}{\hbar}\hat{n}_{ba} p x}$$

$$J_{ba} = \frac{\hbar}{m} \, \Im\left(\phi^{\star}_{ba}(x) \, \frac{\partial \phi_{ba}(x)}{\partial x}\right)$$

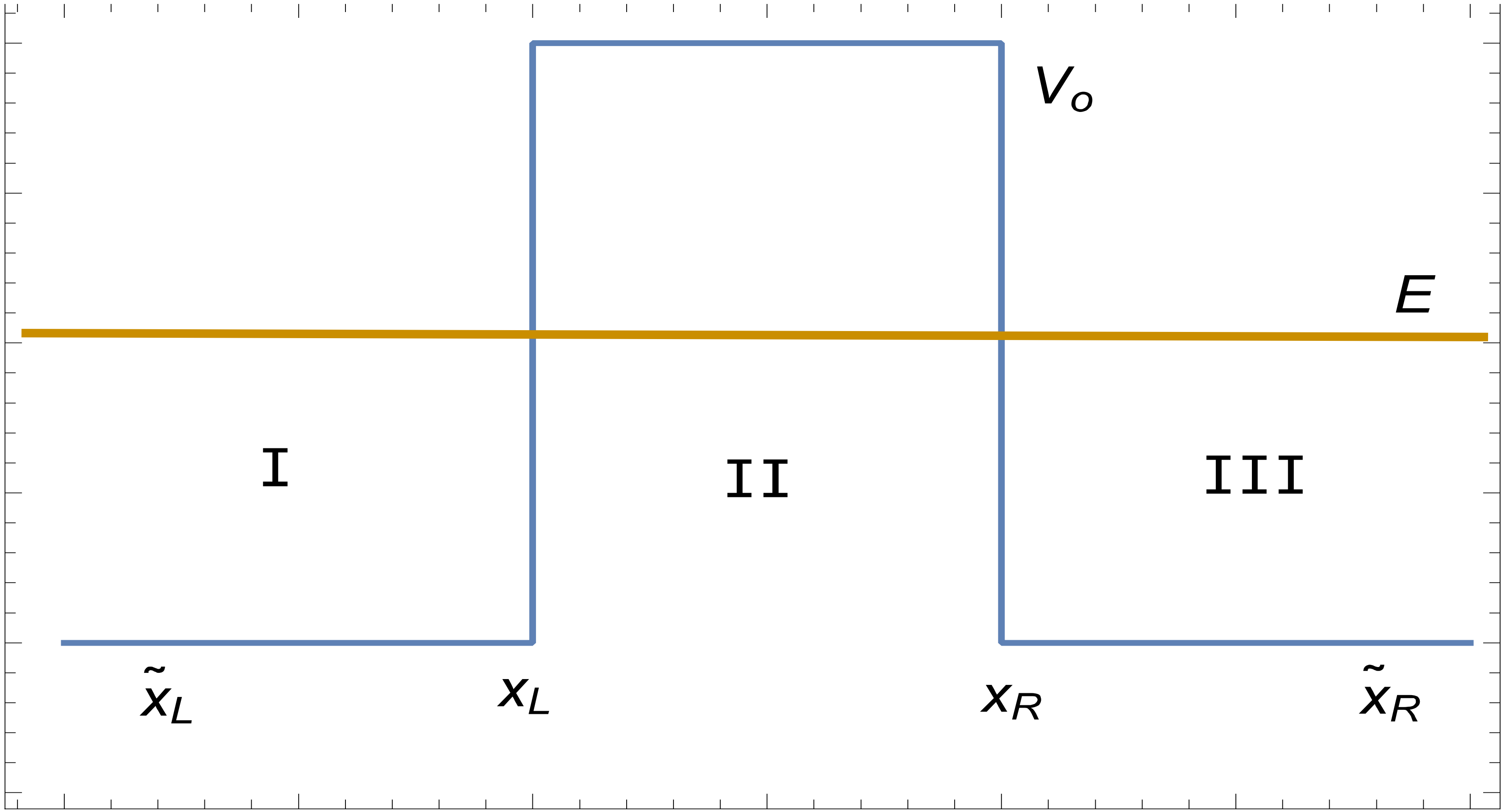


QTT in Rectangular Potential:

$$\phi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & \text{(region I)} \\ Ce^{-\kappa x} + De^{\kappa x} & \text{(region II)} \\ e^{ikx} & \text{(region III)} \end{cases}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

