

PARTICLE PHYSICS WITH BLACK HOLES

Durmuş Demir

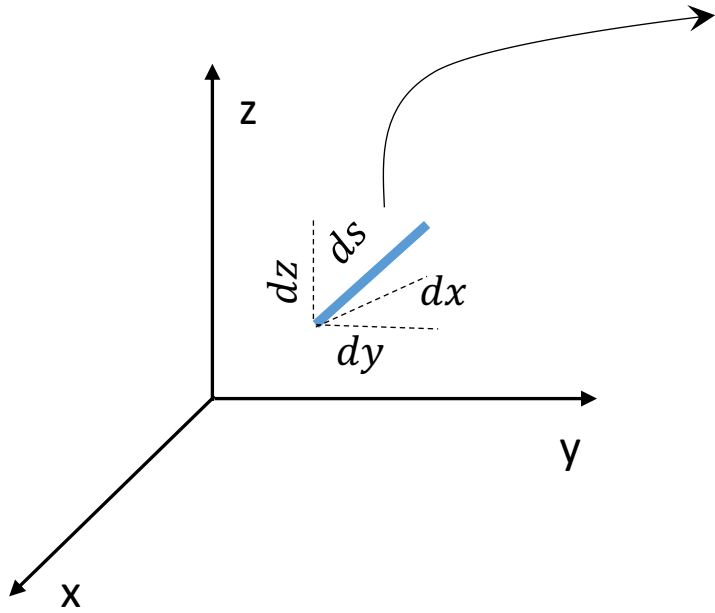
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Boğaziçi Üniversitesi Fizik ve Astronomi Kulübü (April 29, 2023)

FLAT SPACE

3Dtwo events separated by a distance ds

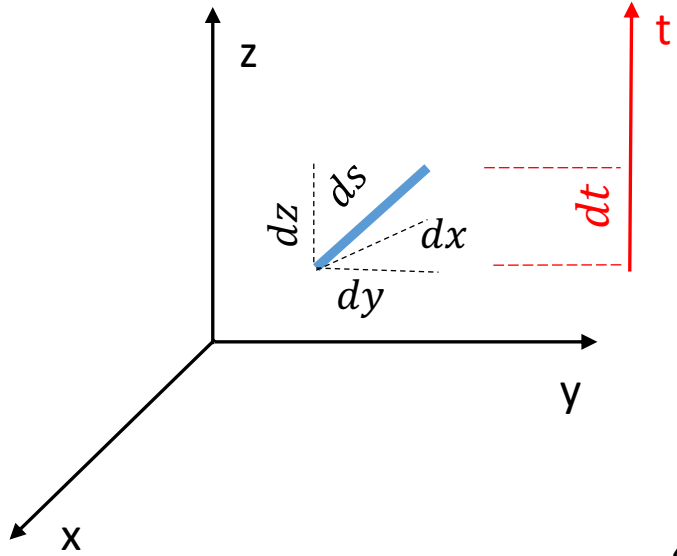
$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$$

space is flat

two events can have zero separation ($ds = 0$)
if they coincide ($dx = 0, dy = 0, dz = 0$)

FLAT SPACETIME

4D



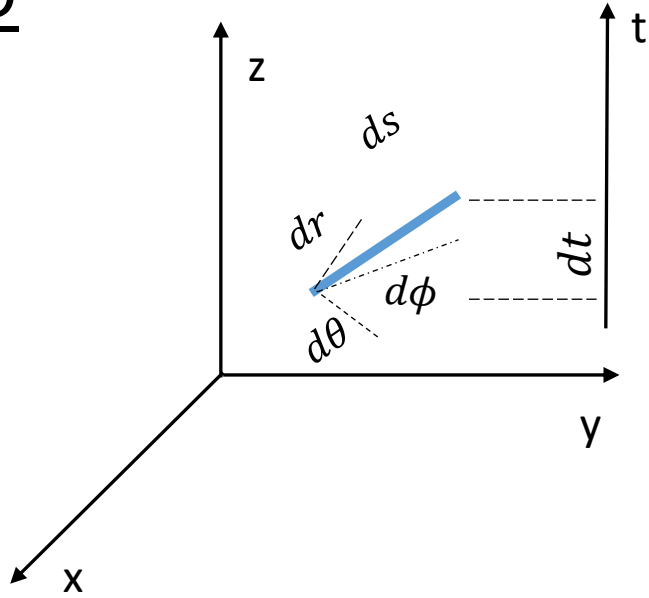
$$(ds)^2 = -(cdt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

“minus” sign is needed
for light to propagate

two events can have zero separation ($ds = 0$)
even if they do not coincide ($dt \neq 0, dx \neq 0,$
 $dy \neq 0, dz \neq 0$)

spacetime is flat

FLAT SPACETIME

4D

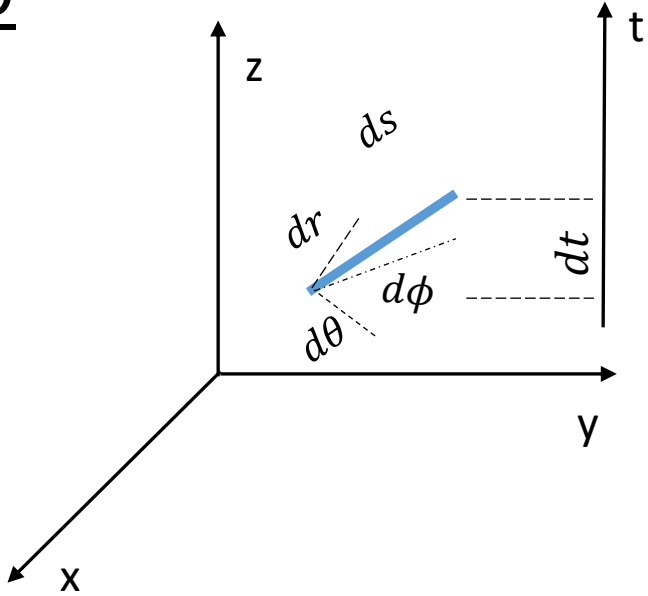
in spherical coordinates:

$$(ds)^2 = -(cdt)^2 + (dr)^2 + r^2((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

spacetime is flat
(coordinate change has no effect)

FLAT DILATED SPACETIME

4D



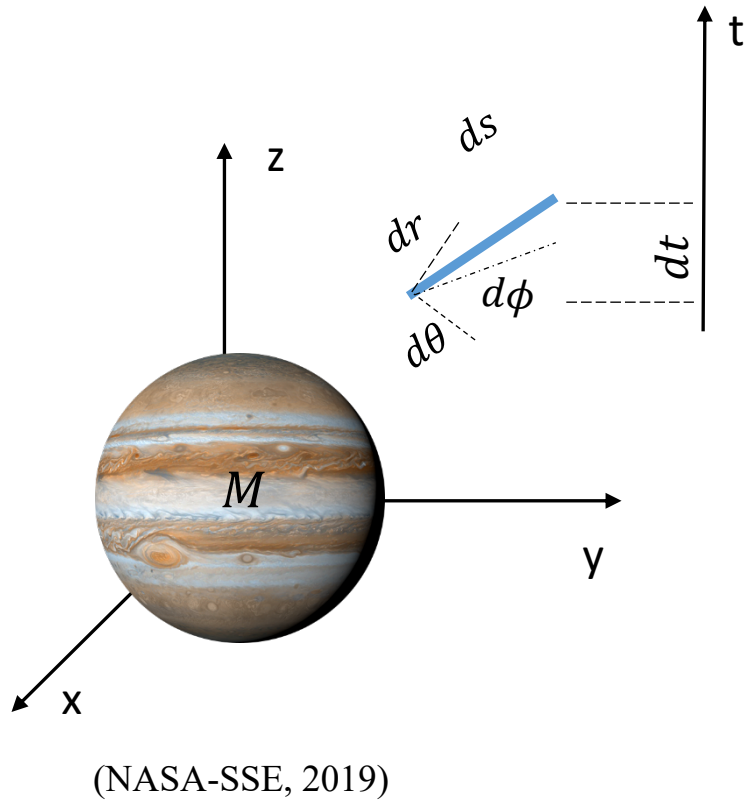
in an environment with intrinsic speed v_{int} :

$$(ds)^2 = -\left(1 - \frac{v_{int}^2}{c^2}\right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{int}^2}{c^2}} + r^2((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

dilated time

contracted distance

CURVED SPACETIME



in the presence of a body (Jupiter) of mass M , the escape speed v_{es} turns out to be the most natural intrinsic speed ($v_{int} = v_{es}$):

➤ escape speed from the body:

$$\frac{1}{2} m v_{es}^2 = \frac{G_N M m}{r} \implies v_{es}^2 = \frac{2 G_N M}{r}$$

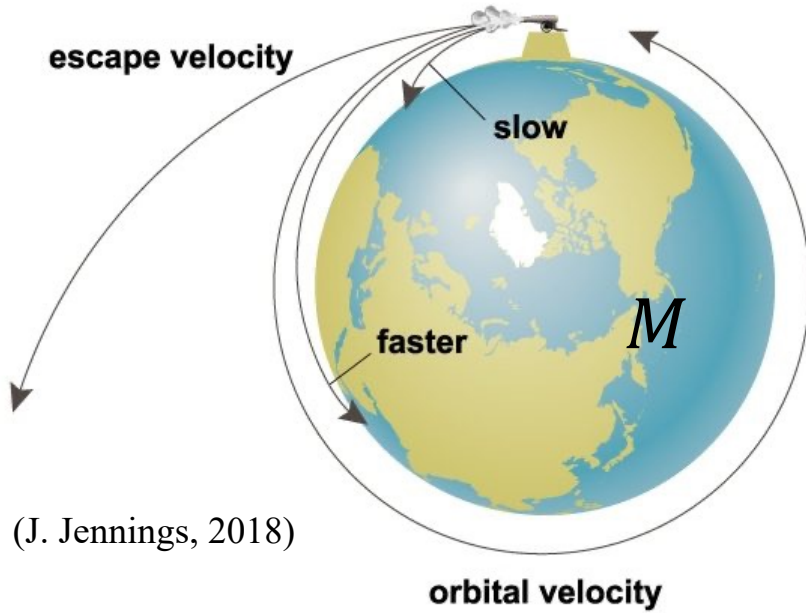
➤ distance between two points outside the body:

$$(ds)^2 = - \left(1 - \frac{v_{es}^2}{c^2} \right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{es}^2}{c^2}} + r^2 ((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

gravitational
time dilation

gravitational
space contraction

CURVED SPACETIME



(J. Jennings, 2018)

