Quantum Tunneling Time:
New Approaches and Potential Applications

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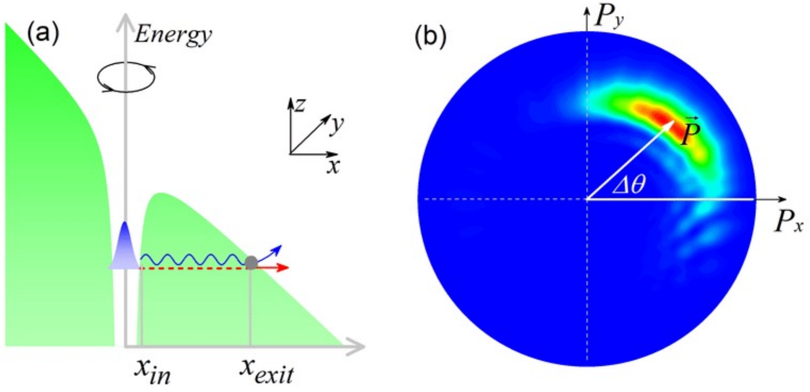
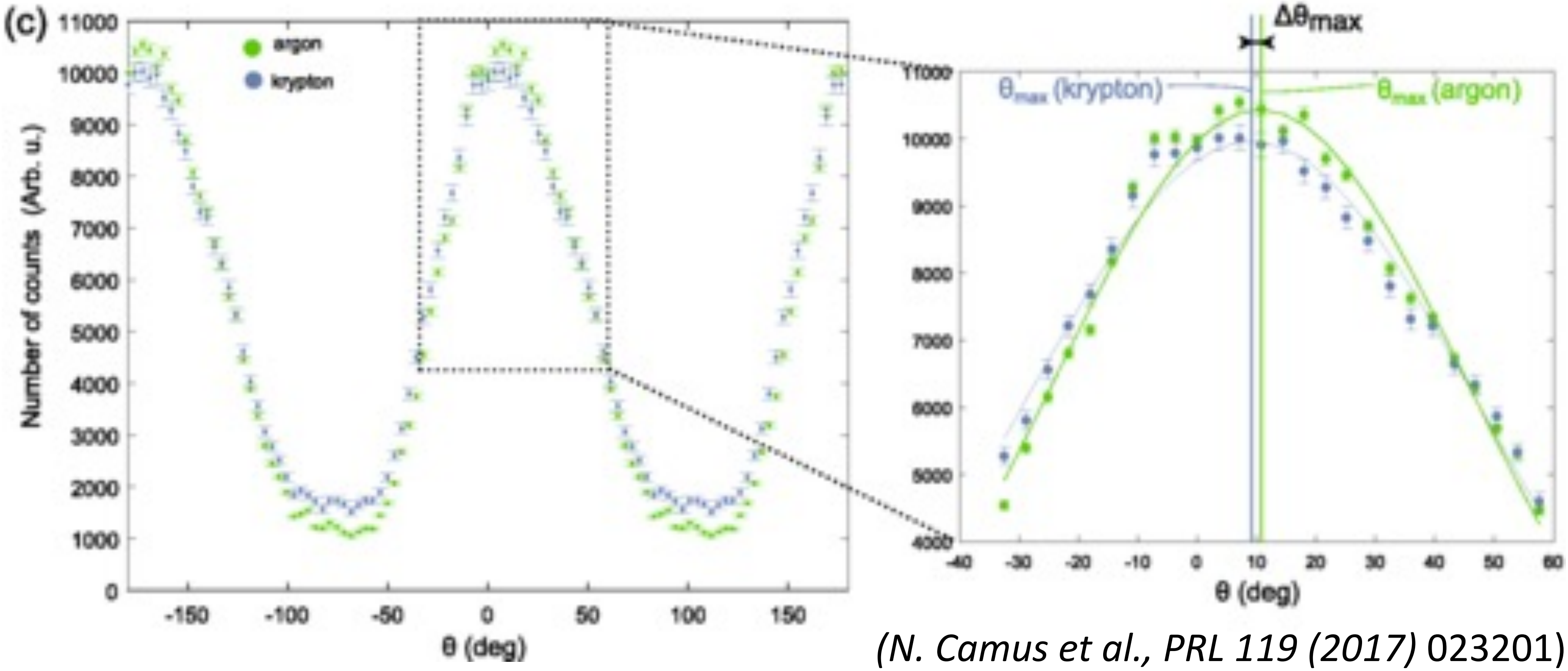
- Tunneling is a textbook topic, tunneling time is not.
  - In quantum theory time is not an operator so time, more specifically, tunneling time needs be modeled from the scratch.
    - Experiments give a finite tunneling time for quantum particles (atoms).
      - Strong-field tunnel ionization experiments have sidelined most of the past tunneling time models.
    - Statistical and Bohmian approaches to tunneling time happen to yield new testable time models.
  - These time models give realistic predictions for tunnel ionization of atoms, DNA point mutation, and free-fall time.
- Tunneling time, in general, can have important implications for physical, biological, chemical and technological processes. Annealing quantum computer is one clear example.

TUNNELING TIME IS NONZERO



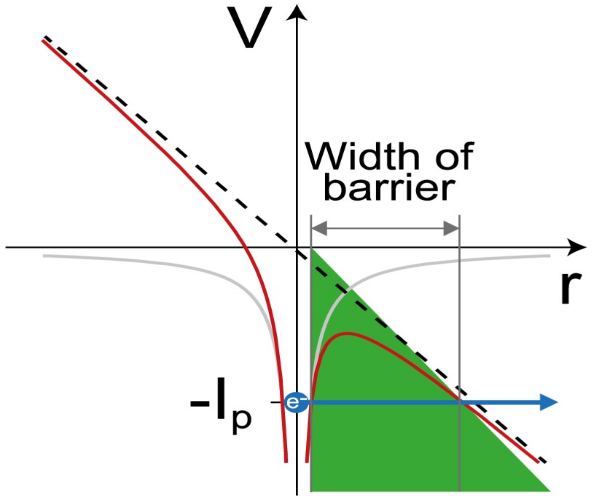
(M. Yuan, Optics Express 27 (2019) 6502)

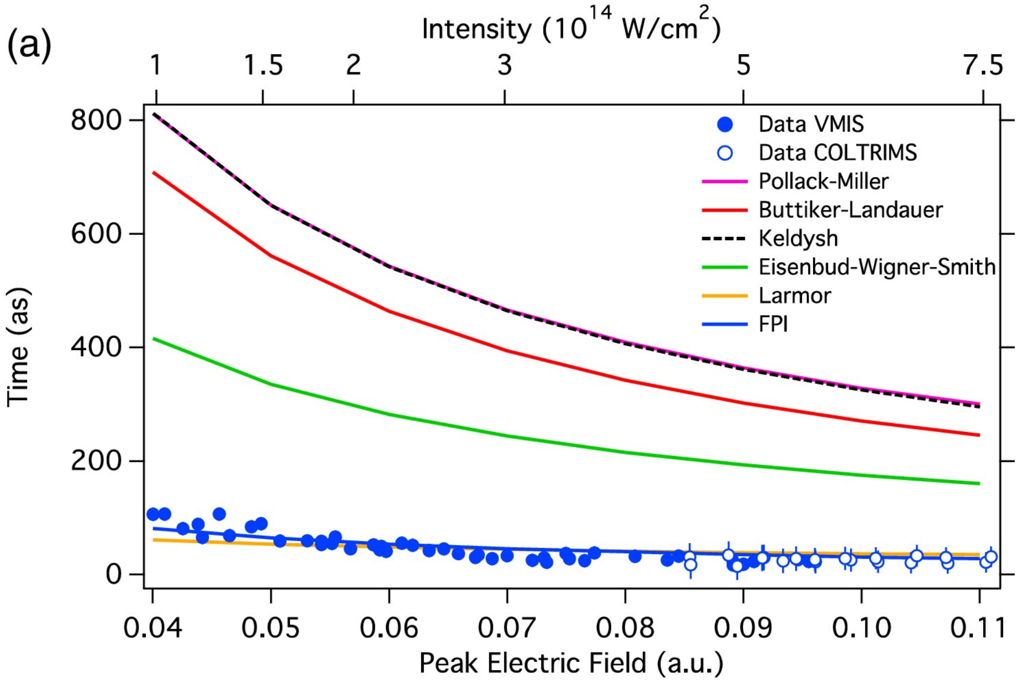
Ar vs Kr tunnel ionization times:



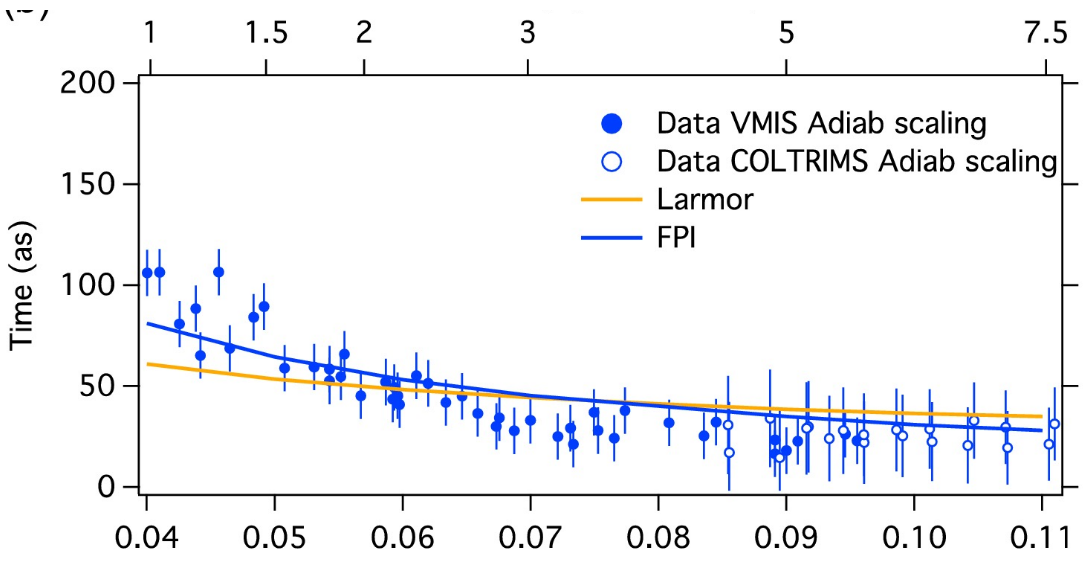
(N. Camus et al., PRL 119 (2017) 023201)

KNOWN TIME MODELS DISAGREE WITH EXPERIMENT

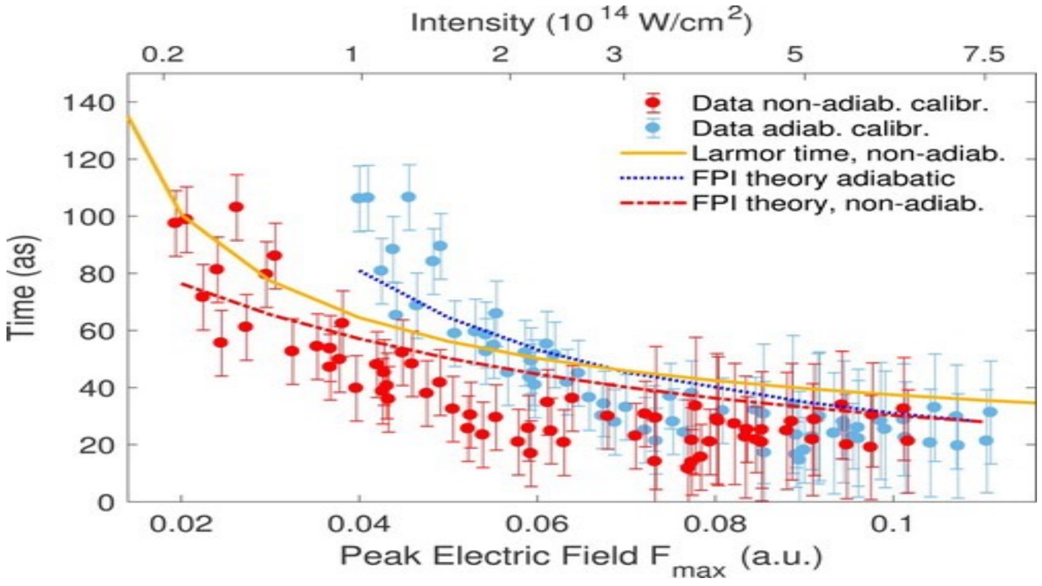




(A. Landsman et al., Optica 1 (2016) 343)