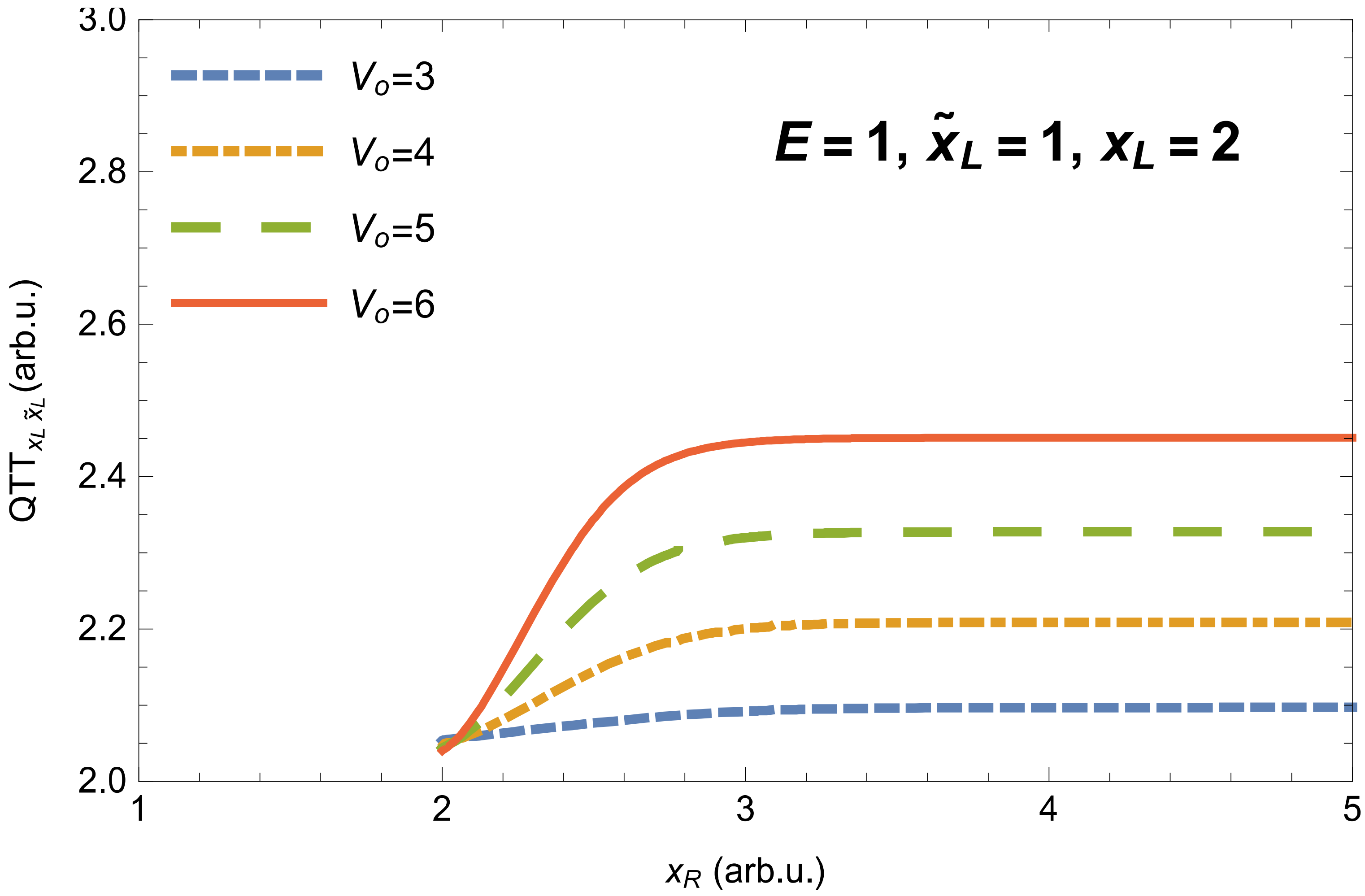


QTT in Region I: Reflection matters!

$$R = 1 \Rightarrow (QTT)_{x_L \tilde{x}_L} = \frac{2m(x_L - \tilde{x}_L)}{\sqrt{2mE}} - \frac{\hbar}{4E}\Big(\tan\big(\theta(x_L)\big) - \tan\big(\theta(\tilde{x}_L)\big)\Big)$$