➤ Schwarzschild radius:

$$\frac{v_{es}^2}{c^2} = \frac{2G_N M}{rc^2} \equiv \frac{r_s}{r} \implies r_s = \frac{2G_N M}{c^2}$$

➤ ds can be expressed in terms of Schwarzschild radius r_s :

$$(ds)^2 = -\left(1 - \frac{r_s}{r}\right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

- $r_s(\text{Sun}) = 3 \text{ km}$
- $r_s(\text{Jupiter}) = 3 \text{ m}$
- $r_s(\text{Earth}) = 10 \text{ mm}$

EINSTEIN GRAVITY

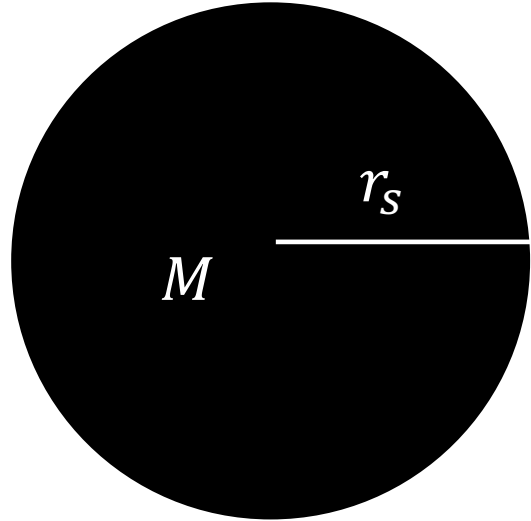
- distance ds for the escape speed turns out to be the solution of the Einstein equations for gravity of a compact body of mass M :

gravity theory	solution of Einstein equations
Einstein gravity action: $\int d(\text{Vol})_4 \left(\frac{R}{16\pi G_N} - V_0 \right)$	Schwarzschild solution (zero vacuum energy $V_0 = 0$) : $(ds)^2 = - \left(1 - \frac{r_s}{r} \right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 ((d\theta)^2 + \sin^2 \theta (d\phi)^2)$

(K. Schwarzschild, 1916 arXiv:physics/9905030)

- true measure of curving is the Riemann curvature:

$$(\text{Riemann curvature})^2 = \frac{12 r_s^2}{r^6}$$

BLACK HOLE

- if the body M is too massive to require an escape speed bigger than the speed of light:

$$v_{es} > c$$

(or $r < r_s$) then a **black hole** forms:

- $r = r_s \Rightarrow$ event horizon (the point of no return)
- $r = r_s \Rightarrow$ event horizon (infinite time dilation)
- $r < r_s \Rightarrow$ black hole interior (cannot be probed from outside)
- $r < r_s \Rightarrow$ black hole interior (effectively: time \Leftrightarrow radial distance)

PHOTON MOTION AROUND BLACK HOLE

➤ Photon: $(ds)^2 = 0 \Rightarrow \left(\frac{ds}{d\tau}\right)^2 = 0$

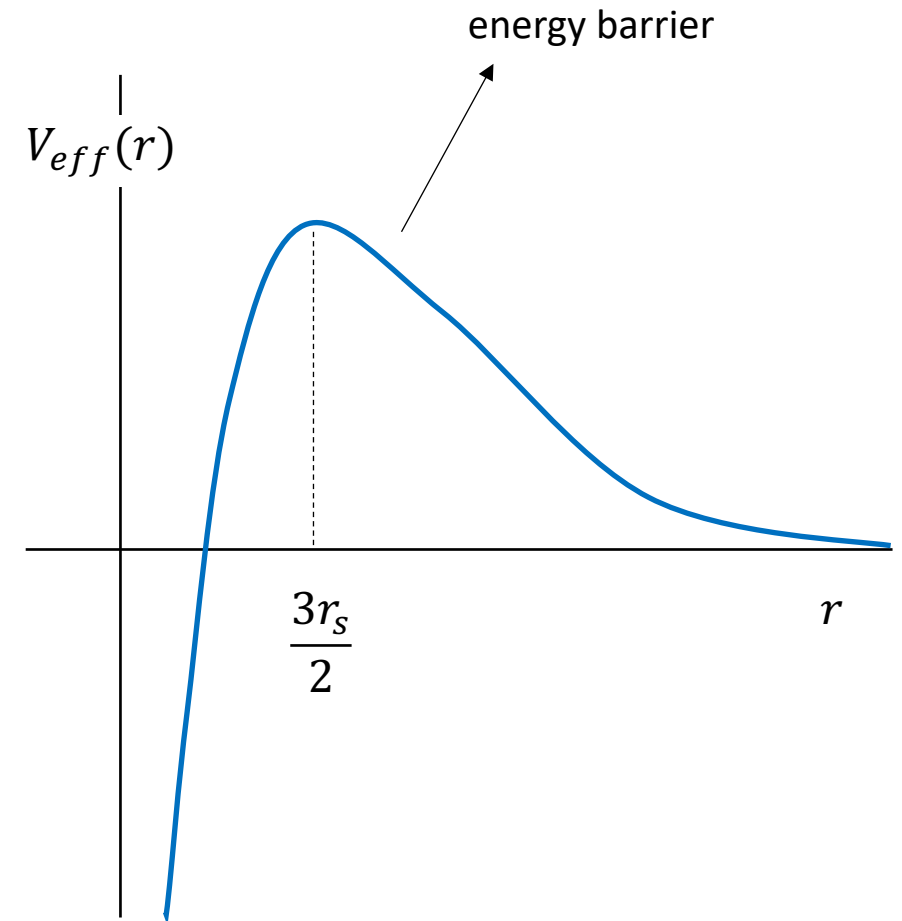
➤ Azimuthal plane ($\theta = \frac{\pi}{2}$): $-\left(1 - \frac{r_s}{r}\right) c^2 \dot{t}^2 + \frac{\dot{r}^2}{1 - \frac{r_s}{r}} + r^2 \dot{\phi}^2 = 0$

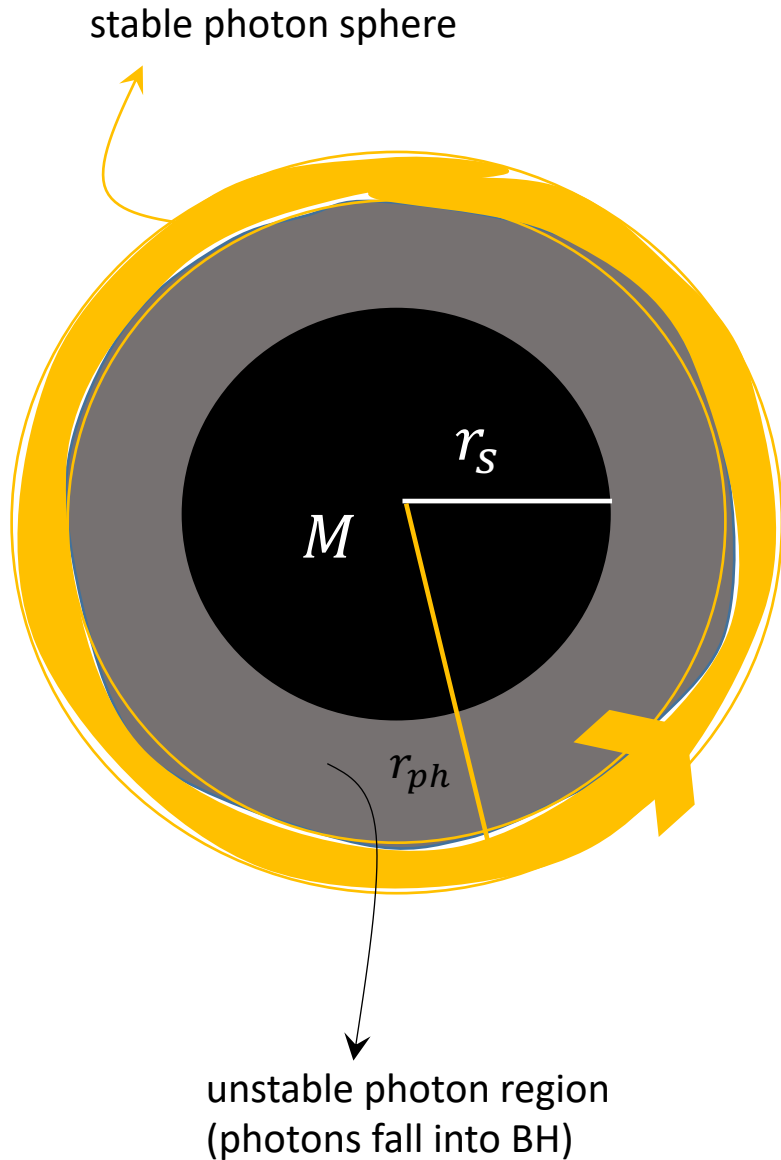
➤ Energy is conserved: $\dot{t} = \frac{\epsilon}{1 - \frac{r_s}{r}}$

➤ Ang. Mom. is conserved: $\dot{\phi} = \frac{\ell}{r^2}$

➤ Photon is a unit-mass particle: $\frac{\dot{r}^2}{2} + \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\epsilon^2}{2}$

➤ $V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}$



PHOTON SPHERE

- photons orbiting in xy -plane ($r = r_{ph} = \text{constant}$, $\theta = \frac{\pi}{2}$):