(A. Landsman et al., Optica 1 (2016) 343)

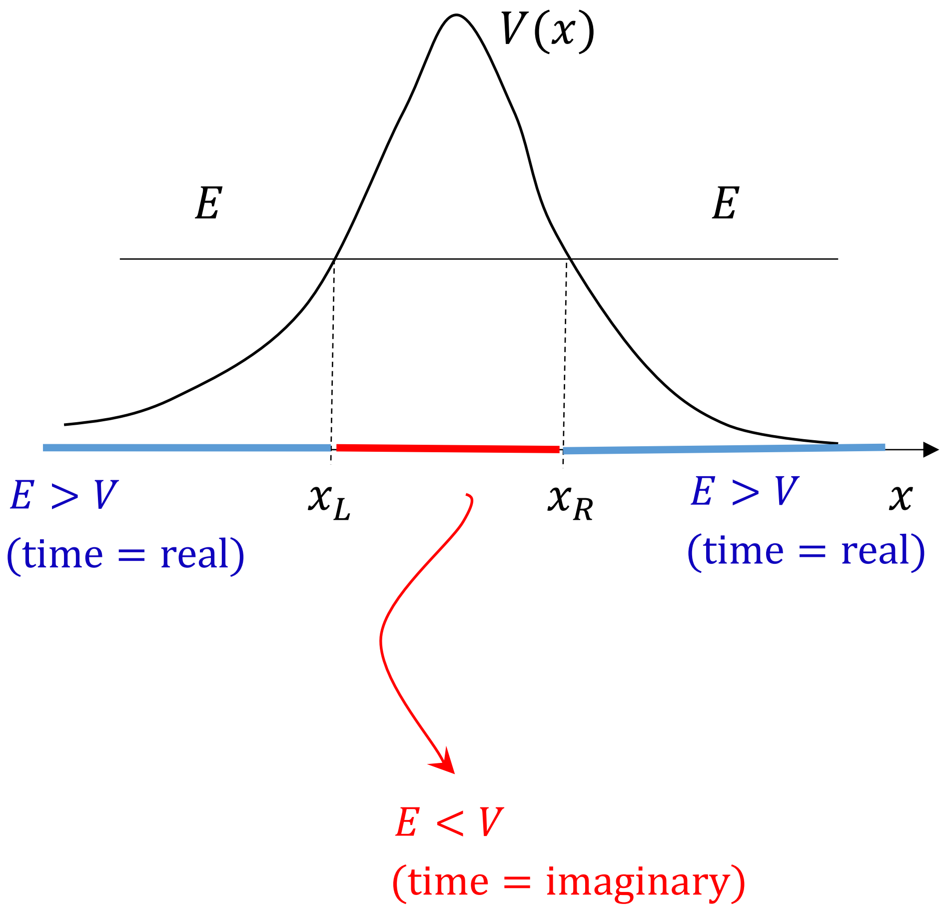


(C. Hoffmann et al., J. Mod. Optics 66 (2019) 1052)

ENTROPIC TUNNELING TIME

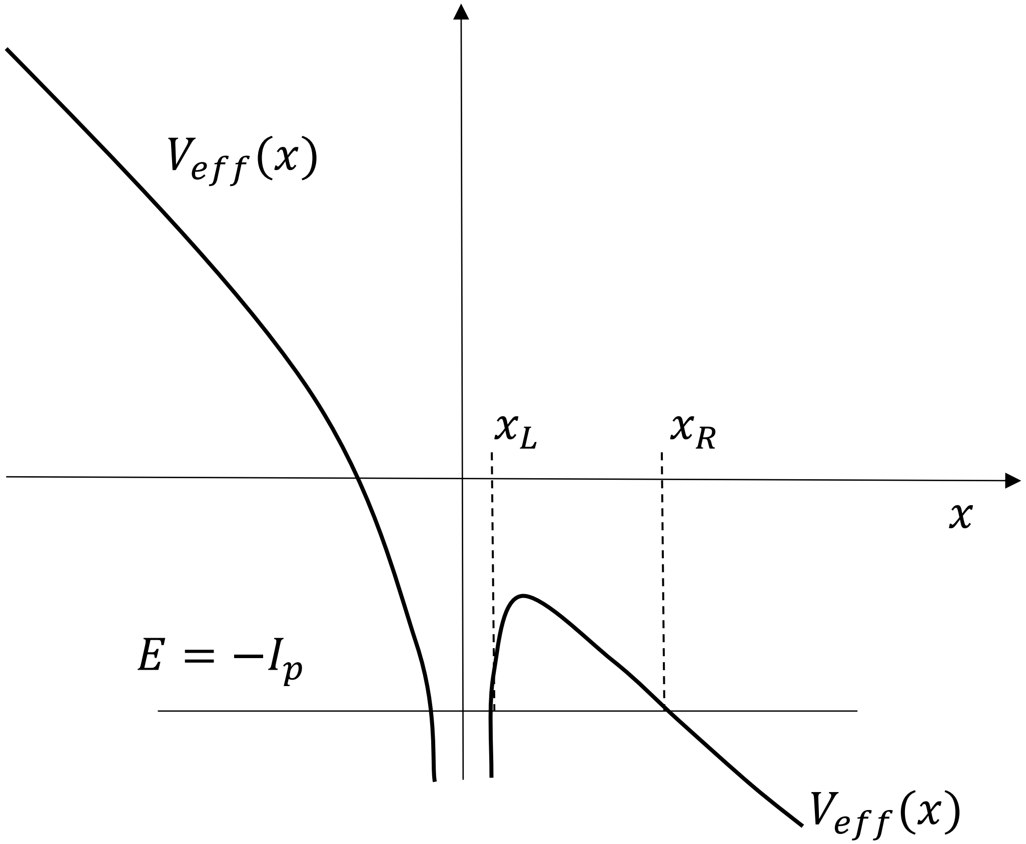
- Time is imaginary in tunneling region (classical dynamics).
- Imaginary time is equivalent to inverse temperature (QM ≡ eq. Stat Mech.)
- Energy in quantum fluctuations (no real propagation) should pertain to (useless) entropic energy.
- Uncertainty product with thermal sets the time scale of the tunneling transition.
- Entropic tunneling time:

$$(\Delta t)_{ETT} \equiv \frac{k_B \tau_c}{S}$$



- $\tau_c = \int_{x_L}^{x_R} \frac{m\, dx}{\sqrt{2m\,(V(x)-E}}$
- $S \equiv -k_B P \log P$
- $P \equiv \int_{x_L}^{x_R} \psi^\star(x)\psi(x)dx$

ENTROPIC TUNNELING TIME: He IONIZATION



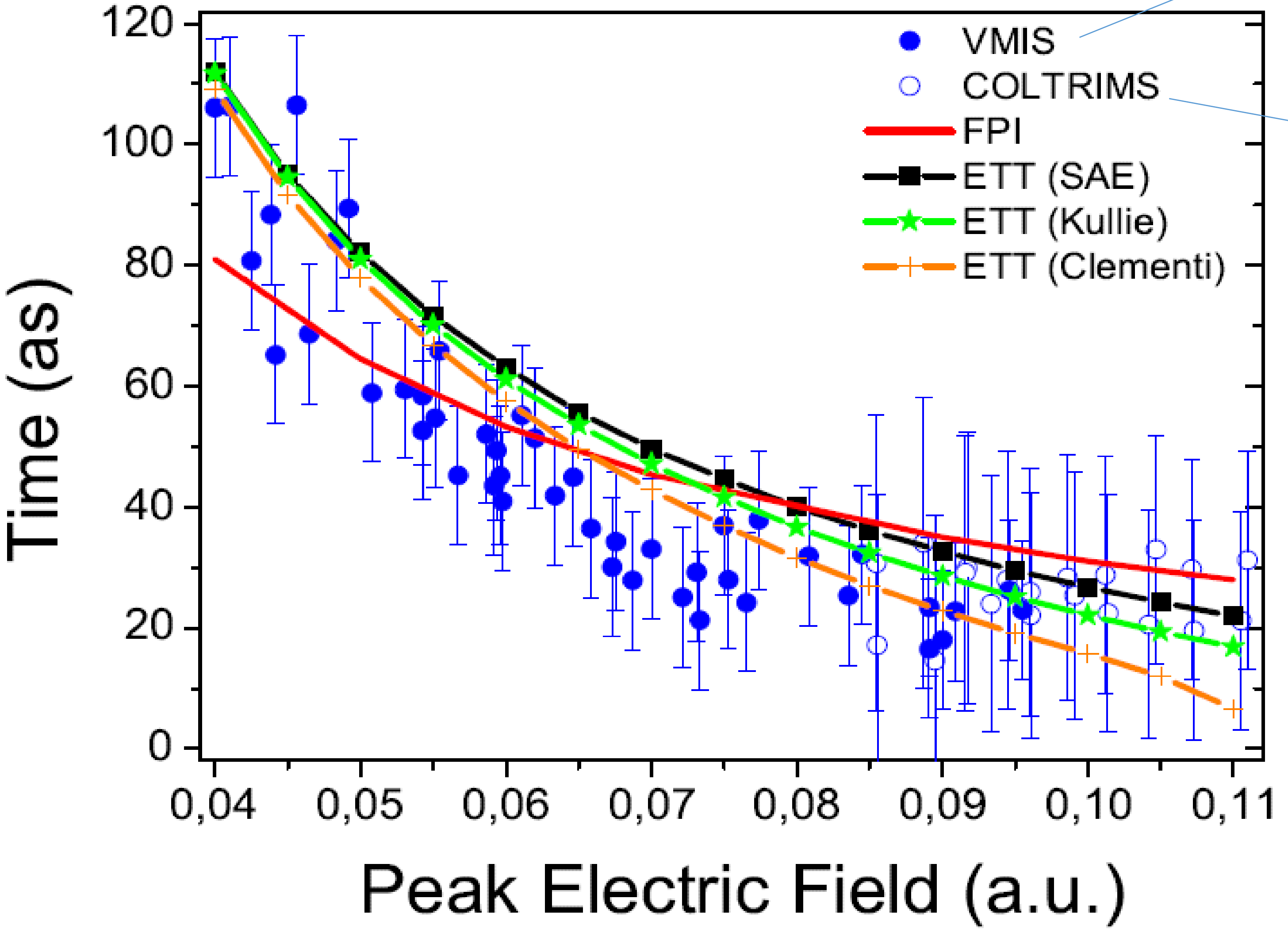
Effective potential at a radius x from the He^+ ion:

$$V_{eff}(x) = -\frac{Z_{eff}(x)}{x} - \mathcal{E}\, x$$

- $Z_{eff}(\mathrm{SAE}) = 1 + 1.231\, e^{-0.662x} - 1.325\, e^{-1.236x} - 0.231\, e^{-0.48x}$
- $Z_{eff}(\mathrm{Kullie}) = 1.375$
- $Z_{eff}(\mathrm{Clementi}) = 1.6875$

(SAE: X. Tong et al., J. Phys. B 38 (2005) 2593)
(Kullie: O. Kullie, J. Phys. B 49 (2016) 095601)
(Clementi: E. Clementi et al., J. Chem. Phys. 49 (1963) 2686)

ENTROPIC TUNNELING TIME: He IONIZATION



velocity map spectrometer (VMIS)

momentum spectrometer (COLTRIMS)