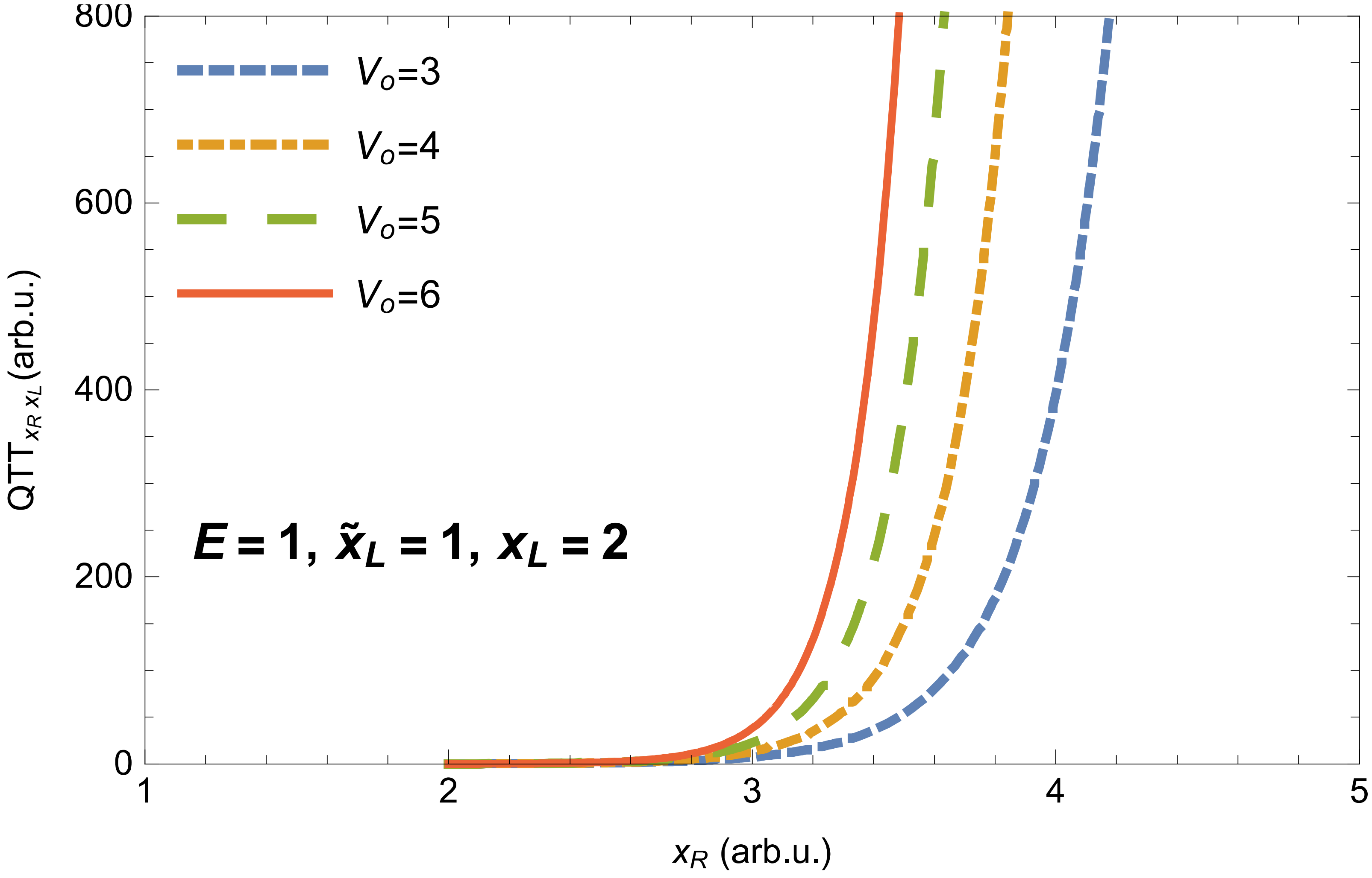
$$R = 0 \Rightarrow (QTT)_{x_L \tilde{x}_L} = \frac{m(x_L - \tilde{x}_L)}{\sqrt{2mE}}$$