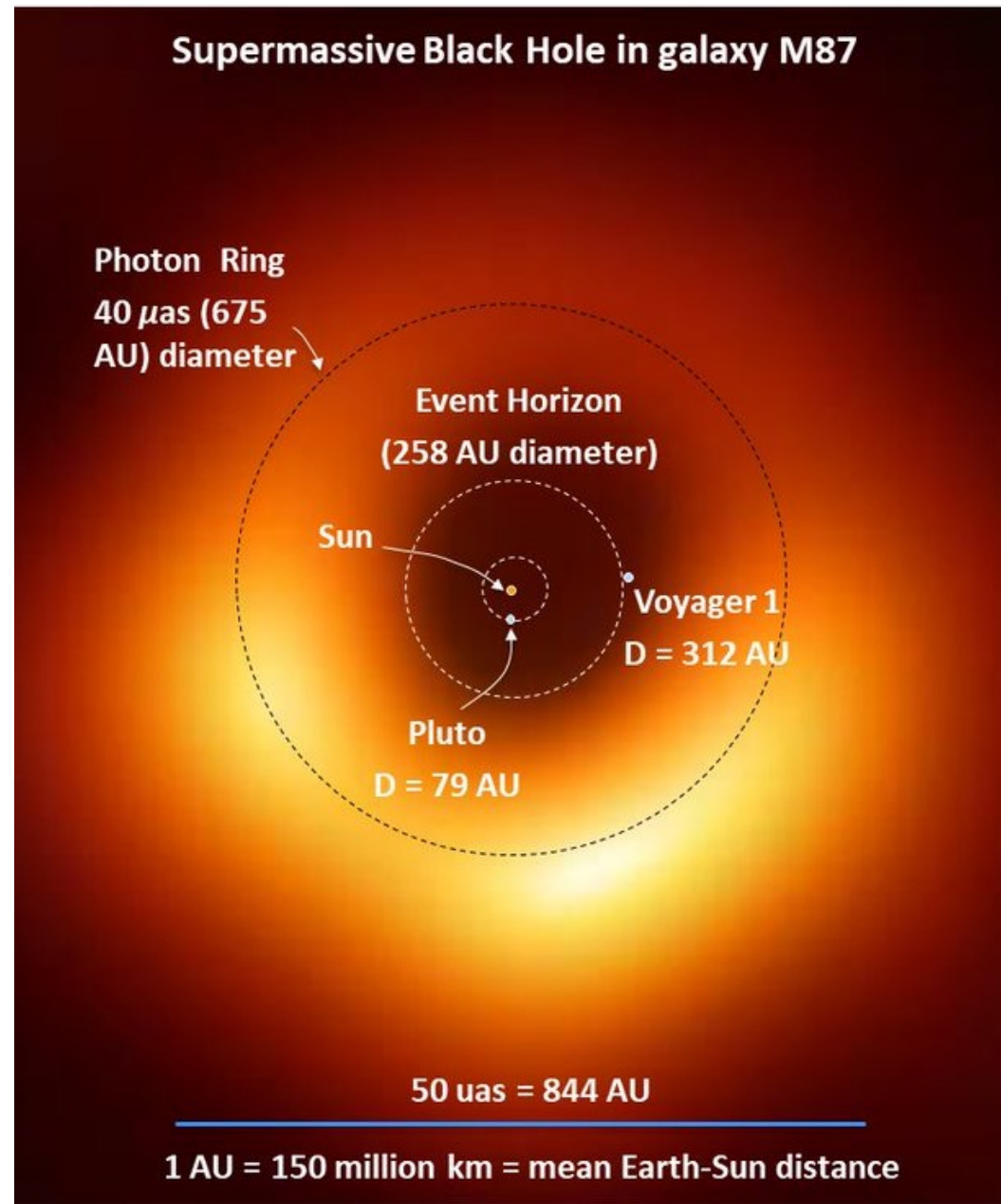
$$ds = 0 \Rightarrow \frac{d\phi}{dt} = \frac{c}{r_\gamma} \left(1 - \frac{r_s}{r_\gamma}\right)^{1/2}$$

$$\text{EoM} \Rightarrow \frac{d\phi}{dt} = \frac{c}{r_\gamma} \left(\frac{r_s}{2r_\gamma}\right)^{1/2}$$

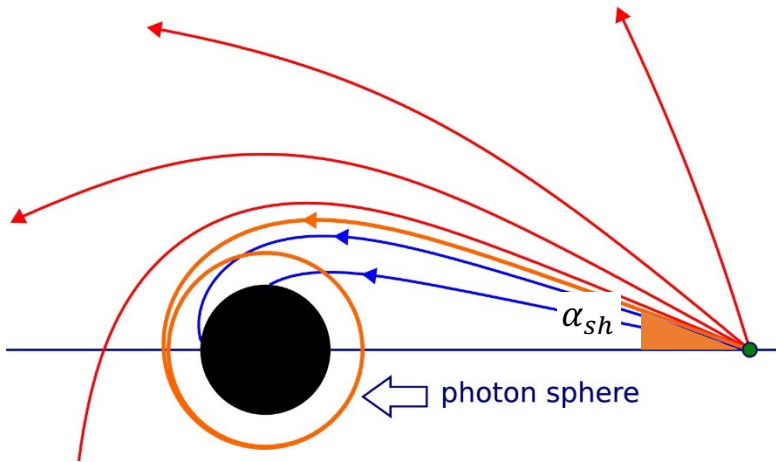
$$\left. \begin{array}{l} \\ \\ \end{array} \right\} r_{ph} = \frac{3}{2} r_s$$

- photon radius $r = r_{ph}$ is the last stable orbit.
- photon radius $r = r_{ph}$ depends on the underlying gravity theory.

Mass (Solar Masses)	6.54 billion
Event Horizon diameter (AU)	258
Distance (Light Years)	55 million



BLACK HOLE SHADOW



(Perlick and Tsupko 2022 *Phys Rep* 947 1)

➤ photons falling within the photon sphere fall into black hole – a large shadow!

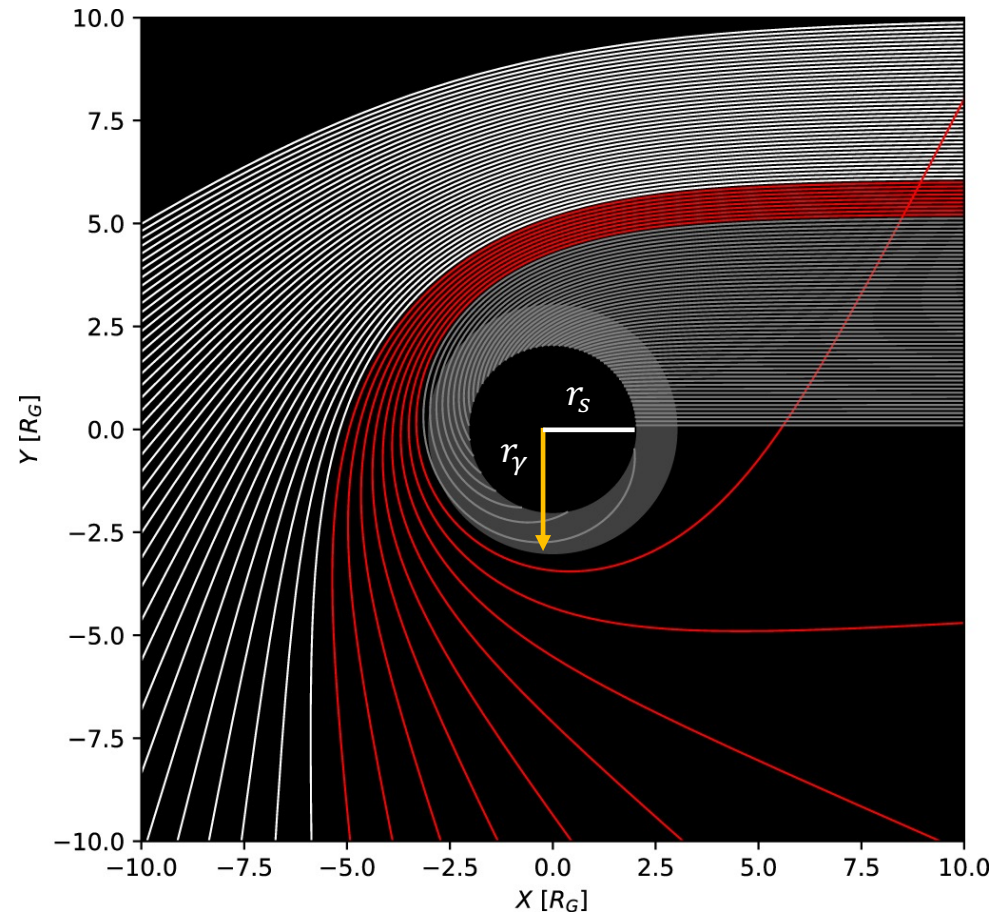
➤ consider a general spacetime:

$$(ds)^2 = -A(r)(c dt)^2 + B(r)(dr)^2 + D(r)((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

➤ shadow is characterized by “gravitational capture angle” \equiv shadow angle α_{sh} :