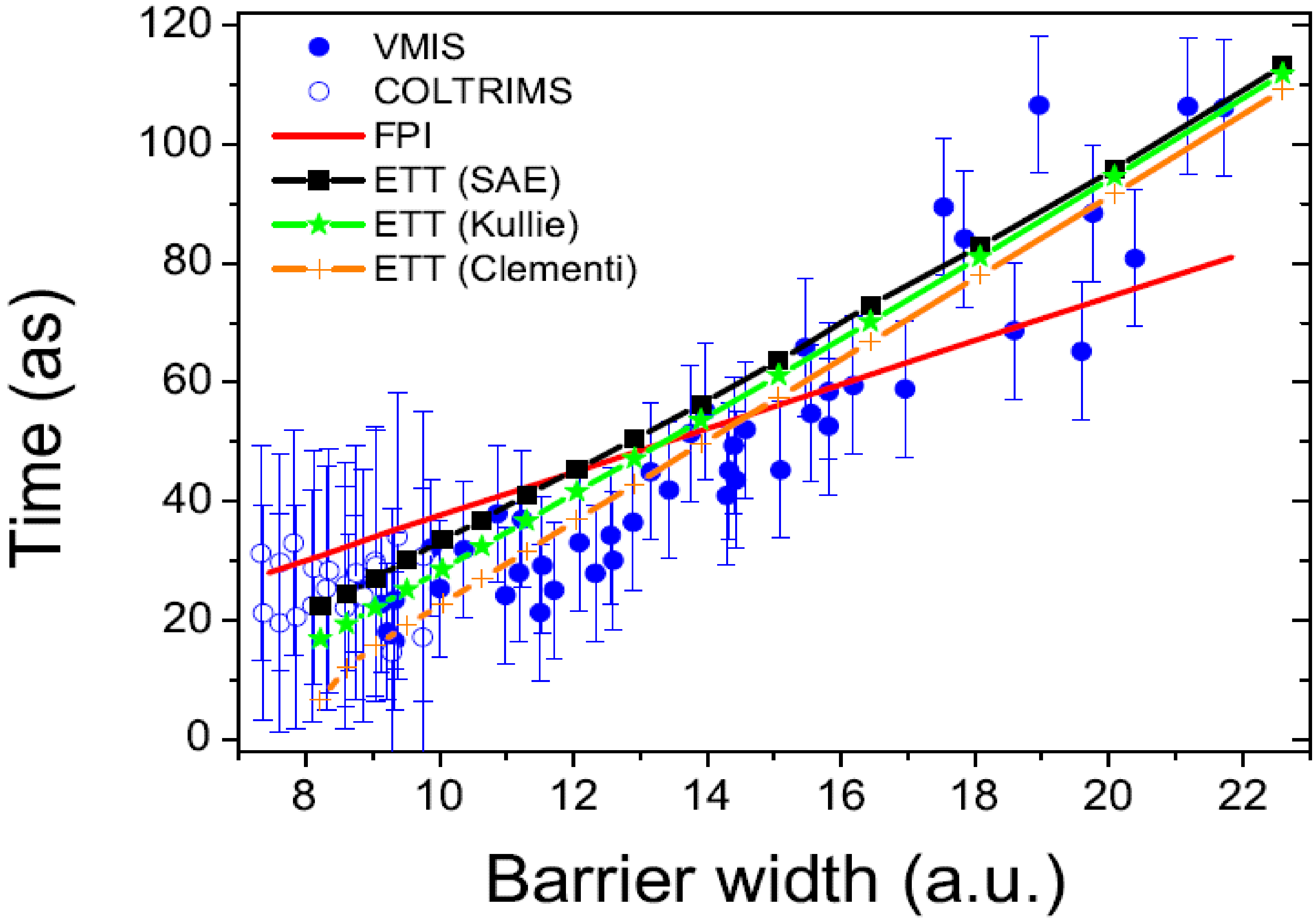
Experiment:

(A. Landsman et al, Optica 1 (2016) 343)

Model:

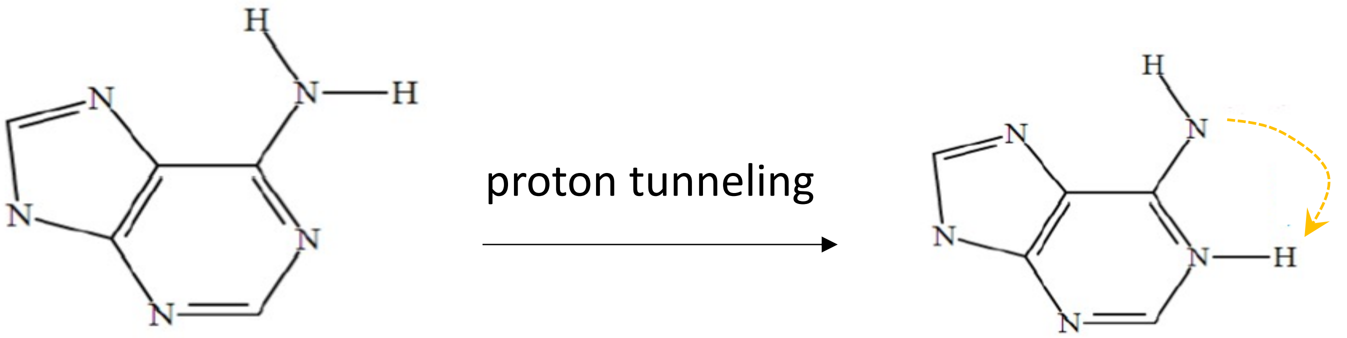
(DD & T. Güner, Annals of Physics 386 (2017) 291)

ENTROPIC TUNNELING TIME: He IONIZATION

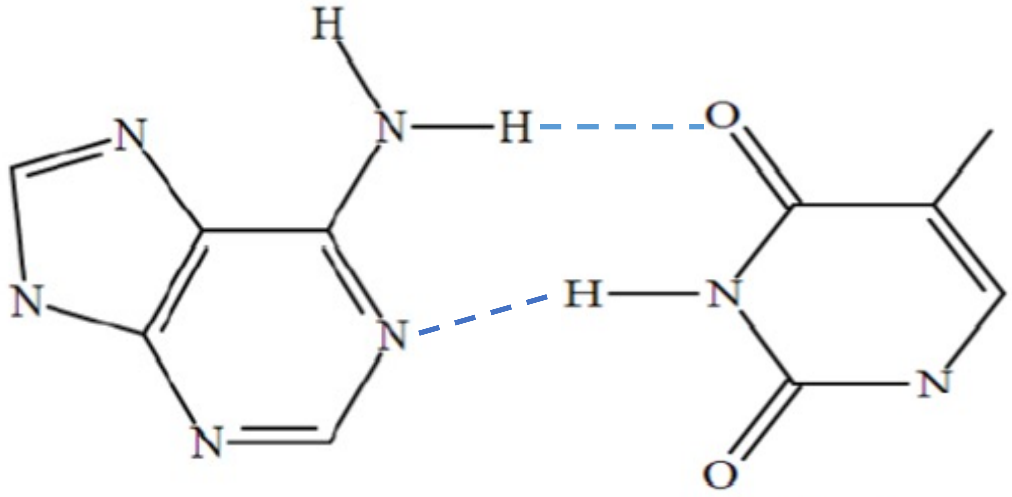


Experiment: *(A. Landsman et al., Optica 1 (2016) 343)* Model: *(DD & T. Güner, Annals of Physics 386 (2017) 291)*

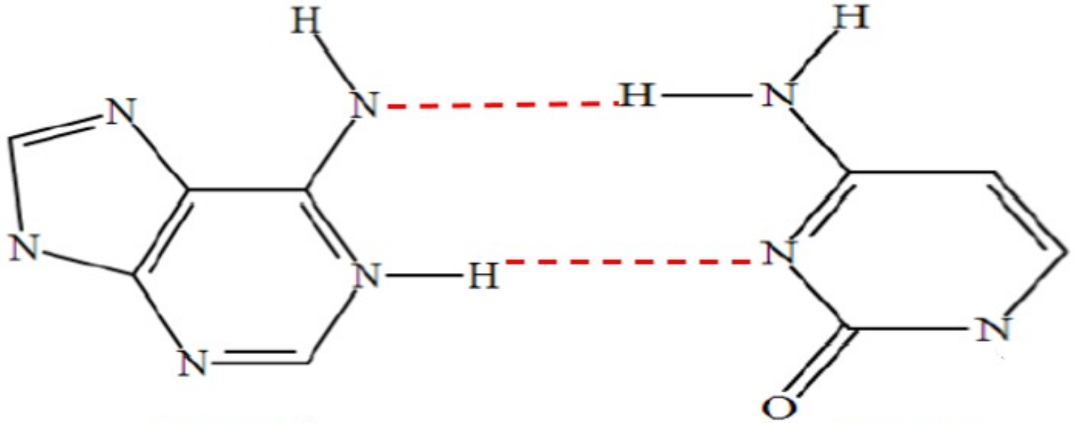
ENTROPIC TUNNELING TIME: DNA MUTATION



Adenine　　　　Adenine*



Adenine　　　　Thymine



Adenine*　　　　Cytosine

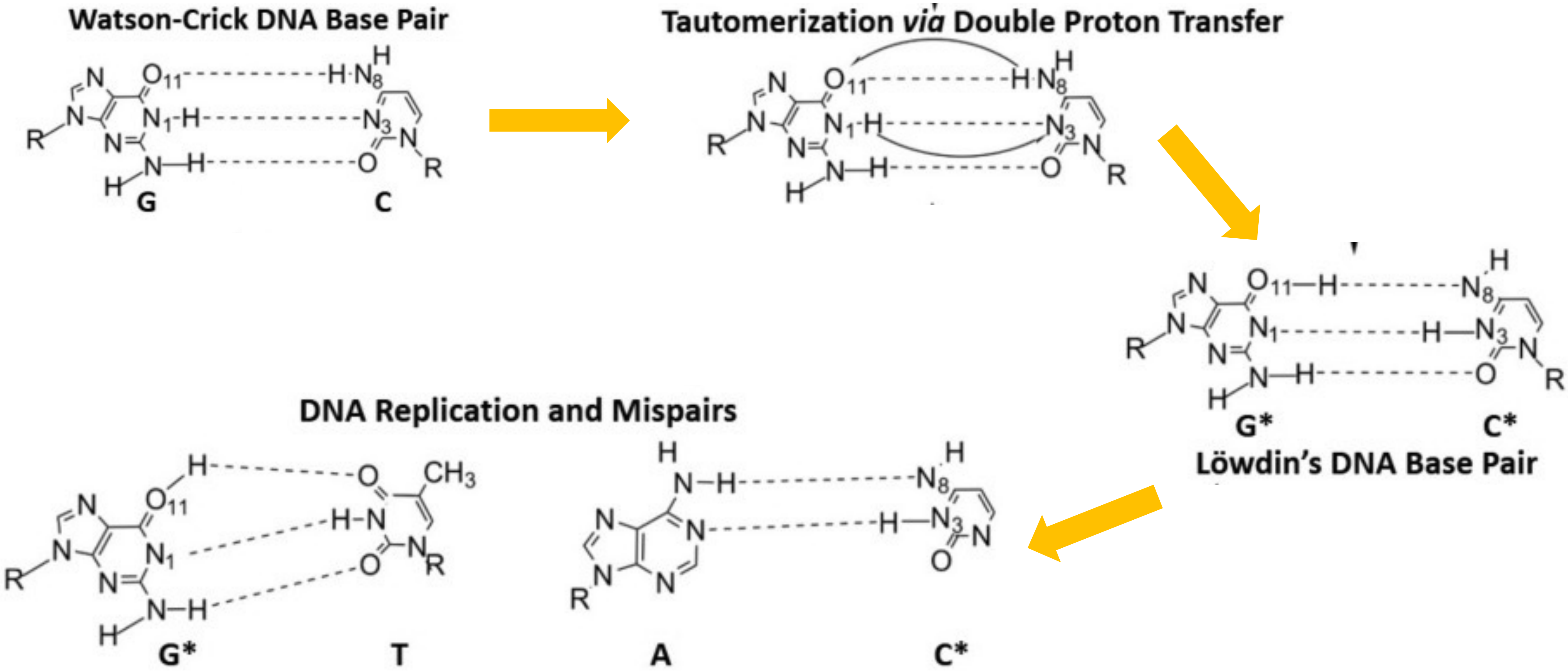
A | C
T | G

A* — C
T — G*

(P. Löwdin, Rev. Mod. Phys. 35 (1963) 724)