$$\theta(x) = kx + \varphi_{AB}$$

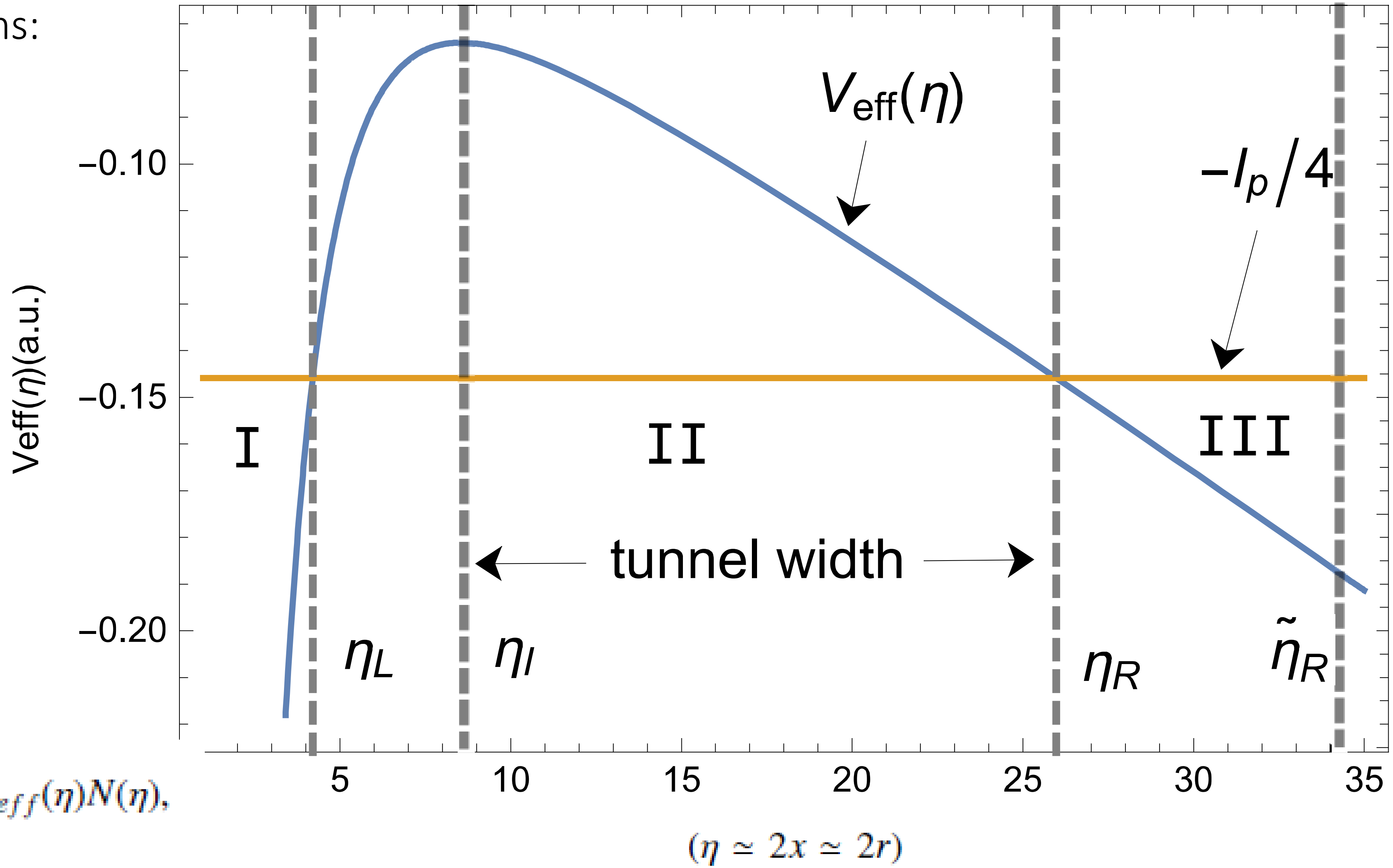


QTT in Region II: Reflection matter!

$$(QTT)_{x_R\, x_L} = \frac{m(x_R - x_L)}{\sqrt{2mE}} \frac{V_0 - 2E}{V_0 - E} + \frac{\hbar V_0}{8\sqrt{E(V_0 - E)^3}} \sinh\big(\kappa(x_R - x_L)\big)\, e^{\kappa(x_R - x_L)}$$



