Quantum Tunneling Time

with Applications to Point Mutations in DNA

Durmuş A. Demir

Sabancı University, İstanbul

HUMBOLDT KOLLEG 2023 ANKARA (Ankara University, 12-14 April 2023)
MEMBERS

Durmuş Demir
[PI – CV & Publications]

Elham Ghorani
[Doc – CV & Publications]

Elif Özçelik
[Bech – CV & Publications]

Beyhan Pulıçe
[PostDoc – CV & Publications]

Ozan Sargin
[PostDoc – CV & Publications]

Emre Vardar
[Bech – CV & Publications]

FORMER MEMBERS

Gizem Çelebi
[Doc – CV & Publications]
Tunneling Time: Which interpretation of the quantum theory is realized in nature?

Tunneling Time: What is the true transmission time?

Tunneling Time: Is there an entropic equivalent of the imaginary time under the barrier?
  - Entropic Proton Tunneling Time and Point Mutations in DNA

Future Prospects
Tunneling is a textbook topic, tunneling time is not.

In quantum theory, time is not an observable so tunneling time needs be modeled by some additional methods/ideas.

Experiments have already measured a finite tunneling time for quantum particles (electron, proton, atoms).

Strong-field tunnel ionization experiments have sidelined most of the past tunneling time models.

It is time to build more complete tunneling time models, test them with the experimental data, and make predictions for other phenomena.
EXPERIMENTS: TUNNELING TAKES TIME

(M. Yuan, Optics Exp. 27 (2019) 6502)
EXPERIMENTS: TUNNELING TAKES TIME

Ar vs Kr tunnel ionizations:

(N. Camus et al., PRL 119 (2017) 023201)

He tunnel ionization:

(C. Hoffmann et al., J. Mod. Optics 66 (2019) 1052)
EXPERIMENTS: TUNNELING TAKES TIME

Rb atom tunneling across an optical potential:

Proton tunneling in \( \text{H}_2 + \text{D}^- \rightarrow \text{H}^- + \text{HD} \):

(R. Ramos et al., Nature 583 (2020) 569)

(R. Wild et al., Nature 615 (2023) 425)
KNOWN TIME MODELS DISAGREE WITH EXPERIMENT

(A. Landsman et al., Optica 1 (2016) 343)

(C. Hoffmann et al., J. Mod. Optics 66 (2019) 1052)
WHICH INTERPRETATION OF THE QUANTUM THEORY?

- Imagine an ultra-high vacuum (pressures about $10^{-5}$ Pa or mean free paths about $10^5$ m).

- Throw quantum particles upwards and measure their return time.

- This process enables us to answer two crucial questions:
  - Which interpretation of quantum theory is realized in nature? Copenhagen or Bohmian?
  - What is the tunneling time formula?

(taken from https://equis.org)
Imagine an ultra-high vacuum (pressures about $10^{-5}$ Pa or mean free paths about $10^5$ m).

Model particles by a wavepacket of width $d$.

<table>
<thead>
<tr>
<th></th>
<th>Copenhagen</th>
<th>Bohmian</th>
</tr>
</thead>
<tbody>
<tr>
<td>particle trajectory</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>probability backflow</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

\[
\frac{(\Delta t)_q}{(\Delta t)_c} = 1 + \frac{\hbar^2}{4m^2d^2v_i^2} + \vartheta(h^4)
\]

\[
1 + \frac{\hbar}{m\sqrt{2gd^3}} + \vartheta(h^2)
\]


(DD, Phys. Rev. A106 (2022) 022215)
- Imagine an ultra-high vacuum (pressures about $10^{-5}$ Pa or mean free paths about $10^5$ m).
- Consider stationary-state mono-energetic particles (states with no classical analogue).

\[
\zeta = \frac{2}{3} \left( \frac{|z - z_n|}{L_q} \right)^{\frac{3}{2}} \quad \text{with} \quad L_q = \left( \frac{\hbar^2}{2m^2g} \right)^{\frac{1}{3}}
\]

(DD, Phys. Rev. A106 (2022) 022215)
\[
\psi_f(z) = \psi_p(z) + \psi_w(z)
\]
\[
\psi_p(z) = N i \frac{\xi}{3} \left( e^{\frac{i \pi}{6}} J_1(\frac{\xi}{3}) + e^{-\frac{i \pi}{6}} J_{-1}(\frac{\xi}{3}) \right)
\]
\[
\psi_w(z) = -N \frac{\xi}{3} \left( (1 - e^{-\frac{i \pi}{3}}) J_1(\frac{\xi}{3}) - (1 - e^{\frac{i \pi}{3}}) J_{-1}(\frac{\xi}{3}) \right)
\]
\[
\psi_a(z) = \psi_i(z) + \psi_r(z)
\]
\[
\psi_i(z) = N \frac{\xi}{3} \left( e^{-\frac{i \pi}{6}} J_1(\frac{\xi}{3}) + e^{\frac{i \pi}{6}} J_{-1}(\frac{\xi}{3}) \right)
\]
\[
\psi_r(z) = N \frac{\xi}{3} \left( (1 - e^{-\frac{i \pi}{3}}) J_1(\frac{\xi}{3}) + (1 - e^{\frac{i \pi}{3}}) J_{-1}(\frac{\xi}{3}) \right)
\]
\[
\rho = |\psi_f|^2
\]
\[
\rho = |\psi_a|^2
\]