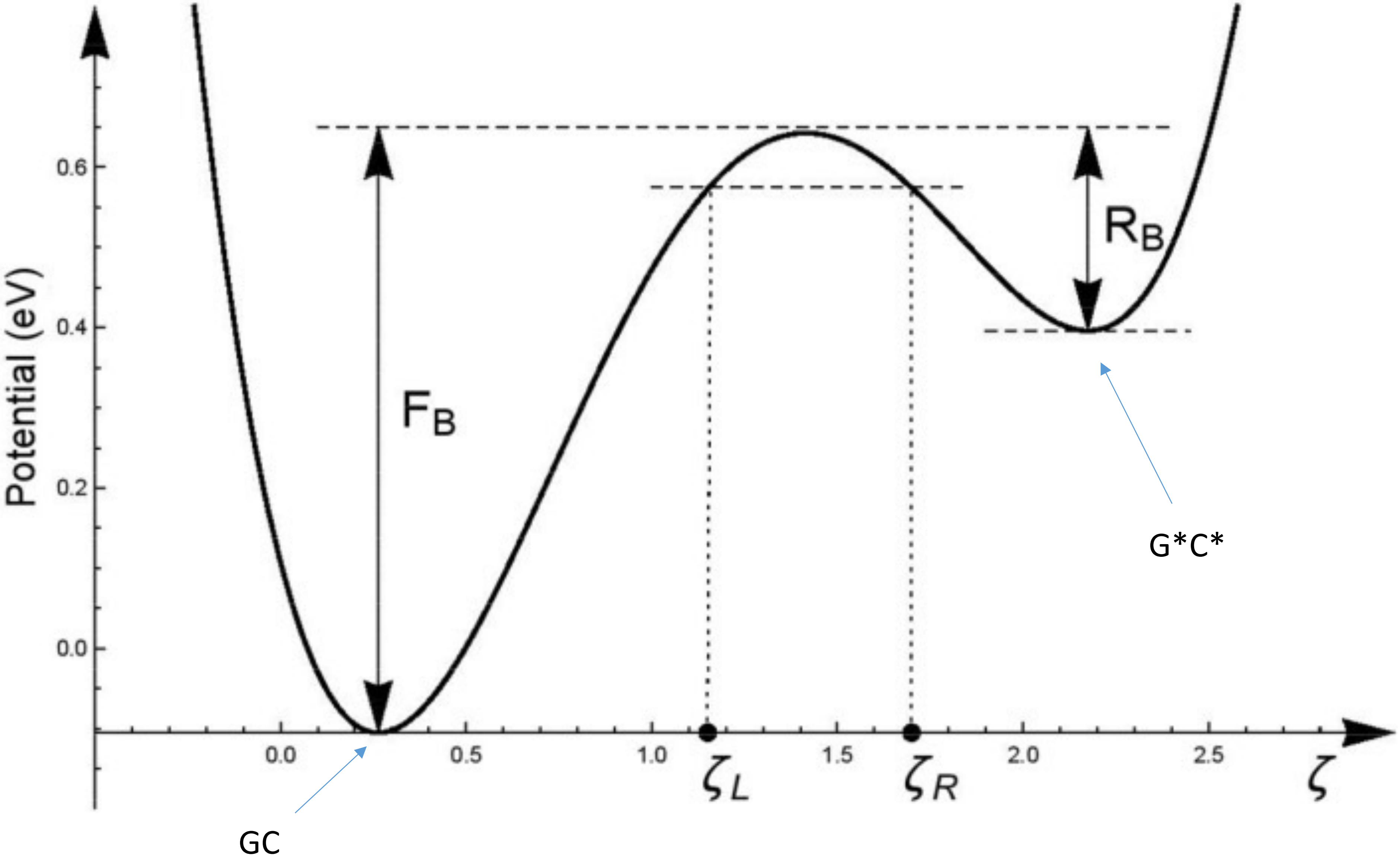
ENTROPIC TUNNELING TIME: DNA MUTATION

Inter-base proton tunneling:



(G. Çelebi, E. Özçelik, E. Vardar, DD, Prog. Biophysics and Molecular Biology 167 (2021) 96)

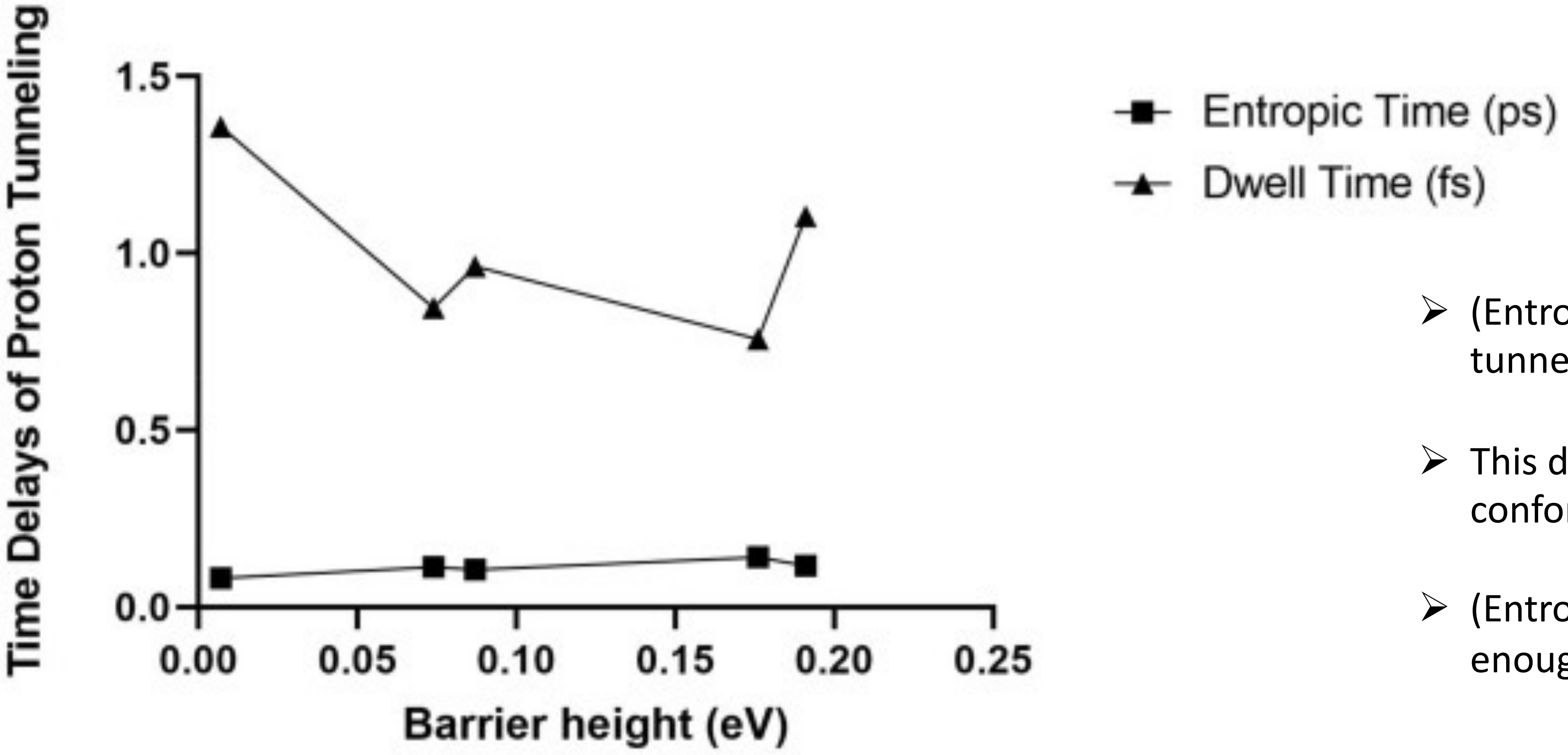
ENTROPIC TUNNELING TIME: DNA MUTATION



(D. Soler-Polo et al., J. Chem. Theor. Comput. 15 (2019) 6984)

ENTROPIC TUNNELING TIME: DNA MUTATION

Inter-base proton tunneling:

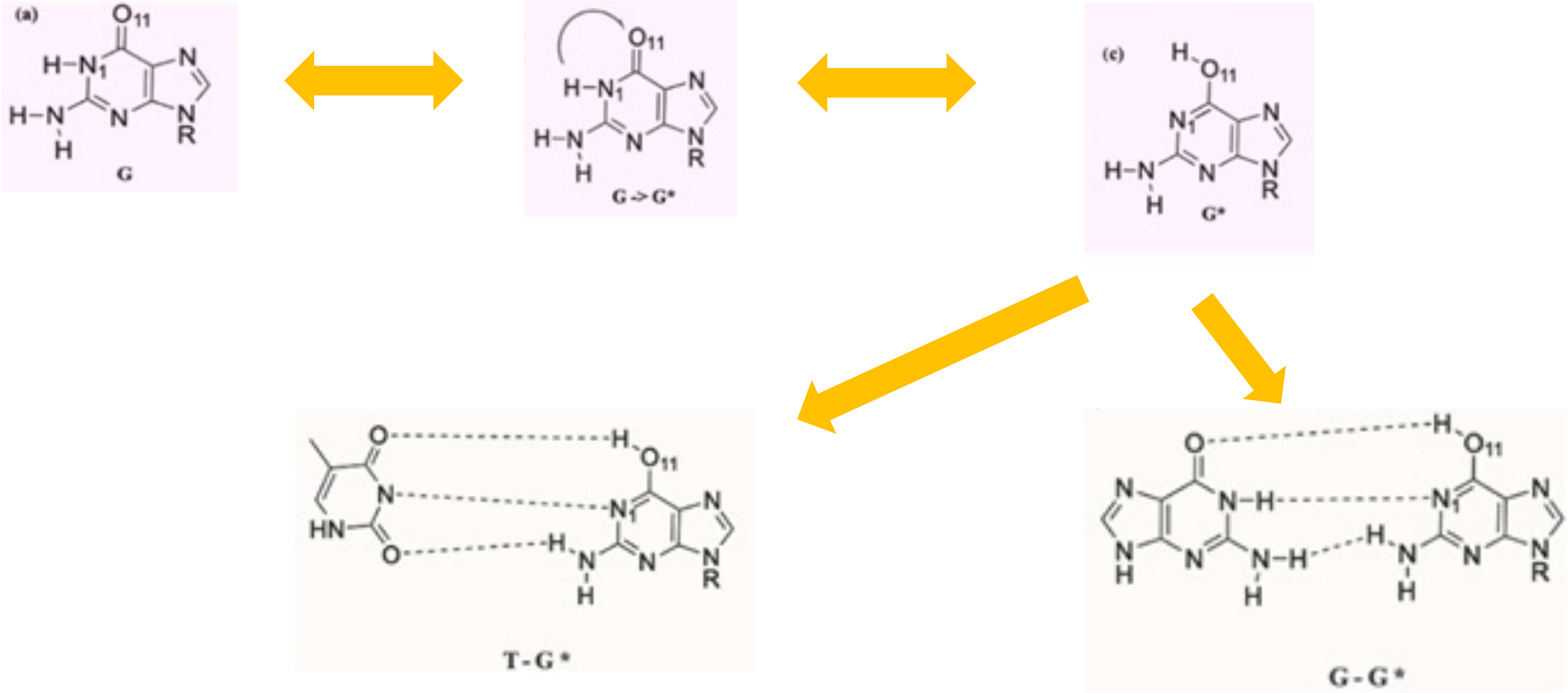


- (Entropic) time delay during the proton tunneling is about picoseconds.
- This delay is close to the time scale of conformational changes in biosystems.
- (Entropic) time delay could be long enough to start DNA point mutations.

(G. Çelebi, E. Özçelik, E. Vardar, DD, Prog. Biophysics and Molecular Biology 167 (2021) 96)