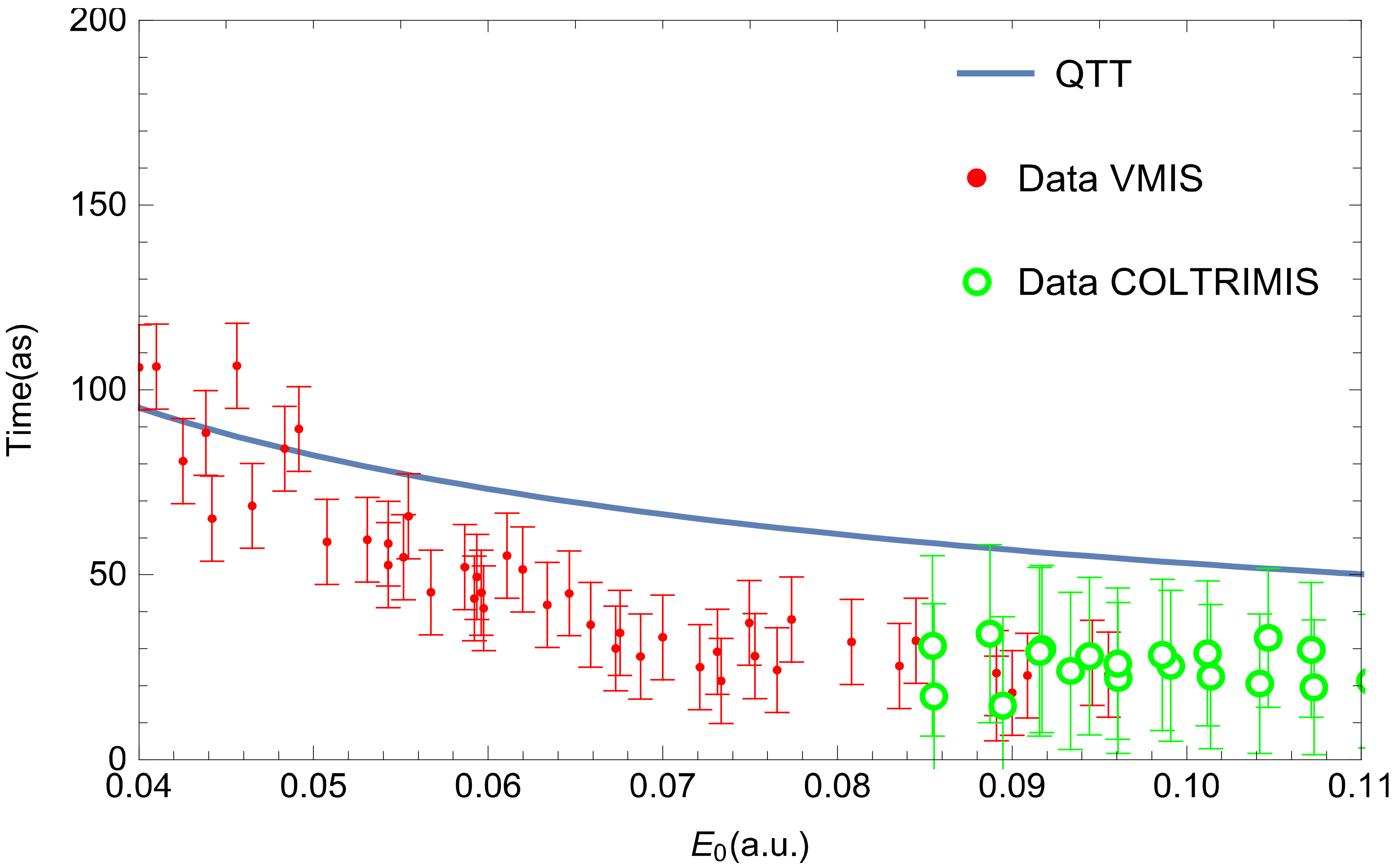
QTT in Ionization of Atoms:



$$-\frac{I_p}{4}N(\eta) = -\frac{1}{2}\frac{\partial^2 N(\eta)}{\partial \eta^2} + V_{eff}(\eta)N(\eta),$$

$$V_{eff}(\eta) = -\frac{1}{8\eta^2} - \frac{1}{2\eta} - \frac{\Phi(\eta/2)}{2\eta} + \alpha_I\frac{E_0}{\eta^2} - \frac{E_0\eta}{8} + \frac{\sqrt{2I_p}}{4\eta}$$