(DD, Phys. Rev. A106 (2022) 022215)
\[(\Delta t)_q^{\text{(penetrate)}} = \int_{z_i}^{z_f} \left[ \frac{\psi_f(z)}{2j_p} \right]^2 dz = \frac{2\pi T_q}{\left[ \frac{1}{3} \Gamma \left( \frac{1}{3} \right) \right]} = (\Delta t)_q^{\text{(withdraw)}} \]

\[(\Delta t)_q^{\text{(rise)}} = \int_{z_i}^{z_f} \left[ \frac{\psi_a(z)}{2j_i} \right]^2 dz = -\frac{2\pi T_q}{\left[ \frac{1}{3} \Gamma \left( \frac{1}{3} \right) \right]} + 2\pi T_q \left( \beta_q [Ai(-\beta_q)]^2 + [Ai'(-\beta_q)]^2 \right) = (\Delta t)_q^{\text{(fall)}} \]

\[\beta_q = \left( \frac{(\Delta t)_c}{4T_q} \right)^2 \quad \text{with} \quad T_q = \left( \frac{\hbar}{4mg^2} \right)^{\frac{1}{3}}\]
\[(\Delta t)_q = (\Delta t)_q^{(\text{rise})} + (\Delta t)_q^{(\text{penetrate})} + (\Delta t)_q^{(\text{withdraw})} + (\Delta t)_q^{(\text{fall})}\]

- Low \((\Delta t)_c\): \((\Delta t)_q\) fluctuates strongly. It could be smaller or larger than \((\Delta t)_c\).
- High \((\Delta t)_c\): \((\Delta t)_q\) relaxes in an oscillatory fashion towards \((\Delta t)_c\).
- Short-height flights can better extract quantum effects.
- Equivalence principle is attained for long-height flights.

(DD, Phys. Rev. A106 (2022) 022215)
Particle spin precesses when passing through magnetic field regions.

Given a barrier covered by a feeble magnetic field $\vec{B} = B \hat{z}$ and initial particle spin $\vec{S} = \frac{\hbar}{2} \hat{z}$:

- Particle spin acquires a $y$-component, even when potential $= 0$!
- Particle spin acquires a $z$-component only when potential $\neq 0$!
- (Energy) state of the particle remains intact for feeble magnetic fields.


(R. Ramos et al., Nature 583 (2020) 529)
There are two times:
- $\tau_y$ for precession about $y$ axis, and
- $\tau_z$ for precession about $z$ axis.

Question: What is the actual tunneling time?

- Büttiker: $(\text{ATT})_B = \sqrt{\tau_y^2 + \tau_z^2}$
- Steinberg: $(\text{ATT})_S = \tau_y$

(R. Ramos et al., Nature 583 (2020) 529)
- Uncertainty product:
  \[ (\Delta S_x)^2 (\Delta S_y)^2 \geq \frac{\hbar^2}{4} \langle S_z \rangle^2 \]

- Fano factor \( (\Delta S_x)^2 / \langle S_x \rangle \) = a measure of spin dispersion (Poisson, clustered, uniform).

- Use Fano factor to define the actual tunneling time:
  \[ (\text{ATT})_F = \omega L^{-1} \frac{(\Delta S_x)^2 (\Delta S_y)^2}{\frac{\hbar}{2} \langle S_x \rangle \frac{\hbar}{2} \langle S_y \rangle} \]

\[ \Rightarrow (\text{ATT})_F = \tau_y + \frac{\tau_z^2}{\tau_y} \]
(ATT)$_F$ proves to be a “physical transmission” time in all the relevant asymptotics.

A genuine physical time that can be tested new materials to put a (hopefully) end to the question of what the actual tunneling time is.

<table>
<thead>
<tr>
<th>Table 1: The three ATT candidates in the low-barrier, high-barrier, thick-barrier and classical dynamics limits.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_y$</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>low-barrier: $V_0 \ll E$ (fixed $E$)</td>
</tr>
<tr>
<td>high-barrier: $E \ll V_0$ (fixed $V_0$)</td>
</tr>
<tr>
<td>thick-barrier: $L^2 \gg \frac{\hbar}{m} \tau_c(V_0, E)$ (fixed $V_0, E$)</td>
</tr>
<tr>
<td>classical dynamics: $\hbar \to 0$ (fixed $V_0, E, L$)</td>
</tr>
</tbody>
</table>

(DD, Phys. Lett. A448 (2022) 128321)
- Time is imaginary in tunneling region (classical dynamics).