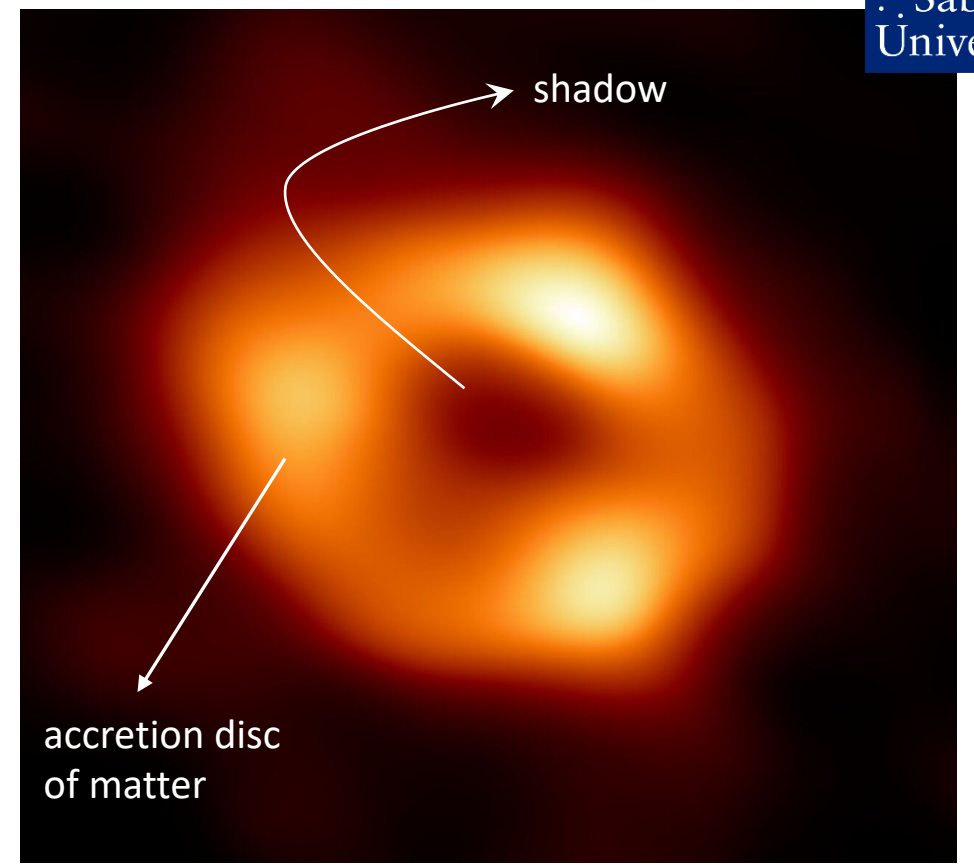
$$\sin^2 \alpha_{sh} = \frac{D(r_{ph}) A(r_o)}{A(r_{ph}) D(r_o)} \xrightarrow{\text{Schwarzschild}} \frac{r_{ph}^2}{1 - \frac{r_s}{r_{ph}}} \times \frac{\left(1 - \frac{r_s}{r_o}\right)}{r_o^2}$$

Distant observer: $\frac{27 r_s^2}{4 r_o^2}$



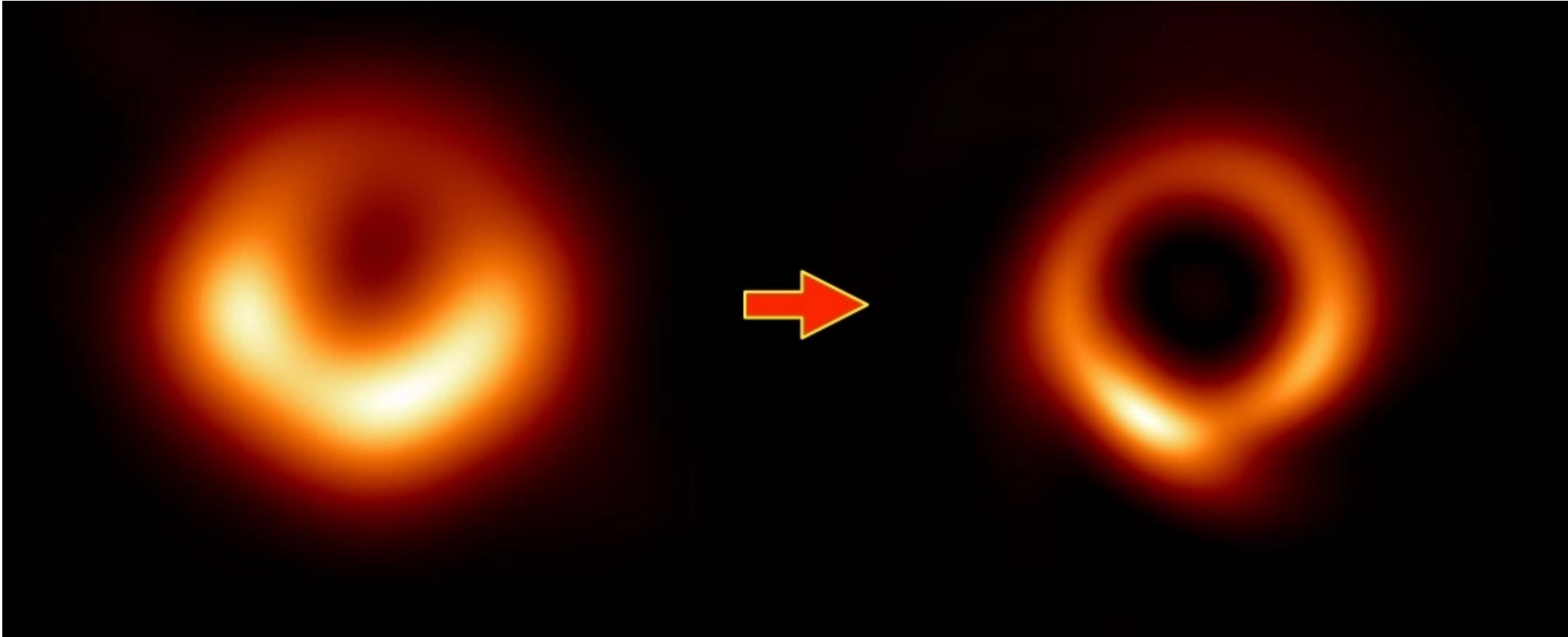
Shadow of a black hole

(Thomas Bronzwaer and Heino Falcke 2021 *ApJ* **920** 155)



Shadow of a black hole

(EHT observation of Sgr. A* in 2019)



Shadow of M87* (2019)

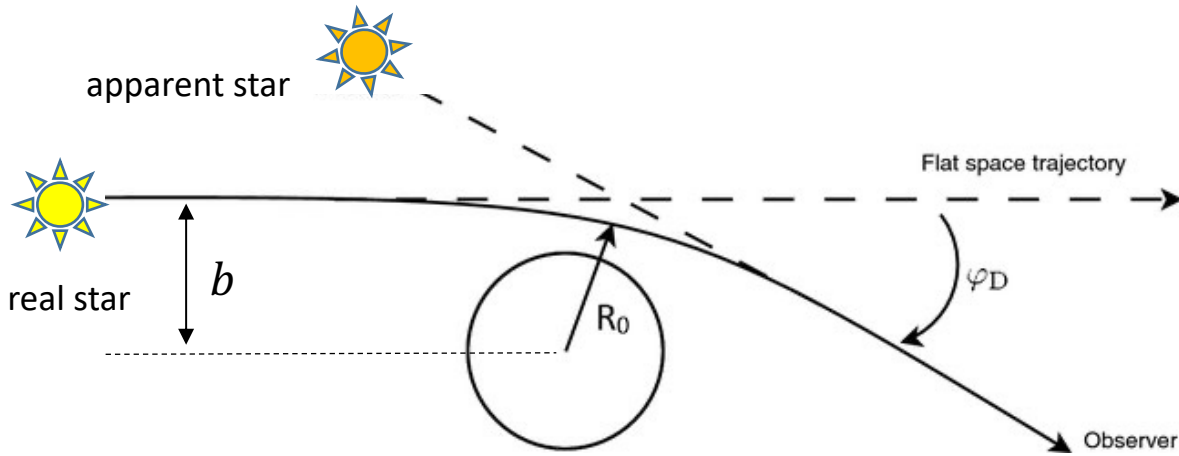
(EHT, 2019)

Shadow of M87* (AI)

(PRIMO, 2023)

(Lia Medeiros et al 2023 ApJL 947 L7)