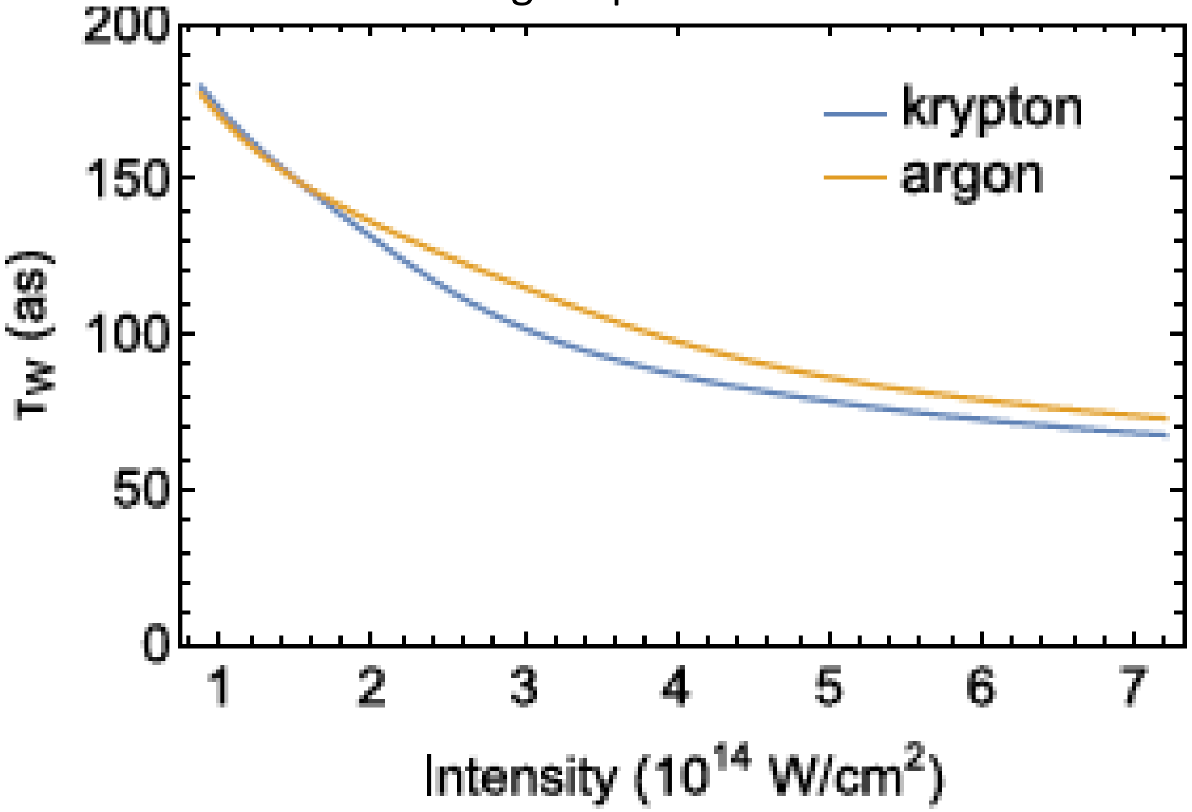
QTT in He Ionization:



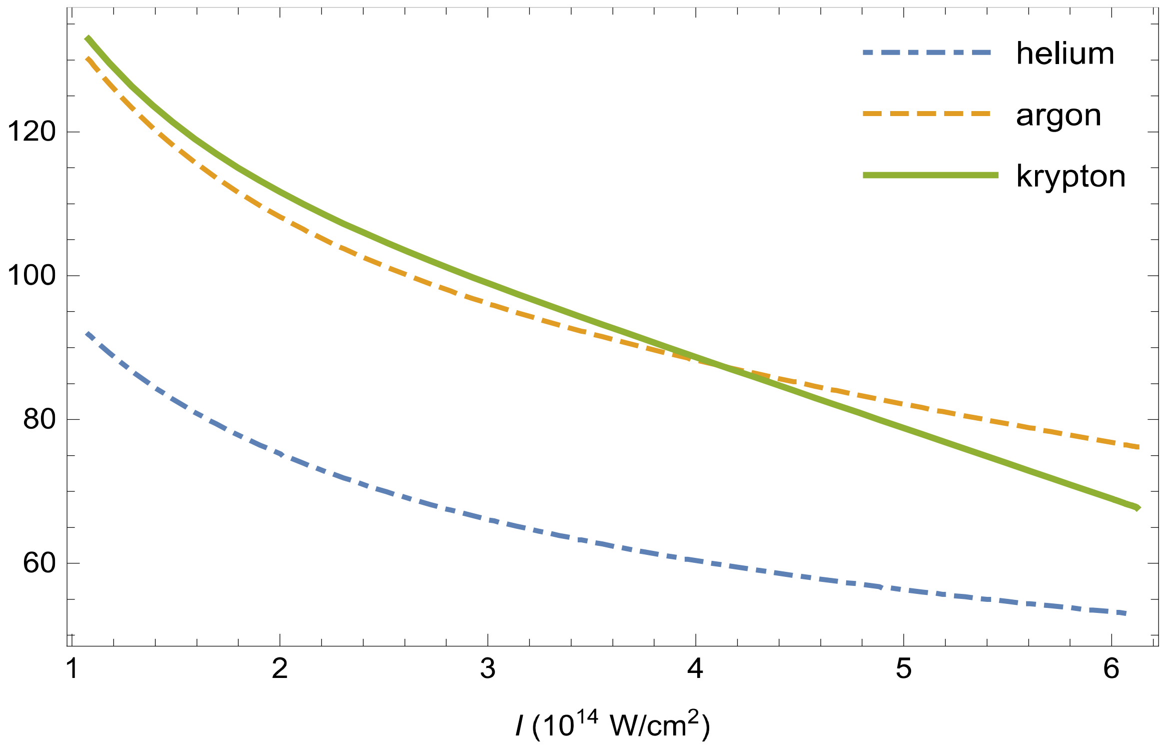
(A. Landsman *et al*, Optica 1 (2016) 343)