ENTROPIC TUNNELING TIME: DNA MUTATION

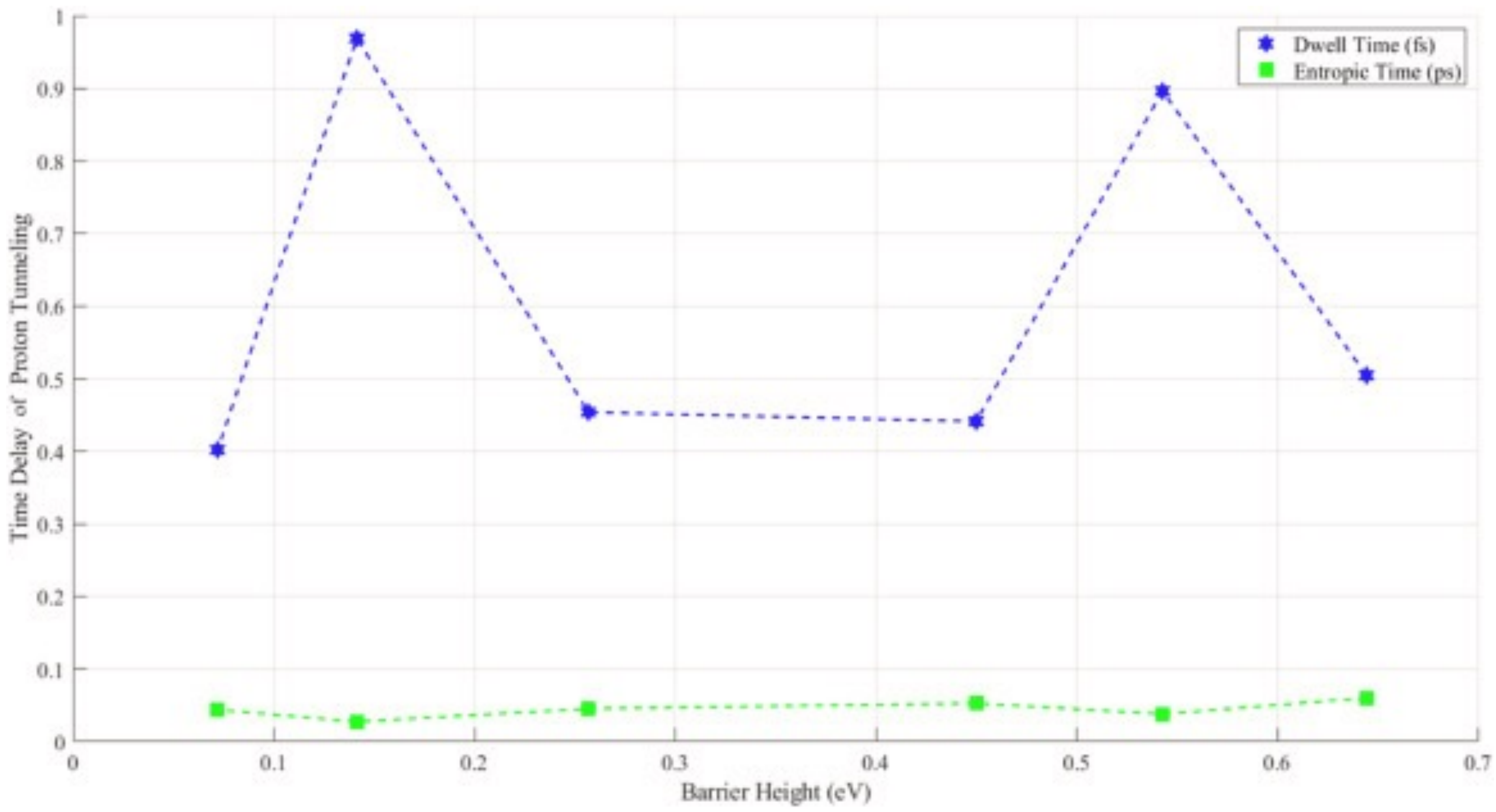
Intra-base proton tunneling:



(E. Özçelik, E. Akar, S. Zaman, DD, Prog. Biophysics and Molecular Biology 173 (2023) 4)

ENTROPIC TUNNELING TIME: DNA MUTATION

Intra-base proton tunneling:



- ➢ (Entropic) time delay during the proton tunneling is about tenth of picoseconds.
- ➢ This delay is close to the time scale of conformational changes in biosystems.
- ➢ (Entropic) time delay could be long enough to start DNA point mutations.

(E. Özçelik, E. Akar, S. Zaman, DD, Prog. Biophysics and Molecular Biology 173 (2023) 4)

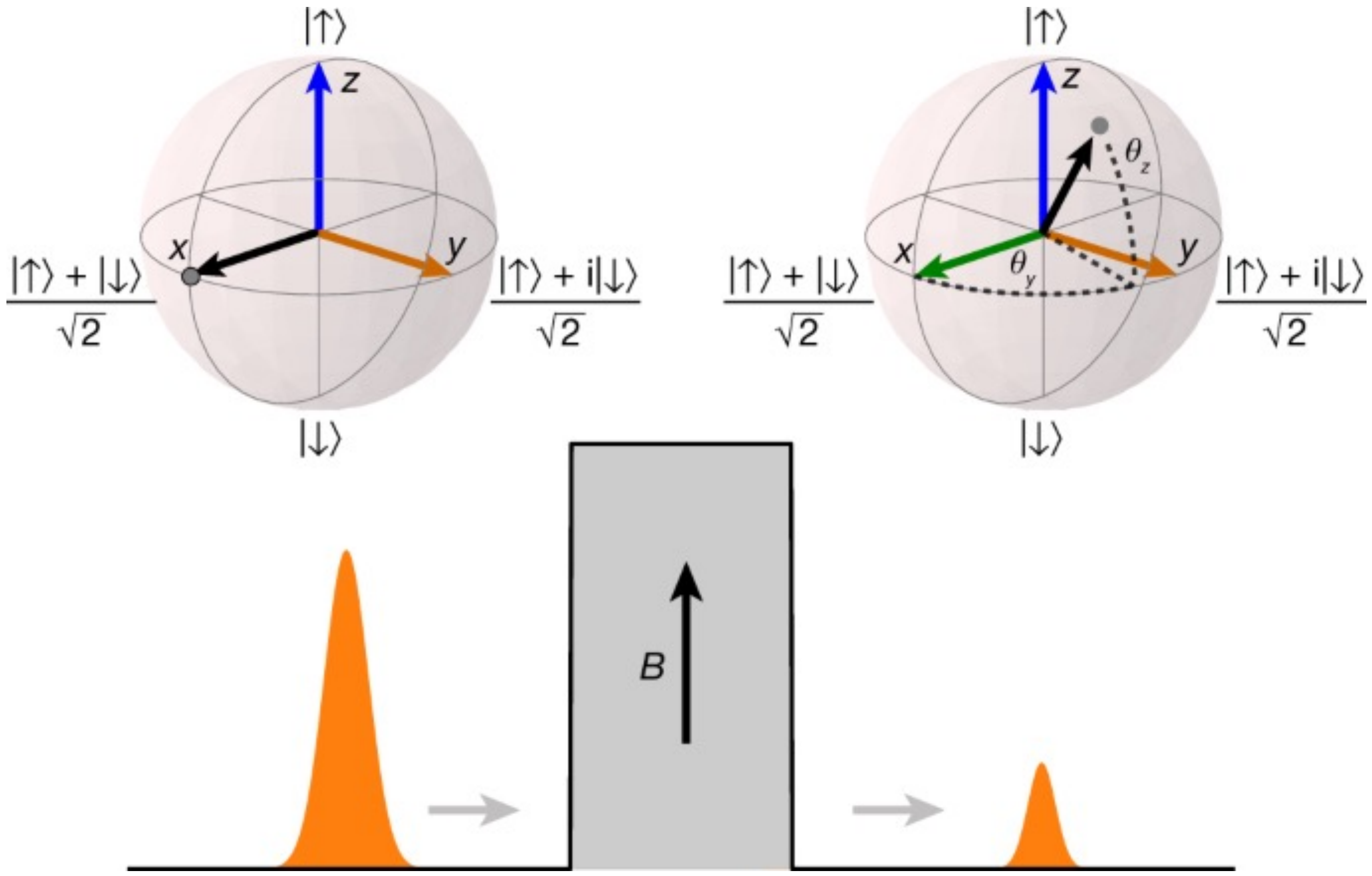
Entropic tunneling time applied to epilepsy: *L. Al-Husinat et al., NeuroQuantology 20 (2022) 7292*.

ACTUAL TUNNELING TIME BY LARMOR CLOCK

- Particle spin precesses when passing through magnetic field regions.

- Given a barrier spanned by magnetic field $\vec{B} = B\,\hat{z}$ and initial particle spin $\vec{S} = \frac{\hbar}{2}\,\hat{x}$:

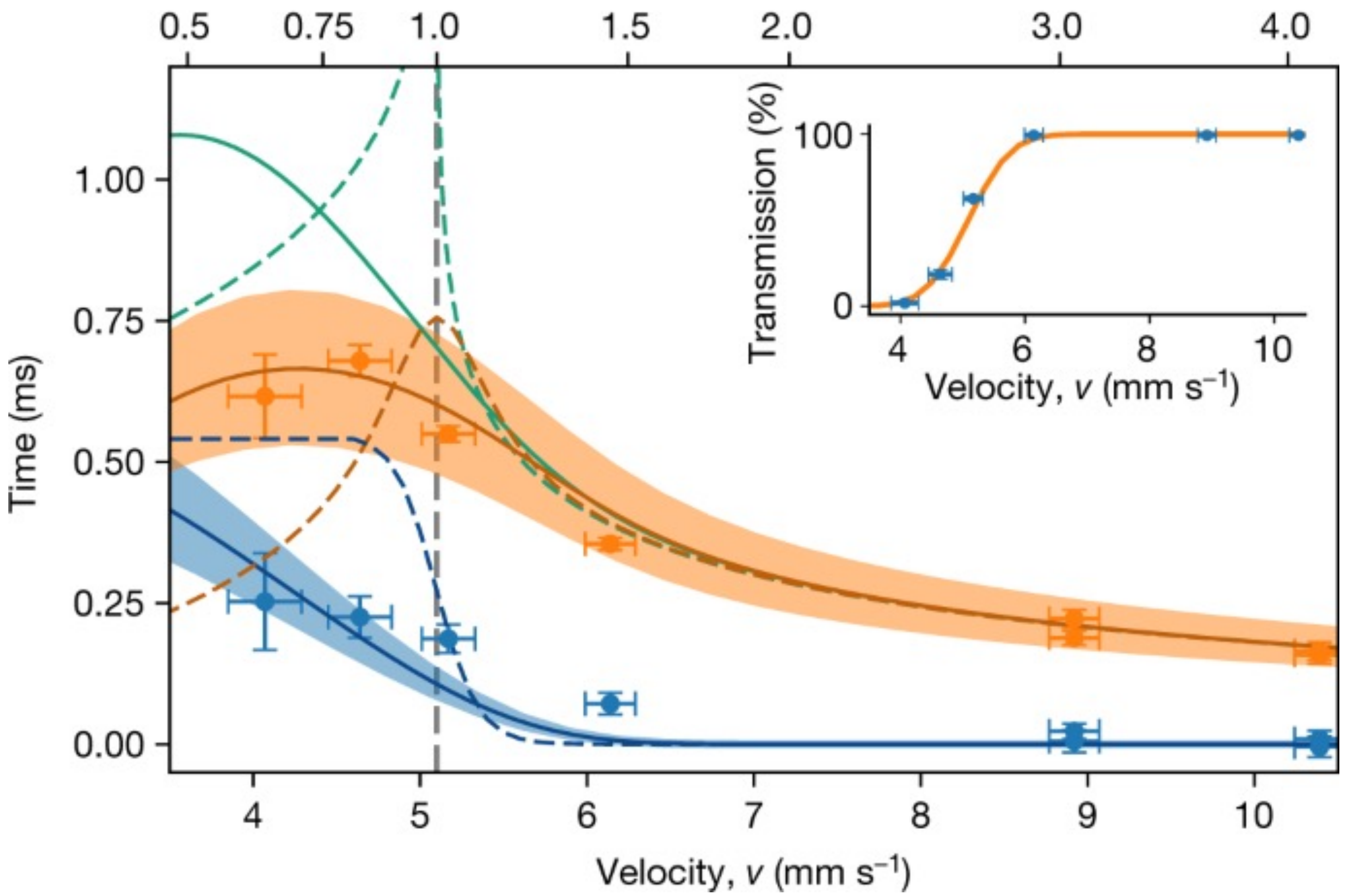
  - Particle spin acquires a y-component, even when potential $= 0$!
  - Particle spin acquires a z-component only when potential $\neq 0$!
  - (Energy) state of the particle remains intact for feeble magnetic fields.