LIGHT BENDING BY BLACK HOLE

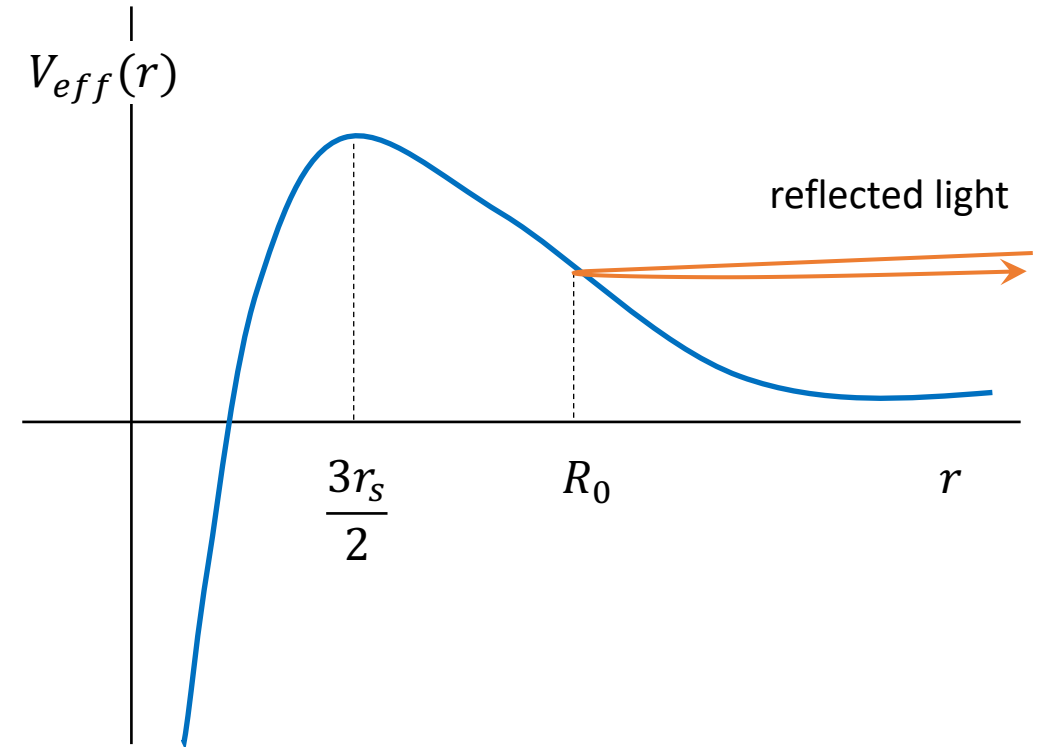


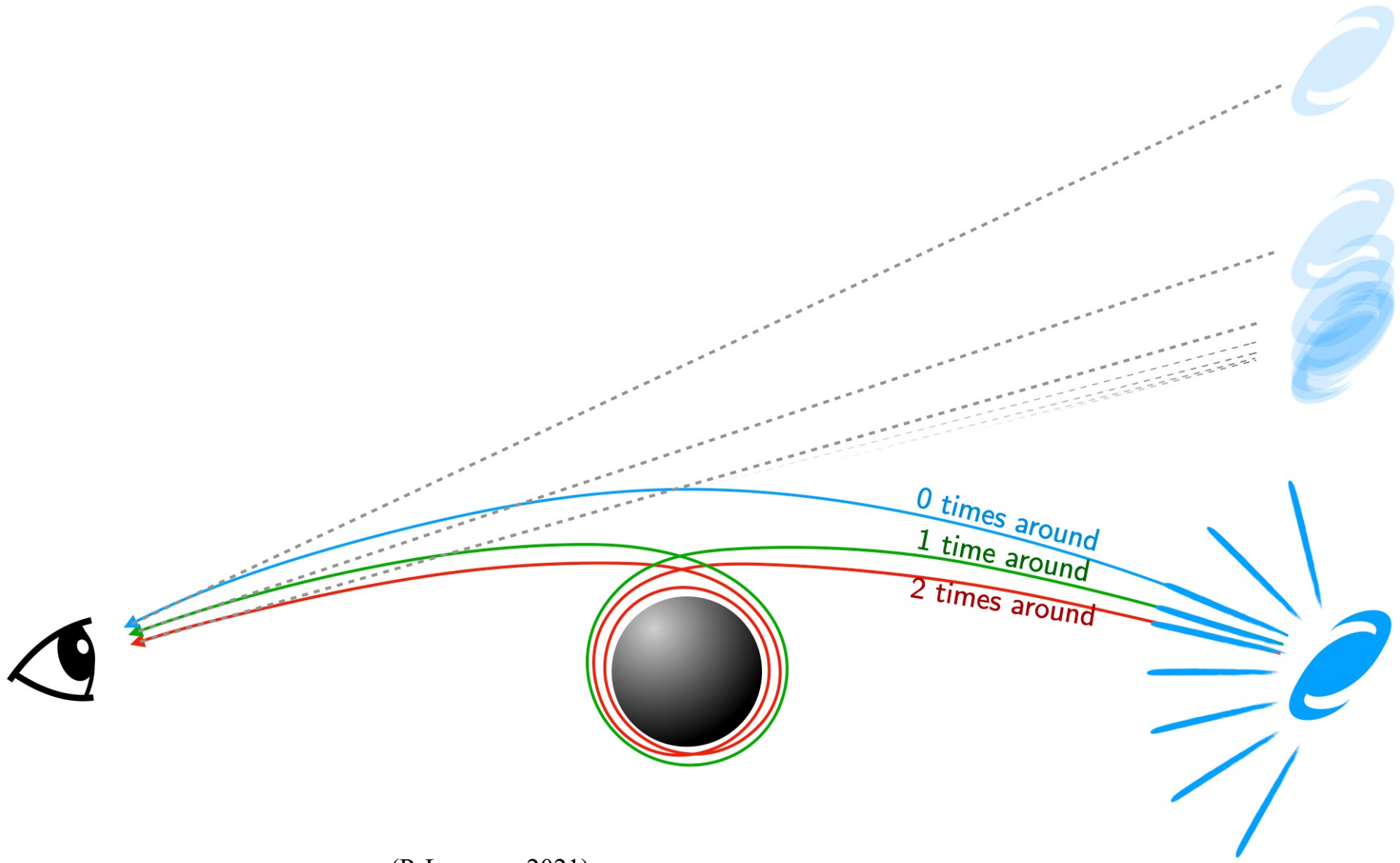
(Burger, D. et al 2018 Gen Relativ Gravit 50, 156)

$$\rightarrow \frac{dr}{d\tau} = \sqrt{\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)}$$

$$\rightarrow \frac{d\phi}{d\tau} = \frac{1}{r^2}$$

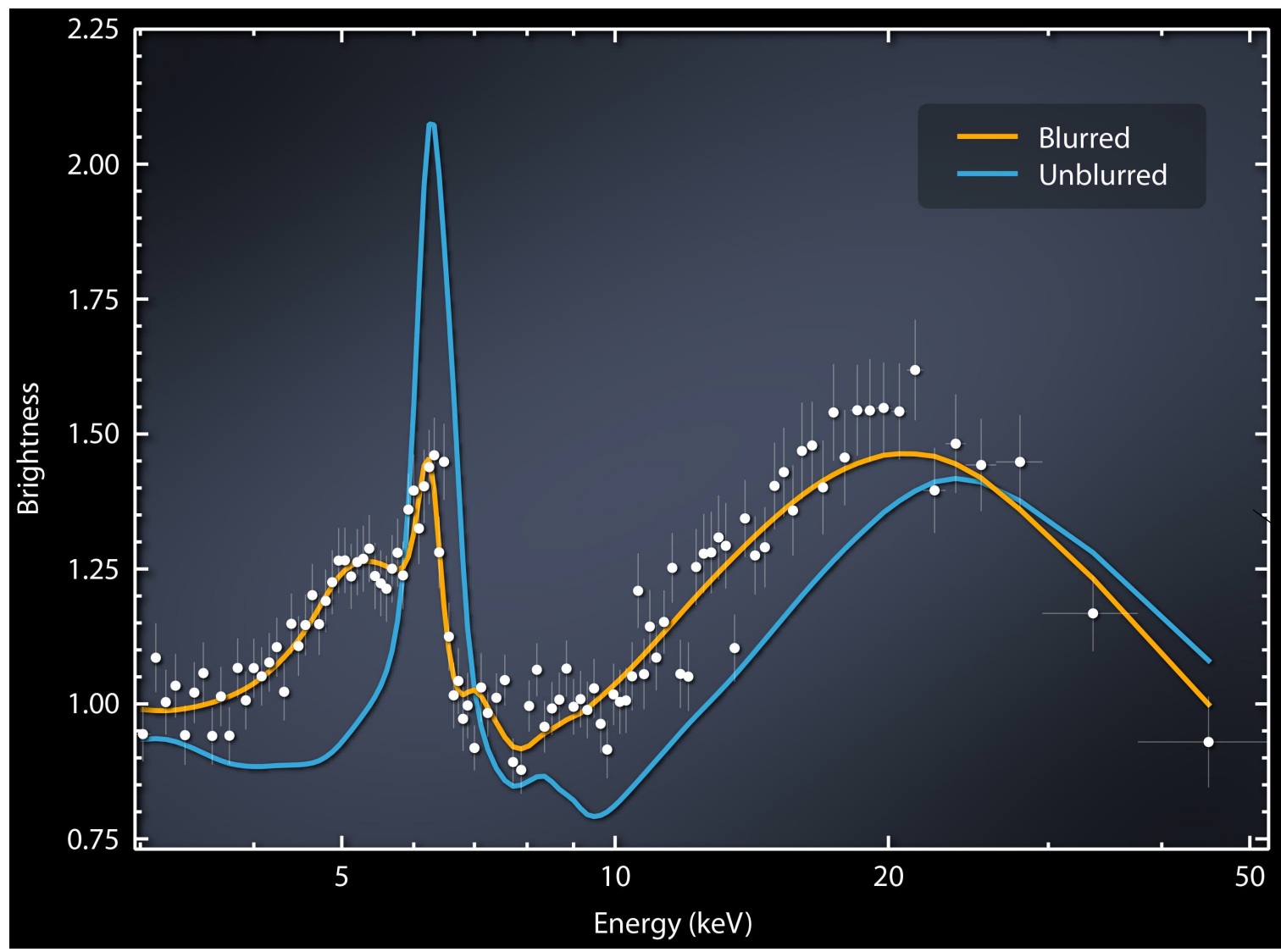
$$\rightarrow \varphi_D = 2 \int_{R_0}^{\infty} \frac{dr}{r^2 \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)\right)^{\frac{1}{2}}} \xrightarrow{\text{small } r_s} \frac{2r_s}{R_0}$$





(P. Laursen, 2021)

bending of light leads to multiple images for objects behind (lensing effect)