QTT in Ionization of Noble Gases:

Wigner phase time



QTT



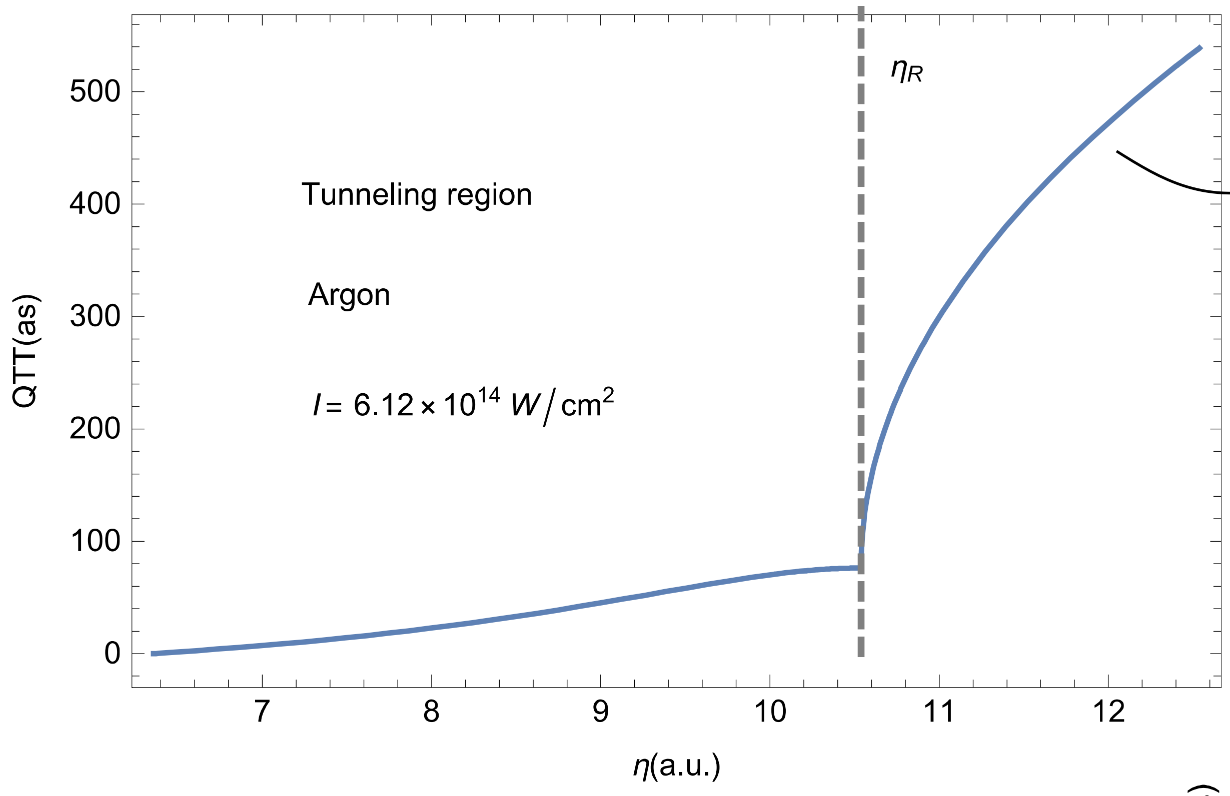
Atom	η_L	η_I	η_R
Intensity 1.08×10^{14} W/cm^2			
He	1.5358	6.2307	42.0210
Ar	4.2036	8.5492	25.9824
Kr	5.0274	9.1864	22.7422
Intensity 6.12×10^{14} W/cm^2			
He	1.5477	4.3271	17.2830
Ar	4.2493	6.3563	10.5383
Kr	5.2817	6.8643	9.2879