(M. Büttiker, Phys. Rev. B27 (1983) 6178)   *(R. Ramos et al., Nature 583 (2020) 529)*

ACTUAL TUNNELING TIME BY LARMOR CLOCK

- There are two times:
  - τ_y for precession about y axis, and
  - τ_z for precession about z axis.
- Question: What is the actual tunneling time?
  - Büttiker: $(\mathrm{ATT})_B = \sqrt{\tau_y^2 + \tau_z^2}$
  - Steinberg: $(\mathrm{ATT})_S = \tau_y$



(M. Büttiker, Phys. Rev. B27 (1983) 6178) (R. Ramos et al., Nature 583 (2020) 529)

ACTUAL TUNNELING TIME BY LARMOR CLOCK

- Uncertainty product:

$$(\Delta S_x)^2 \left(\Delta S_y\right)^2 \geq \frac{\hbar^2}{4}\langle S_z\rangle^2$$

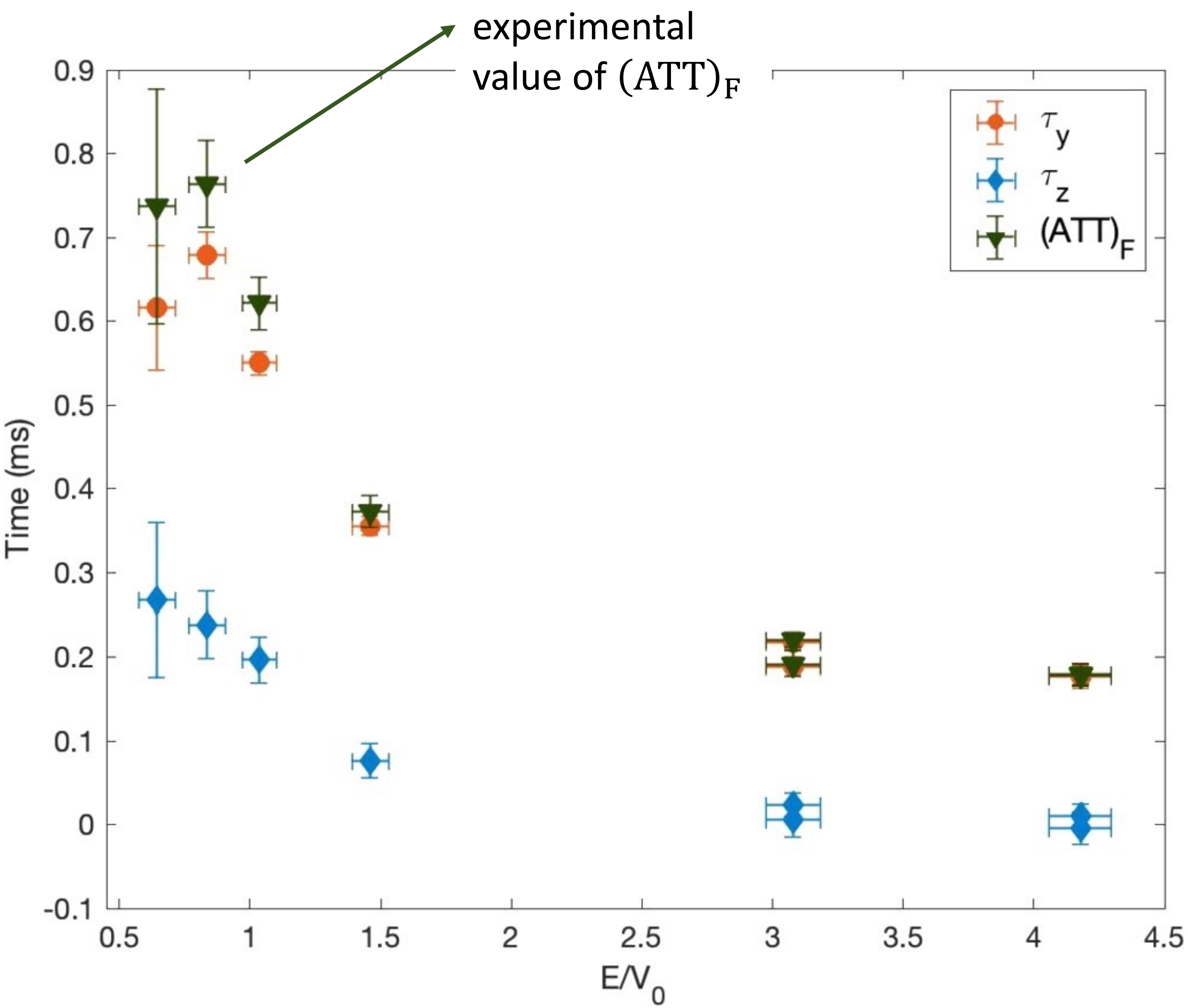
potential

- Fano factor $(\Delta S_x)^2/\langle S_x\rangle$ = a measure of spin dispersion (Poisson, clustered, uniform).

- Use Fano factor to define the actual tunneling time:

$$(\mathrm{ATT})_F = \omega_L^{-1} \frac{(\Delta S_x)^2}{\frac{\hbar}{2}\langle S_x\rangle} \frac{\left(\Delta S_y\right)^2}{\frac{\hbar}{2}\langle S_y\rangle}$$

$$\Rightarrow (\mathrm{ATT})_F = \tau_y + \frac{\tau_z^2}{\tau_y}$$



(DD, Phys. Lett. A448 (2022) 128321)

ACTUAL TUNNELING TIME BY LARMOR CLOCK

- $(\mathrm{ATT})_{\mathrm{F}}$ proves to be a "physical transmission" time in all the relevant asymptotics.

- A genuine physical time that can be tested new materials to put a (hopefully) end to the question of what the actual tunneling time is.

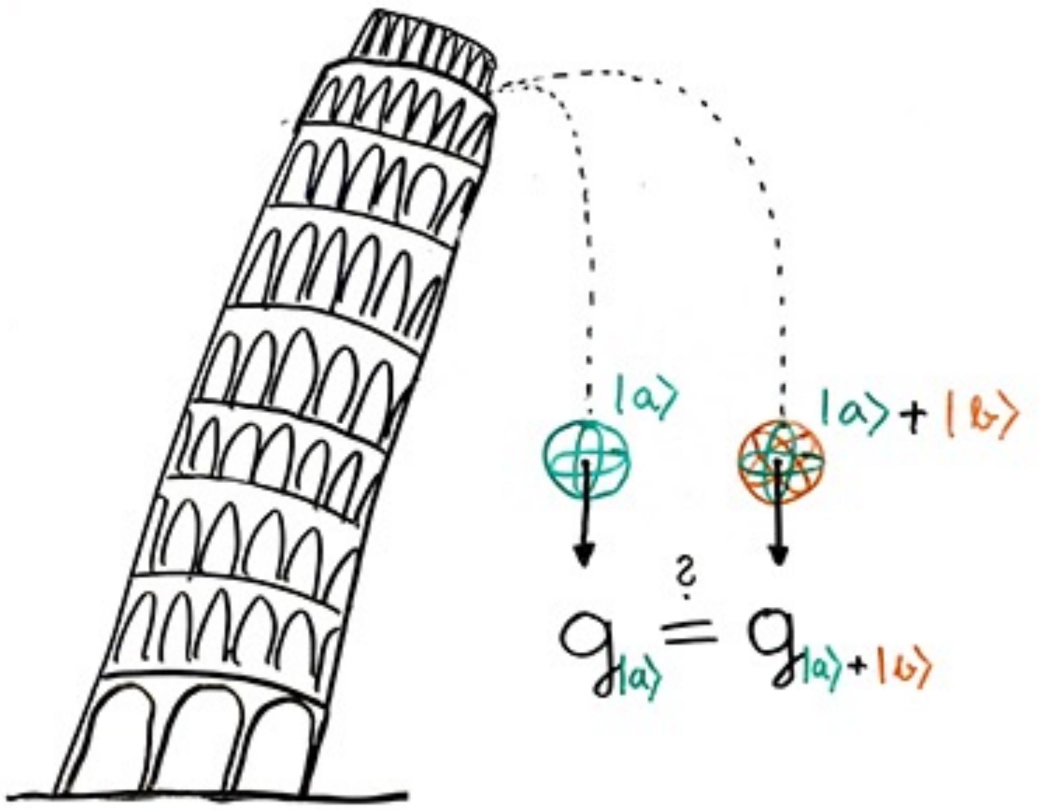
Table 1: The three ATT candidates in the low-barrier, high-barrier, thick-barrier and classical dynamics limits.

	τ_y	τ_z	$(\mathrm{ATT})_B$	$(\mathrm{ATT})_S$	$(\mathrm{ATT})_F$
low-barrier: $V_0 \ll E$ (fixed E)	$\tau_c(0,E)$	0	$\tau_c(0,E)$	$\tau_c(0,E)$	$\tau_c(0,E)$
high-barrier: $E \ll V_0$ (fixed V_0)	0	$\tau_c(V_0,0)$	$\tau_c(V_0,0)$	0	∞
thick-barrier: $L^2 \gg \frac{\hbar}{m}\tau_c(V_0,E)$ (fixed V_0,E)	$\frac{\hbar}{V_0}\frac{\tau_c(V_0,E)}{\tau_c(0,E)}$	∞	∞	$\frac{\hbar}{V_0}\frac{\tau_c(V_0,E)}{\tau_c(0,E)}$	∞
classical dynamics: $\hbar \to 0$ (fixed V_0,E,L)	0	$\tau_c(V_0,E)$	$\tau_c(V_0,E)$	0	∞

(DD, Phys. Lett. A448 (2022) 128321)

TUNNELING TIME FROM FREE-FALL

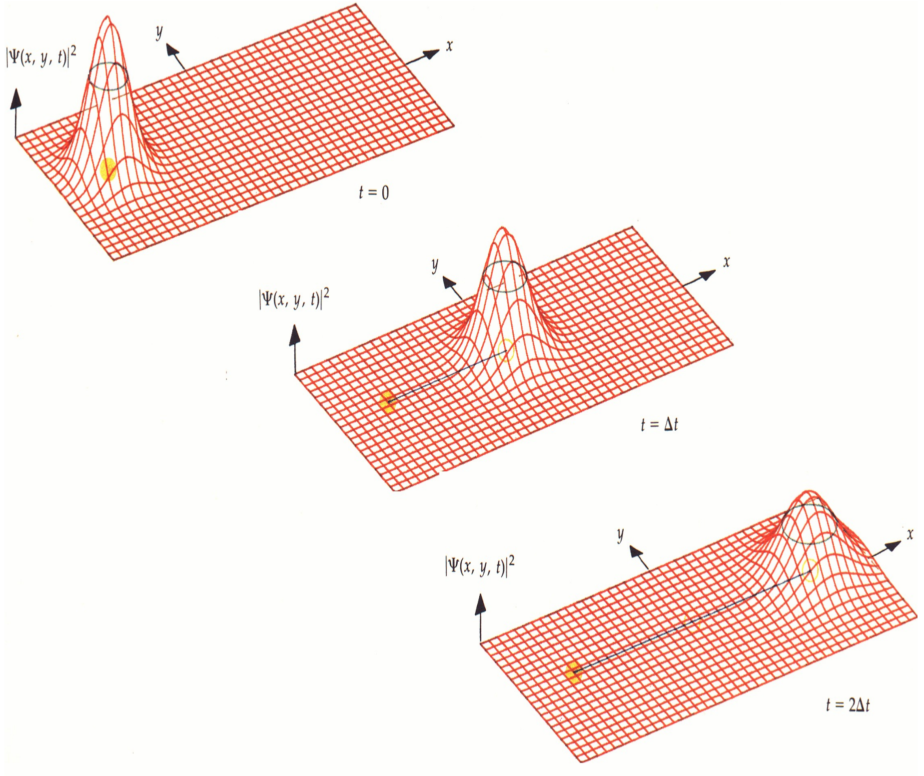
- Imagine an ultra-high vacuum (pressures about 10^{-5} Pa or mean free paths about 10^{5} m).
- Throw quantum particles upwards and measure their return time.
- This process enables us to answer two crucial questions:
  - Which interpretation of quantum theory is realized in nature? Copenhagen or Bohmian?
  - What is the tunneling time formula?



(taken from https://equs.org)

TUNNELING TIME FROM FREE-FALL

- Imagine an ultra-high vacuum (pressures about 10^{-5} Pa or mean free paths about 10^{5} m).
- Model particles by a wavepacket of width d.