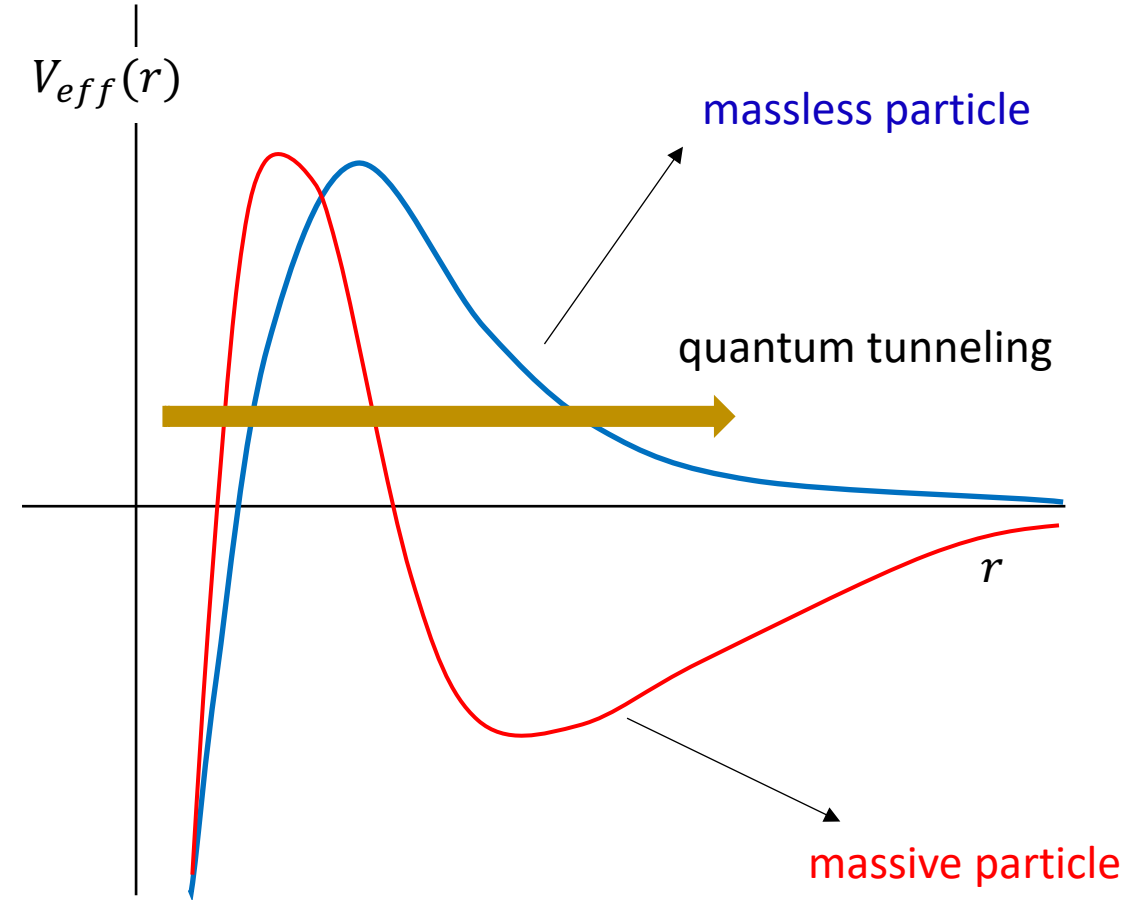
absorption of a corona by
the (spinning) black hole
Markarian (320 mil. lys)

(NASA-NuSTAR, 2014)

- effective potential seen by massless particle (photon):

$$V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}$$

- **potential energy barrier** for massless particles is formed by angular momentum ($1/r^2$) and Schwarzschild radius (r_s/r^3)
- **potential energy barrier** for massive particles involves in addition the Newtonian contribution ($1/r$)
- quantum particles that fell into the black hole can **tunnel out through the barrier**.
- tunneled particles appear as radiation – **the Hawking radiation**.

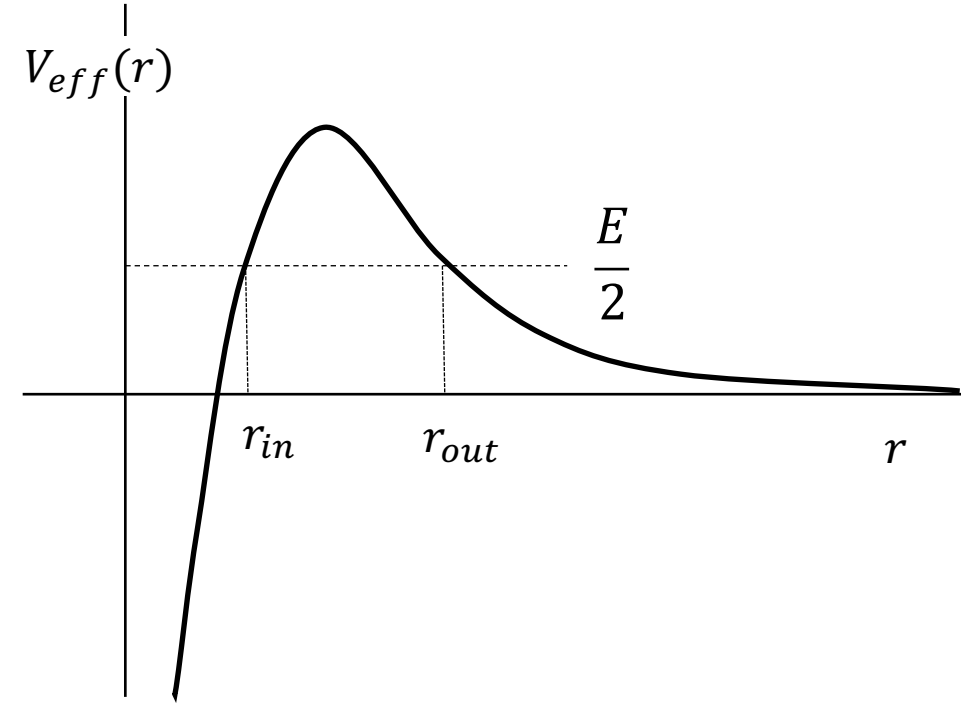


➤ Time it takes to traverse the barrier region:

$$\Delta t = \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - 2V_{eff}(r)}} = \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - \frac{\ell^2}{r^2} + \frac{\ell r_s}{r^3}}} =$$

$$= \int_{r_{in}}^{r_{out}} \frac{r^3 dr}{\sqrt{Er^6 - \ell^2 r^3 + r_s \ell r^2}} = 2\pi i \times \sum_k \text{Res}_k$$

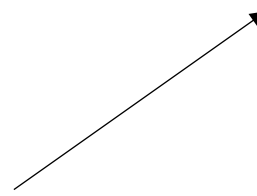
$$\xrightarrow{E \sim \ell \sim 1} 2\pi i \times (r_{out} - r_{in}) \simeq 2\pi i \times 2 r_s$$

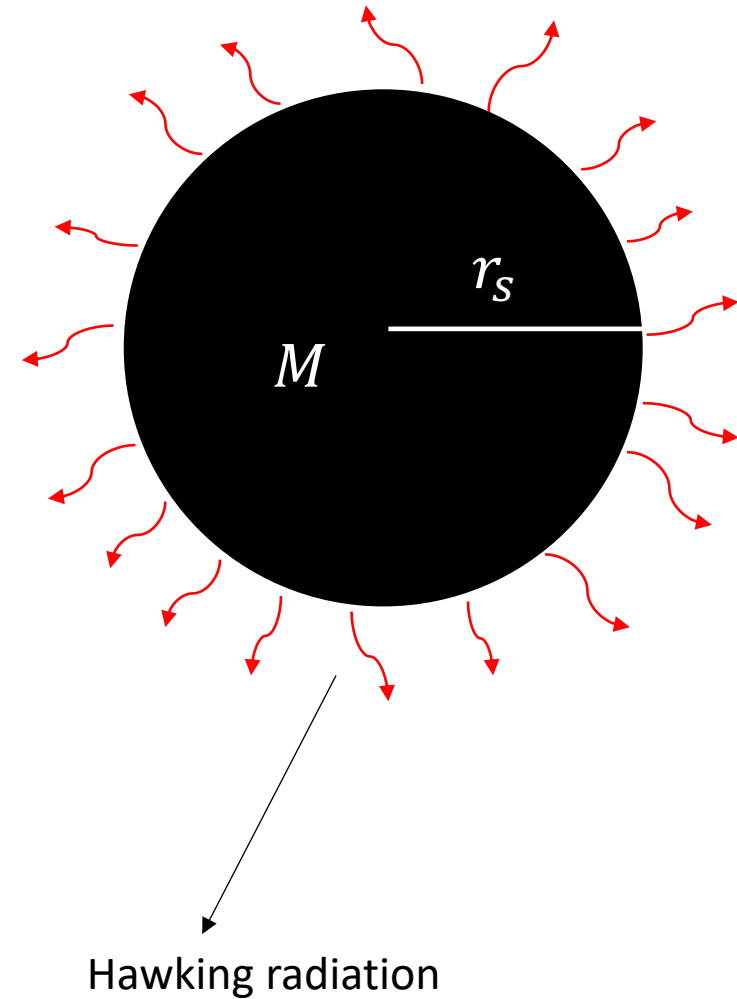


➤ Wavefunction: $\psi \propto e^{iE\Delta t} \simeq e^{-4\pi r_s E}$

➤ Black body spectrum: $T_H \simeq \frac{1}{4\pi r_s} \equiv \frac{\hbar c^3}{8\pi k_B G_N M}$

Hawking temperature





Emitted power (photons only):

$$P = \frac{\hbar c^6}{15360\pi G_N^2 M^2}$$

Evaporation time (photons only):

$$t_{\text{eva}} = \frac{5120 \pi G_N^2 M^3}{\hbar c^4} \approx 10^{67} \text{ years} \times \left(\frac{M}{M_{\text{Sun}}}\right)^3$$