Wigner time = $\frac{d(\text{phase})}{dE} + m\frac{\text{width}}{\sqrt{2mE}}$

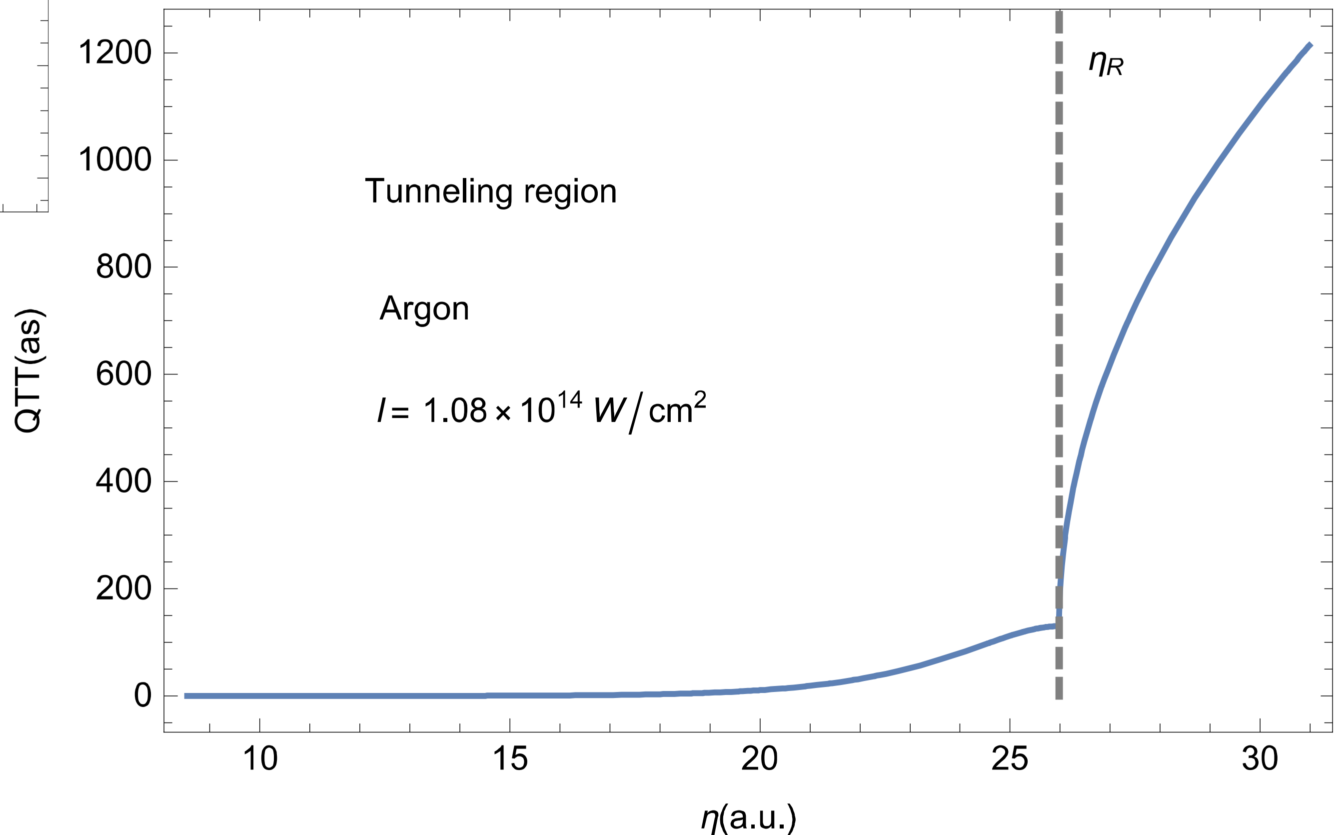
→ by hand

(E. Yakaboylu *et al.*, PRL, 2017)

QTT remains valid inside and outside the barrier:



approximated with classical motion in experiments



- Advances in ultrafast science (lasers) have been enabling us to test “time models” in atomic ionization experiments.

- As a “tunneling time” model Entropic Tunneling Time works fine.

- As a “time” model, however, Quantum Travel Time works fine everywhere, with reasonable agreement with experimental data.

- Quantum Travel Time takes into account “intereference” effects – an important factor in deciding where the tunnel exit is in experiments. (U. Sainadh *et al.*, Nature, 2019)

- Further advances will take us into regimes where even the “wavefunction collapse” might be observable!

Thank You!