(taken from user42076 @ stack exchange)

	Copenhagen	Bohmian
particle trajectory	no	yes
probability backflow	yes	no

	Copenhagen	Bohmian
$\frac{(\Delta t)_q}{(\Delta t)_c}$	$1+\frac{\hbar^2}{4m^2d^2v_i^2}+\vartheta(\hbar^4)$	$1+\frac{\hbar}{m\sqrt{2gd^3}}+\vartheta(\hbar^2)$

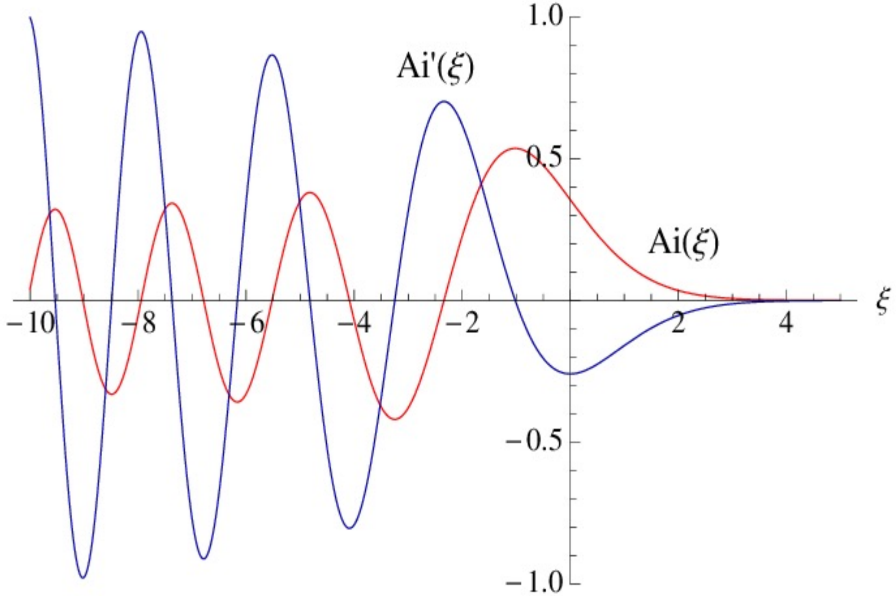
(P. Flores et al., Phys. Rev. A99 (2019) 042113)

(DD, Phys. Rev. A106 (2022) 022215)

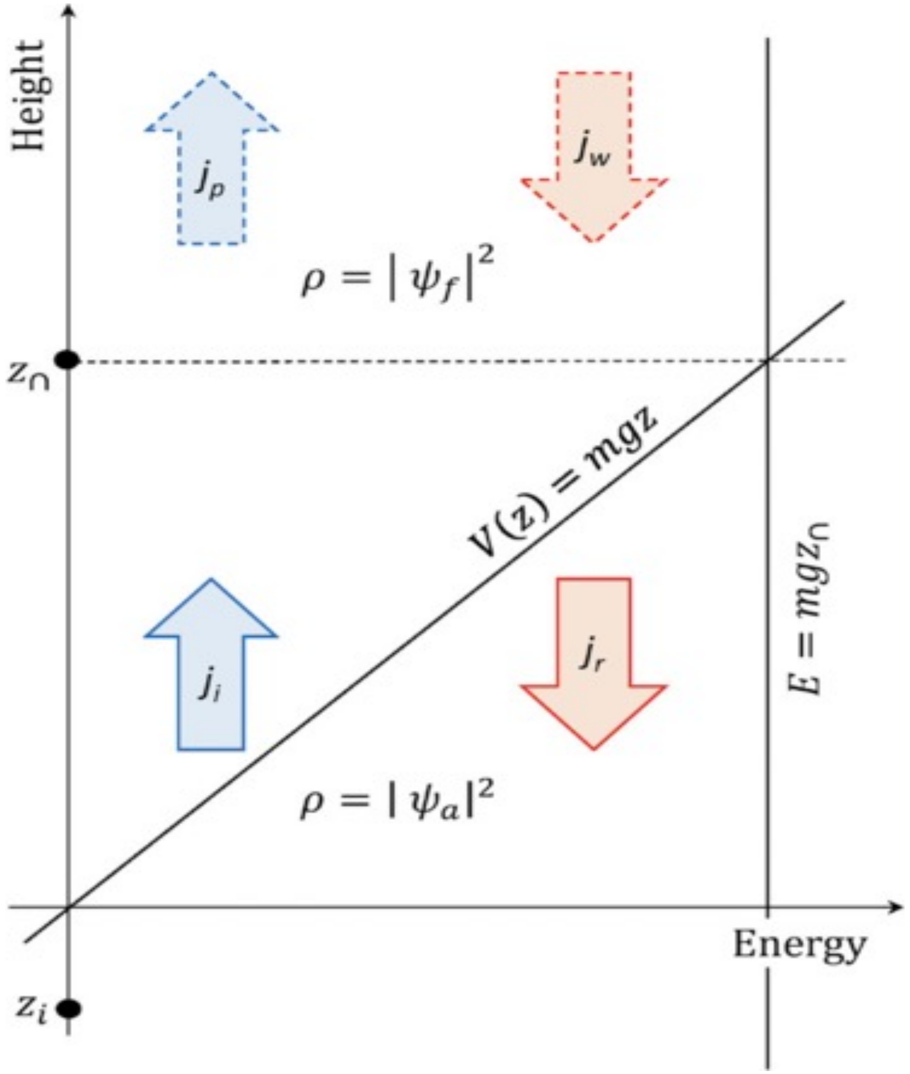
TUNNELING TIME FROM FREE-FALL

- Imagine an ultra-high vacuum (pressures about 10^{-5} Pa or mean free paths about 10^5 m).

- Consider stationary-state mono-energetic particles (states with no classical analogue).



$$\zeta = \frac{2}{3}\left(\frac{|z - z_\cap|}{L_q}\right)^{\frac{3}{2}} \quad \text{with} \quad L_q = \left(\frac{\hbar^2}{2m^2 g}\right)^{\frac{1}{3}}$$



(DD, Phys. Rev. A106 (2022) 022215)

TUNNELING TIME FROM FREE-FALL

$$\psi_f(z) = \psi_p(z) + \psi_w(z)$$

$$\psi_p(z) = Ni\zeta^{\frac{1}{3}}\left(e^{\frac{i\pi}{6}} I_{\frac{1}{3}}(\zeta) + e^{-\frac{i\pi}{6}} I_{-\frac{1}{3}}(\zeta)\right)$$

$$\psi_w(z) = -N\zeta^{\frac{1}{3}}\left(\left(1 - e^{-\frac{i\pi}{3}}\right) I_{\frac{1}{3}}(\zeta) - \left(1 - e^{\frac{i\pi}{3}}\right) I_{-\frac{1}{3}}(\zeta)\right)$$

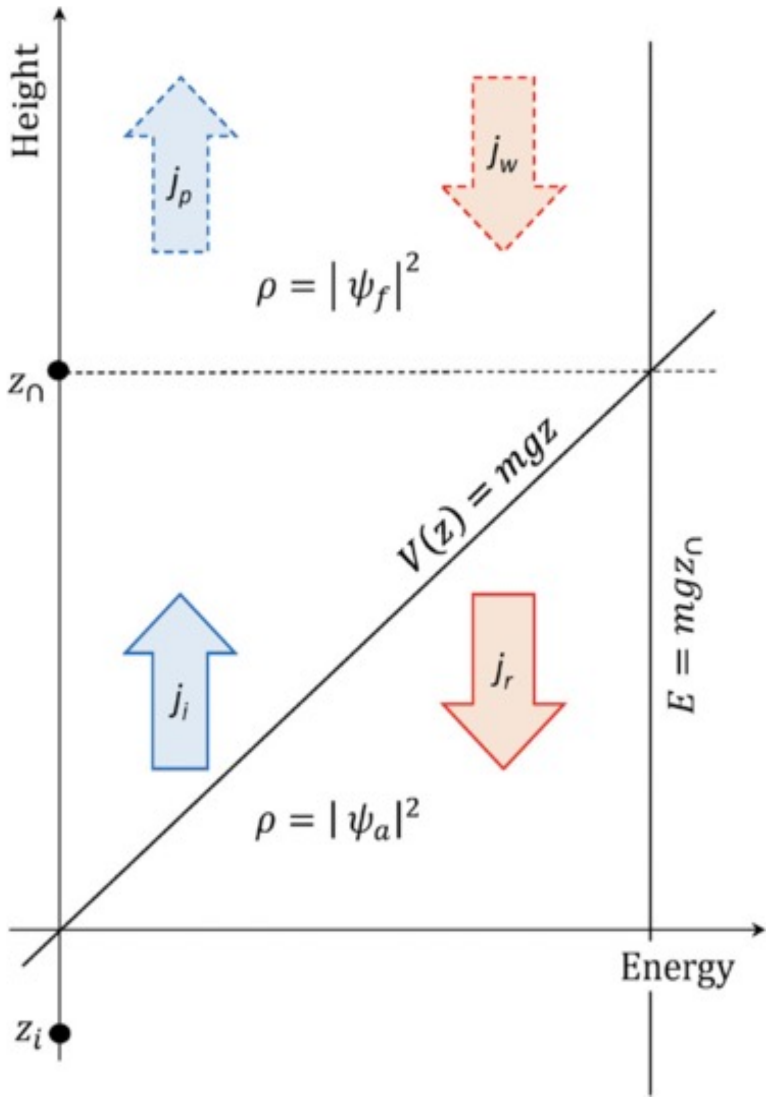
$$j_p = -j_w = \frac{3N^2}{2\pi}\left(\frac{3g\hbar}{m}\right)^{\frac{1}{3}}$$

$$\psi_a(z) = \psi_i(z) + \psi_r(z)$$

$$\psi_i(z) = N\zeta^{\frac{1}{3}}\left(e^{-\frac{i\pi}{3}} J_{\frac{1}{3}}(\zeta) + e^{\frac{i\pi}{3}} J_{-\frac{1}{3}}(\zeta)\right)$$

$$\psi_r(z) = N\zeta^{\frac{1}{3}}\left(\left(1 - e^{-\frac{i\pi}{3}}\right) J_{\frac{1}{3}}(\zeta) + \left(1 - e^{\frac{i\pi}{3}}\right) J_{-\frac{1}{3}}(\zeta)\right)$$

$$j_i = -j_r = \frac{3N^2}{2\pi}\left(\frac{3g\hbar}{m}\right)^{\frac{1}{3}}$$



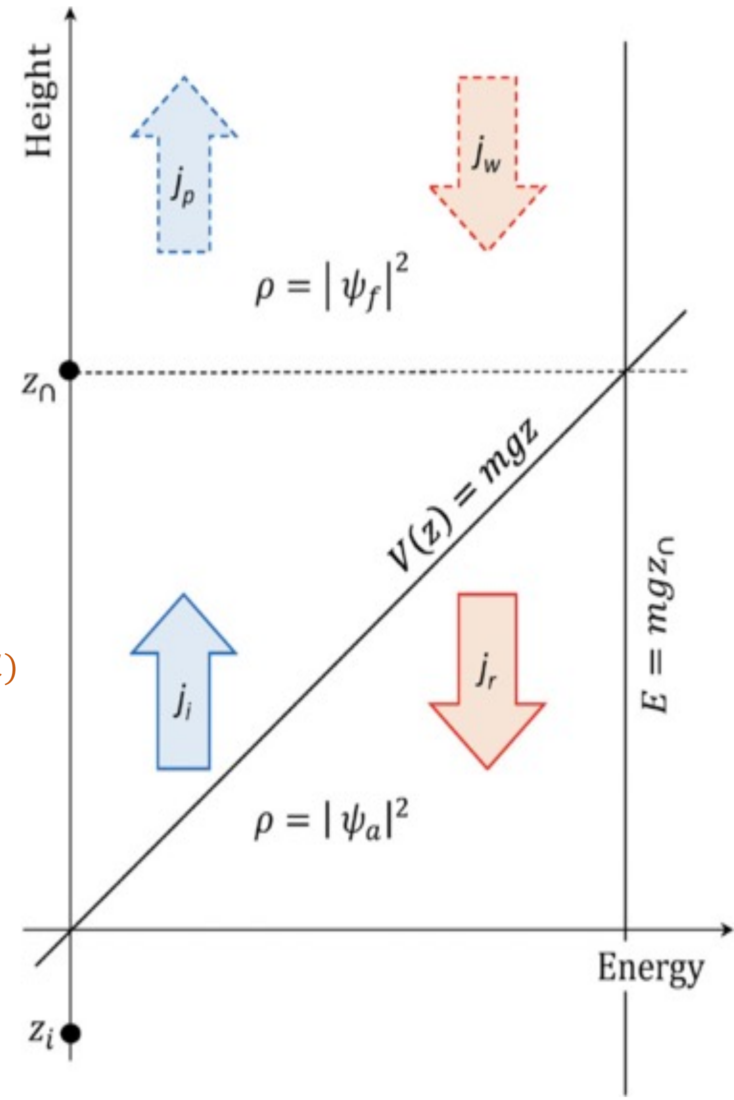
(DD, Phys. Rev. A106 (2022) 022215)

TUNNELING TIME FROM FREE-FALL

$$(\Delta t)_q^{(penetrate)} = \int_{z_i}^{z_\cap} \frac{|\psi_f(z)|^2}{2\,j_p} dz = \frac{2\pi T_q}{\left[3^{\frac{1}{3}}\Gamma\left(\frac{1}{3}\right)\right]^2} = (\Delta t)_q^{(withdraw)}$$

$$(\Delta t)_q^{(rise)} = \int_{z_i}^{z_\cap} \frac{|\psi_a(z)|^2}{2\,j_i} dz = -\frac{2\pi T_q}{\left[3^{\frac{1}{3}}\Gamma\left(\frac{1}{3}\right)\right]^2} + 2\pi T_q \left(\beta_q\left[Ai(-\beta_q)\right]^2 + \left[Ai'(-\beta_q)\right]^2\right) = (\Delta t)_q^{(fall)}$$