QUANTUM TUNNELING AND HAWKING RADIATION

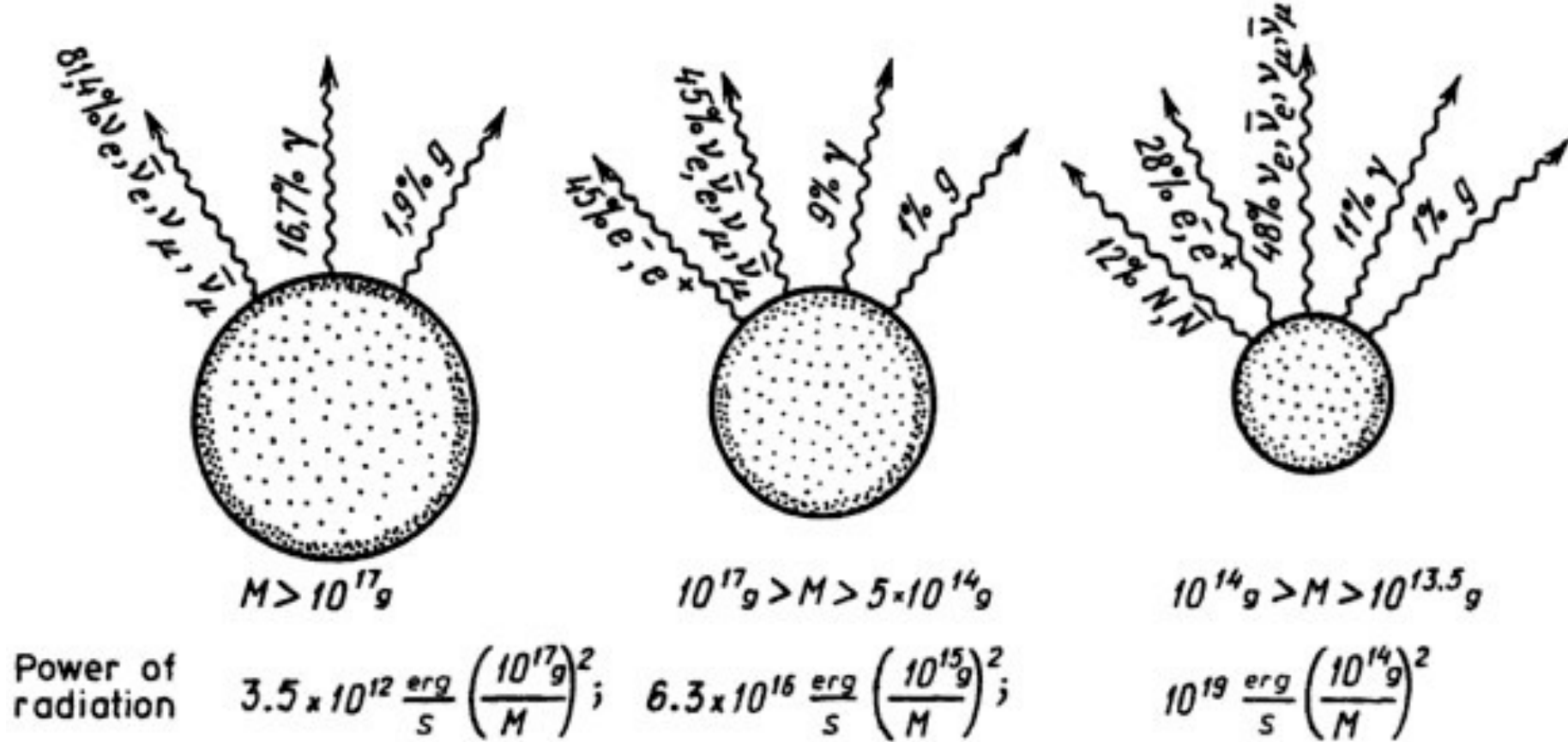


Figure 10.7: Quantum decay of a non-rotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν) and other elementary particles are given in percent of the total number of particles emitted by black holes of different masses.

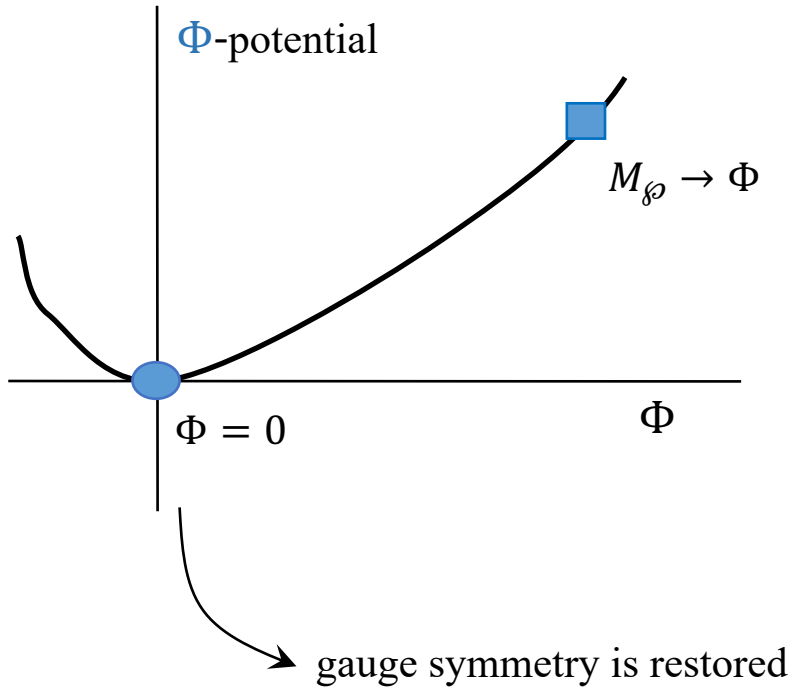
- Quantum field theories (QFTs) exist in flat spacetime and make sense with an ultraviolet (UV) cutoff scale.
- A gauge symmetry breaking cutoff can be:
 - either the **mass of a vector particle** (like Z boson or W boson masses)
 - or **not the mass of a particle** (like QCD scale or Newton's constant)
- A UV cutoff which is the mass of a vector boson **respects translation symmetry** and the gauge symmetries it breaks can be restored by introducing the **Higgs field** (Higgs mechanism).
- A UV cutoff which is not the mass of a particle **breaks translation symmetry** and the gauge symmetries it breaks can be restored by introducing **affine curvature** (Symmergence mechanism).
 - affine curvature leads to the **usual curvature** dynamically (holographically)
 - emergence of general relativity necessitates **new particles** beyond the new ones.

SYMMERGENT GRAVITY

Translation-respecting UV Cutoff M_{\wp}

$$M_{\wp}^2 \text{Tr}[V_{\mu} \eta^{\mu\nu} V_{\nu}] \quad \Phi^{\dagger} V_{\mu} \eta^{\mu\nu} V_{\nu} \Phi \quad \eta^{\mu\nu} (D_{\mu} \Phi)^{\dagger} (D_{\nu} \Phi)$$

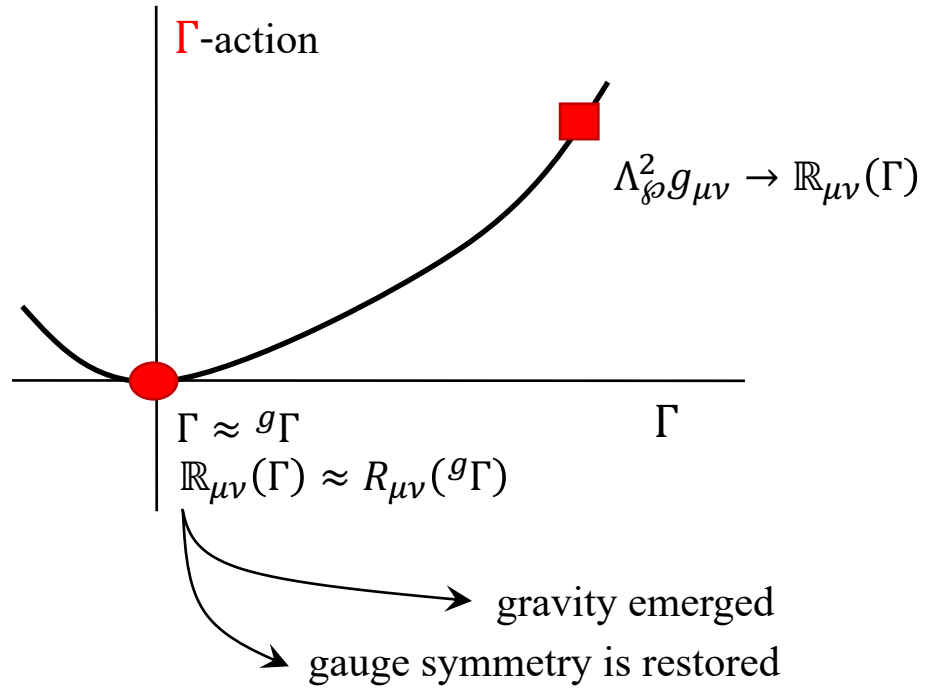
Higgs mechanism:



Translation-breaking UV Cutoff Λ_{\wp}

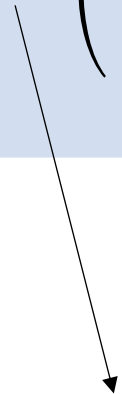
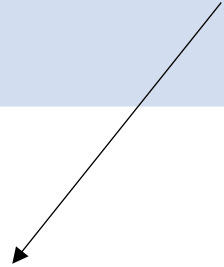
$$\Lambda_{\wp}^2 V_{\mu} \eta^{\mu\nu} V_{\nu} \quad V_{\mu} \mathbb{R}^{\mu\nu}(\bar{\Gamma}) V_{\nu} \quad V_{\mu} (\mathbb{R}^{\mu\nu}(\Gamma) - R^{\mu\nu}(g\Gamma)) V_{\nu}$$

Symmergence mechanism:



ASYMPTOTICALLY-FLAT SYMMERGENT BLACK HOLE

gravity theory	solution of Einstein equations
Symmergent gravity: $\int d(\text{Vol})_4 \left(\frac{R}{16\pi G_N} + \frac{1}{96\pi\gamma} R^2 \right)$	$(ds)^2 = e^{\varphi(r)} \left(- \left(1 - \frac{r_s}{r}\right) c^2 (dt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 ((d\theta)^2 + \sin^2 \theta (d\phi)^2) \right)$