- Imaginary time is equivalent to inverse temperature (QM \( \equiv \) eq. Stat Mech.)

- Energy in quantum fluctuations (no real propagation) should pertain to (useless) entropic energy.

- Uncertainty product with thermal energy sets the time scale of the tunneling transition.

- Entropic tunneling time:

\[
(\Delta t)_{ETT} \equiv \frac{k_B \tau_c}{S}
\]

- \( \tau_c = \int_{x_L}^{x_R} \frac{m}{\sqrt{2m(V(x) - E)}} \)  
- \( S \equiv -k_B P \log P \)  
- \( P \equiv \int_{x_L}^{x_R} \psi^*(x)\psi(x)dx \)
He tunnel ionization:

Effective potential at a radius $x$ from the $He^+$ ion:

$$V_{eff}(x) = - \frac{Z_{eff}(x)}{x} - \mathcal{E} x$$

- $Z_{eff}(SAE) = 1 + 1.231 e^{-0.662x} - 1.325 e^{-1.236x}$
  $- 0.231 e^{-0.48x}$
- $Z_{eff}(Kullie) = 1.375$
- $Z_{eff}(Clementi) = 1.687$

Experiment:
(A. Landsman et al, Optica 1 (2016) 343)

Model:
ENTROPIC TUNNELING TIME

Experiment:  (A. Landsman et al., Optica 1 (2016) 343)

ENTROPIC TUNNELING TIME: DNA MUTATION

(proton tunneling)

A

T

C

G

A*

T

C

G*

(P. Löwdin, Rev. Mod. Phys. 35 (1963) 724)
Inter-base proton tunneling:

Watson-Crick DNA Base Pair

Tautomerization via Double Proton Transfer

DNA Replication and Mispairs

Löwdin’s DNA Base Pair

(G. Çelebi, E. Özçelik, E. Vardar, DD, Prog. Biophysics and Molecular Biology 167 (2021) 96)
ENTROPIC TUNNELING TIME: DNA MUTATION

(Entropic) time delay during the proton tunneling is about picoseconds.

This delay is close to the time scale of conformational changes in biosystems.

(Entropic) time delay could be long enough to start DNA point mutations.
ENTROPIC TUNNELING TIME: DNA MUTATION

Intra-base proton tunneling:

(E. Özçelik, E. Akar, S. Zaman, DD, Prog. Biophysics and Molecular Biology 173 (2023) 4)
Intra-base proton tunneling:

- (Entropic) time delay during the proton tunneling is about tenth of picoseconds.
- This delay is close to the time scale of conformational changes in biosystems.
- (Entropic) time delay could be long enough to start DNA point mutations.

(Entropic tunneling time applied to epilepsy: L. Al-Husinat et al., NeuroQuantology 20 (2022) 7292.)
Potassium ion transfer in G-quadruplex systems:

<table>
<thead>
<tr>
<th>Barrier height (eV)</th>
<th>Dwell entrance rate (mA)</th>
<th>Entropic entrance rate (pA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1254</td>
<td>27.7</td>
<td>423.12</td>
</tr>
<tr>
<td>0.0891</td>
<td>38.4</td>
<td>352.35</td>
</tr>
<tr>
<td>0.0601</td>
<td>32.9</td>
<td>436.45</td>
</tr>
<tr>
<td>0.0268</td>
<td>31.9</td>
<td>482.23</td>
</tr>
<tr>
<td>0.0046</td>
<td>27.4</td>
<td>576.60</td>
</tr>
</tbody>
</table>

(G. Çelebi, G. Demir, DD, J. Biological Inorganic Chemistry, 28 (2023) 213)
Quantum Biology (DNA, enzymes, quadruplexes, ...)

FUTURE PROSPECTS

Quantum Tunneling Makes DNA More Unstable

The freaky physics phenomenon of quantum tunneling may mutate genes

By Lars Fischer, Gary Stix on September 1, 2022

(Demir Group @ Sabancı and Al-Khalili Group @ Surrey)
Quantum Chemistry (reaction rates, interstellar chemistry, ...):

Hydroxyl + Methanol $\rightarrow$ Products
(much faster @ 63 K than at 200 K)

(P. Schreiner et al., Nature 453 (2008) 906)

(Eagle Nebula – interstellar chemistry)

(M. Kara and DD, work in progress)
Quantum Physics (annealing quantum computers, black holes, fusion, ...):

- “flux qubit”
- More than 1 million Josephson junctions
- Even a picosecond delay at each junction leads to nanosecond delays in total ⇒ An important obstacle for future realistic computations.
- Entropic and Bohmian time formulae could lead to a testable framework.

(O. Sargin and DD, work in progress)
THANK YOU!