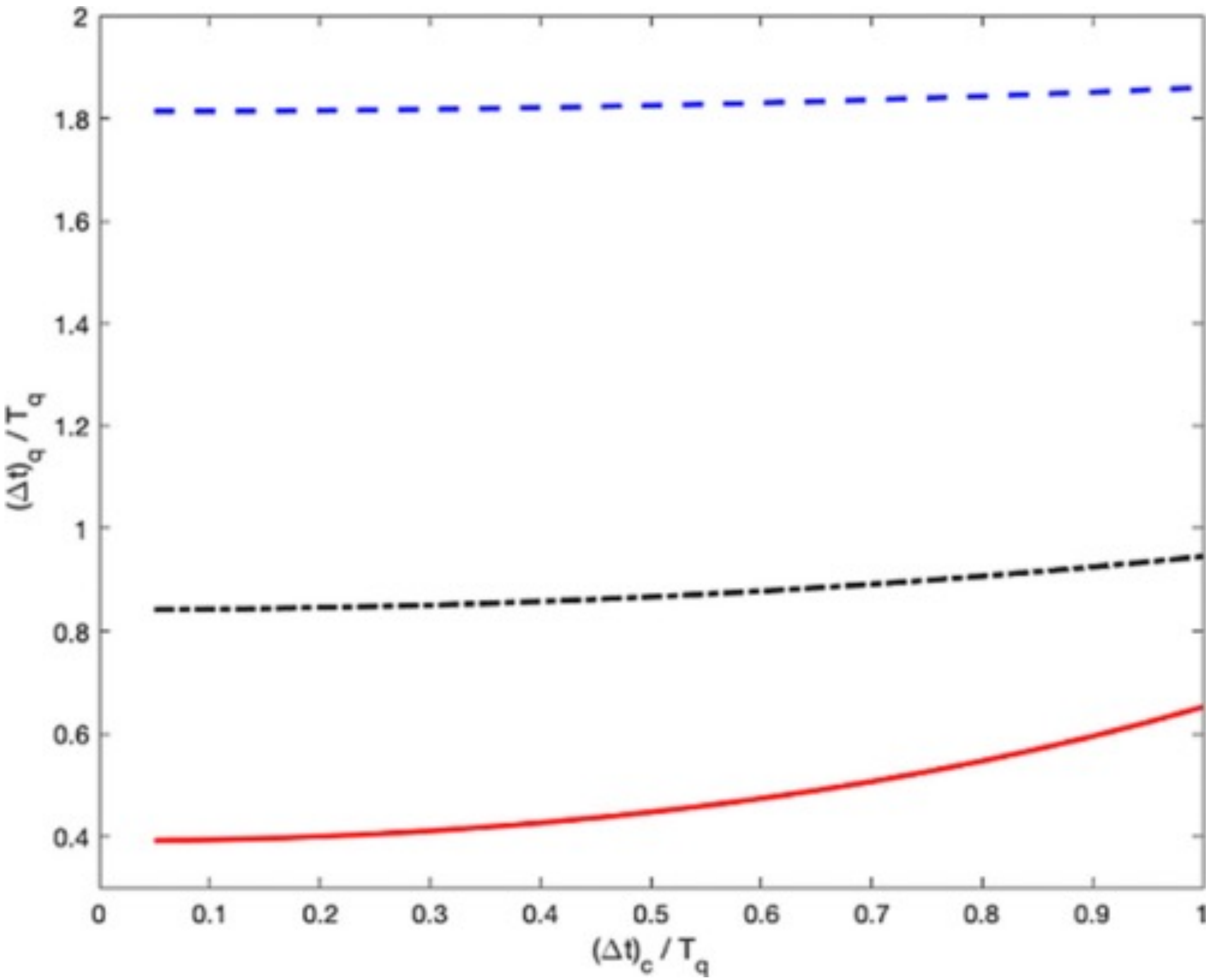
$$\beta_q = \left(\frac{(\Delta t)_c}{4T_q}\right)^2 \quad \text{with} \quad T_q = \left(\frac{\hbar}{4mg^2}\right)^{\frac{1}{3}}$$



(DD, Phys. Rev. A106 (2022) 022215)

TUNNELING TIME FROM FREE-FALL

$$(\Delta t)_q = (\Delta t)_q^{(rise)} + (\Delta t)_q^{(penetrate)} + (\Delta t)_q^{(withdraw)} + (\Delta t)_q^{(fall)}$$

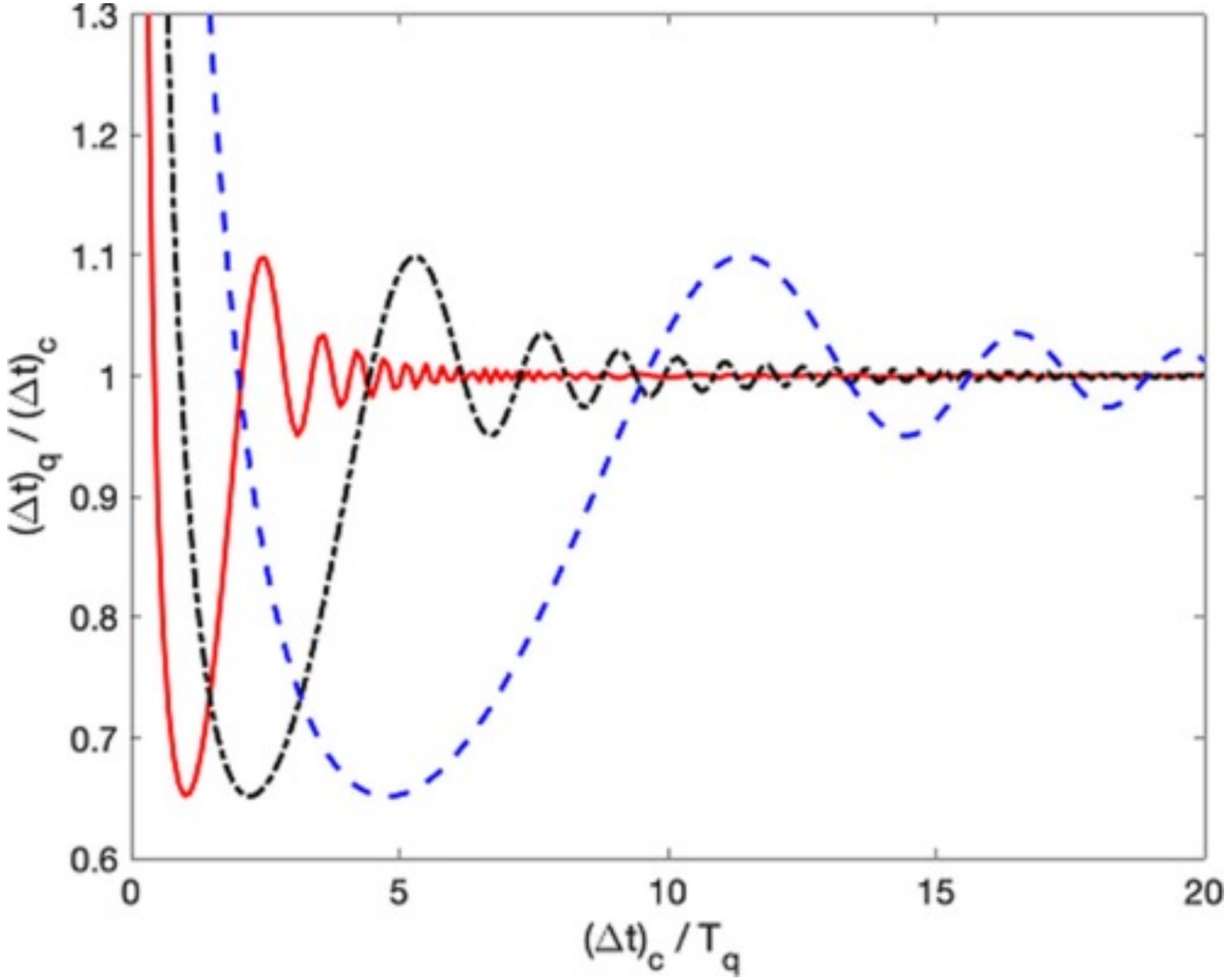


- $z_\cap = z_i \Longrightarrow (\Delta t)_c = 0$
- $z_\cap = z_i$ but $(\Delta t)_q \neq 0$

(DD, Phys. Rev. A106 (2022) 022215)

TUNNELING TIME FROM FREE-FALL

$$(\Delta t)_q = (\Delta t)_q^{(rise)} + (\Delta t)_q^{(penetrate)} + (\Delta t)_q^{(withdraw)} + (\Delta t)_q^{(fall)}$$



- Low $(\Delta t)_c$: $(\Delta t)_q$ fluctuates strongly. It could be smaller or larger than $(\Delta t)_c$.
- High $(\Delta t)_c$: $(\Delta t)_q$ relaxes in an oscillatory fashion towards $(\Delta t)_c$.
- Short-height flights can better extract quantum effects.
- Equivalence principle is attained for long-height flights.

(DD, Phys. Rev. A106 (2022) 022215)

CONCLUSION AND FUTURE PROSPECTS

Quantum Biology (DNA, enzymes, Quadruplexes, …)

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GENETICS

Quantum Tunneling Makes DNA More Unstable

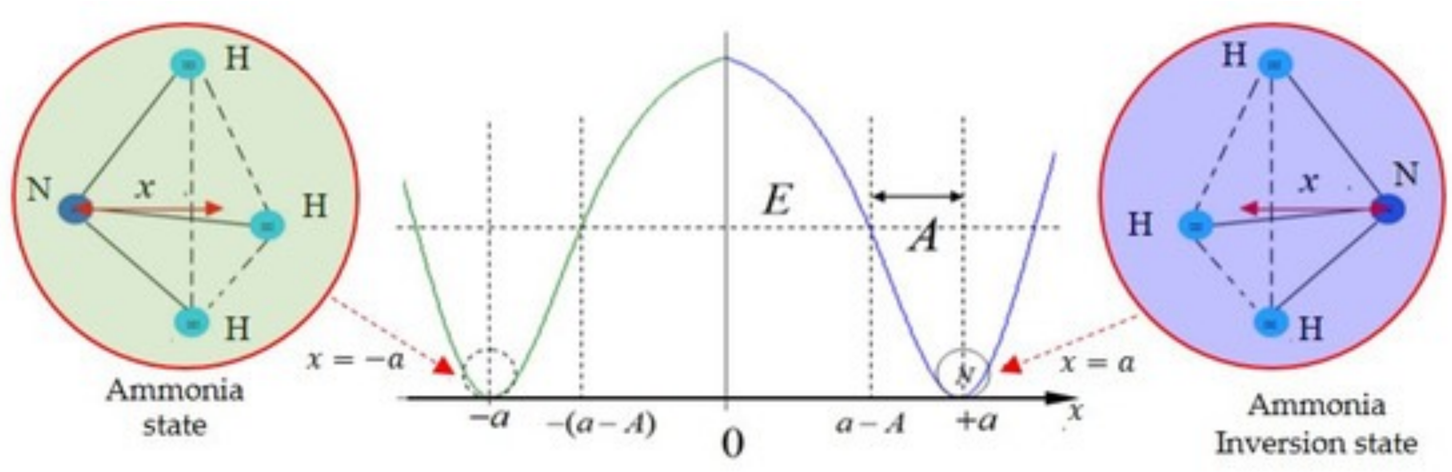
The freaky physics phenomenon of quantum tunneling may mutate genes

By Lars Fischer, Gary Stix on September 1, 2022

(Demir Group @ Sabancı and Al-Khalili Group @ Surrey)

CONCLUSION AND FUTURE PROSPECTS

Quantum Chemistry (reaction rates, ammonia inversion, …):

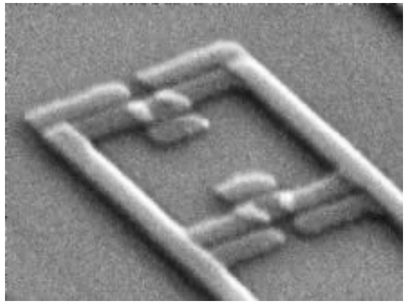


(C. Yang et al., Int. J. Mol. Sci. 22 (2021) 8282)

(M. Kara and DD, work in progress)

CONCLUSION AND FUTURE PROSPECTS

Quantum Physics (annealing quantum computers, black holes, fusion, …):



- ”flux qubit”
- More than 1 million Josephson junctions
- Even a picosecond delay at each junction leads to nanosecond delays in total $\Longrightarrow$ An important obstacle for future realistic computations.
- Entropic and Bohmian time formulae could lead to a testable framework.

(O. Sargın, A. Hayreter, DD, work in progress)

THANK YOU!