Schwarzschild solution

$$\gamma = - \frac{64\pi}{3(n_B - n_F)}$$

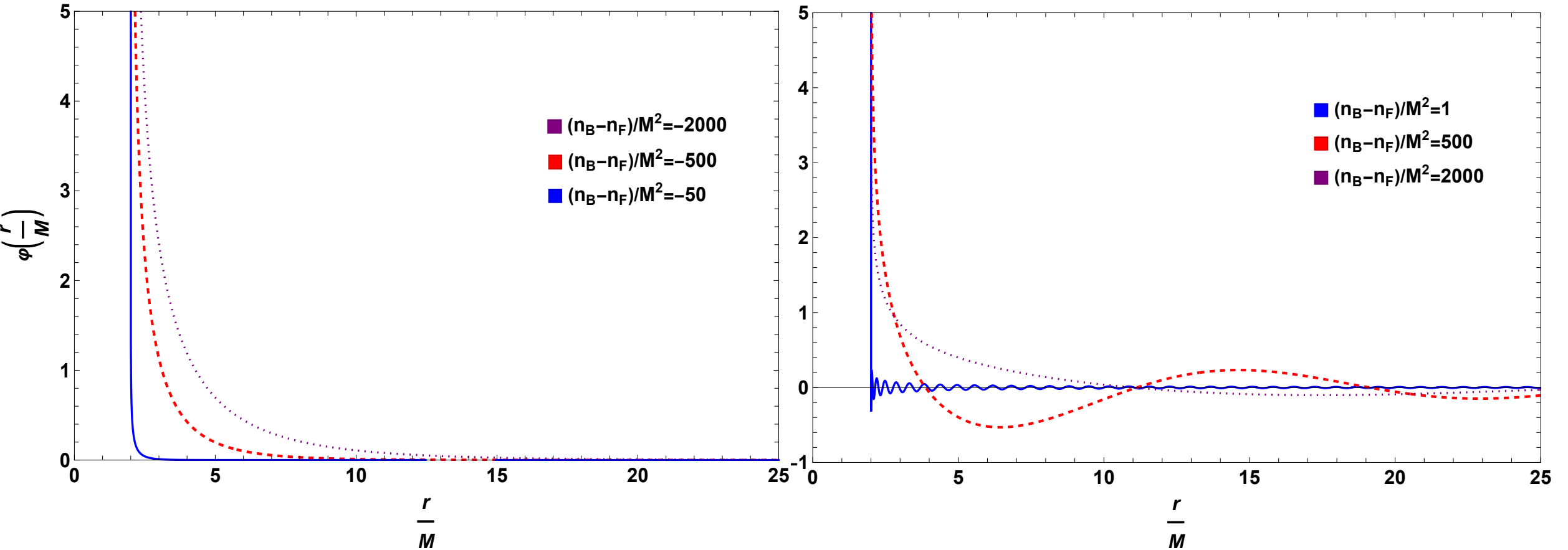


number difference between bosonic and fermionic degrees of freedom

$$\frac{d}{dr} \left((r^2 - rr_s) \frac{d\varphi(r)}{dr} \right) = -\gamma r^2 \varphi(r)$$

(H. K. Nguyen, 2022 arXiv:2211.07380)
 (B. Pulice, R. Pantig, A. Övgün, DD, work in progress)

ASYMPTOTICALLY-FLAT SYMMERGENT BLACK HOLE



- The conformal factor $\varphi(r)$ diverges at the Schwarzschild event horizon $r = r_s \equiv 2M$ and gets diminished exponentially (power-law periodically) at large r for $n_B - n_F < 0$ ($n_B - n_F < 0$).

ASYMPTOTICALLY-FLAT SYMMERGENT BLACK HOLE

it is convenient to write the symmergent metric as:

$$(ds)^2 = -A(r) (cdt)^2 + \frac{(dr)^2}{B(r)} + C(r) ((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

time-dilation potential:

$$A(r) = e^{\varphi(r)} \left(1 - \frac{r_s}{r}\right)$$

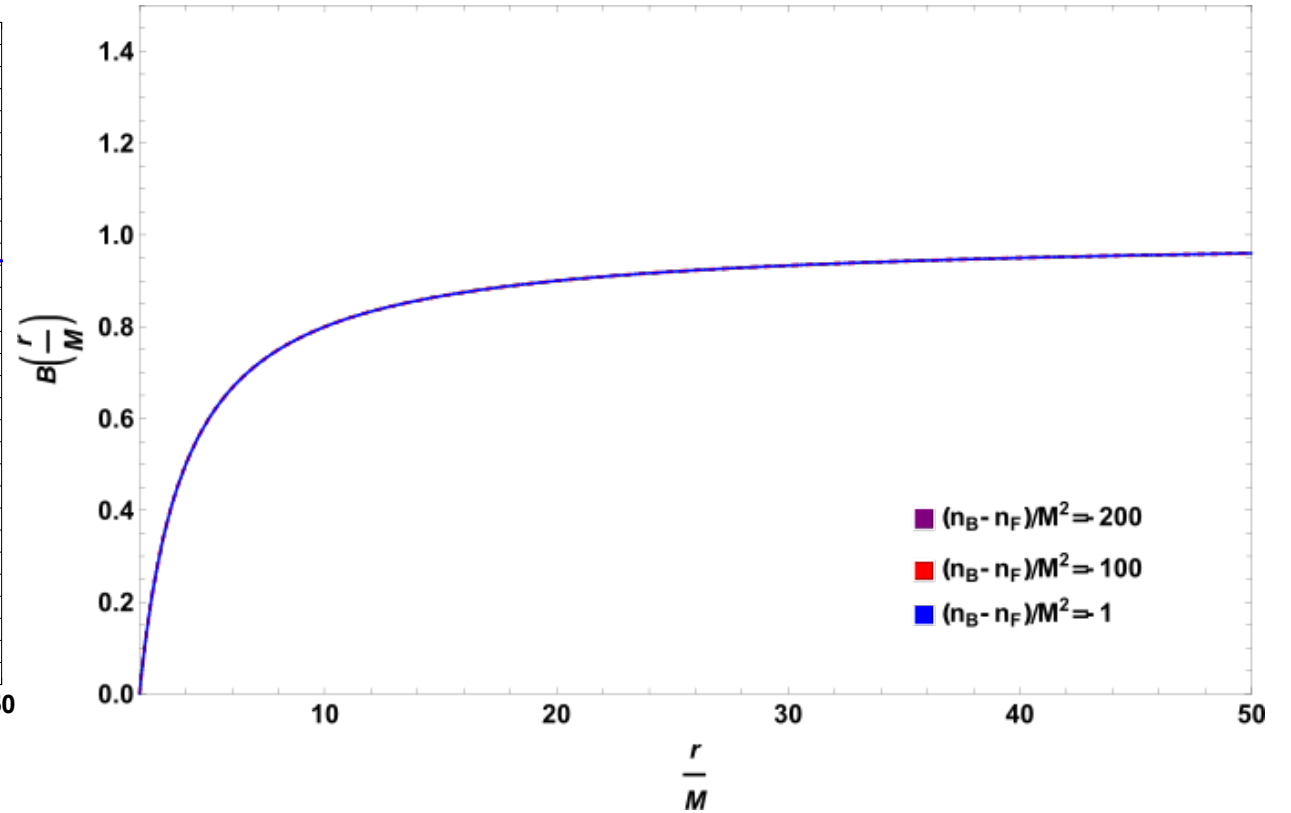
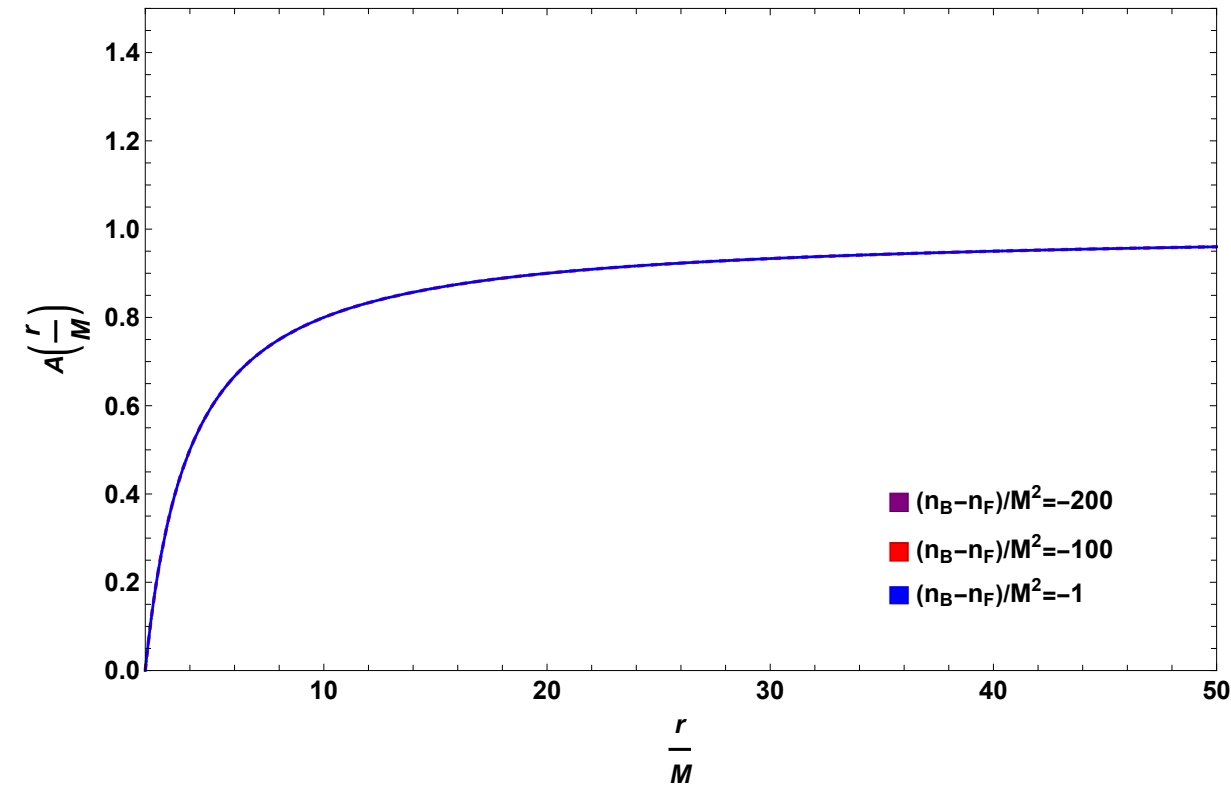
angular potential:

$$C(r) = e^{\varphi(r)} r^2$$

space-contraction potential:

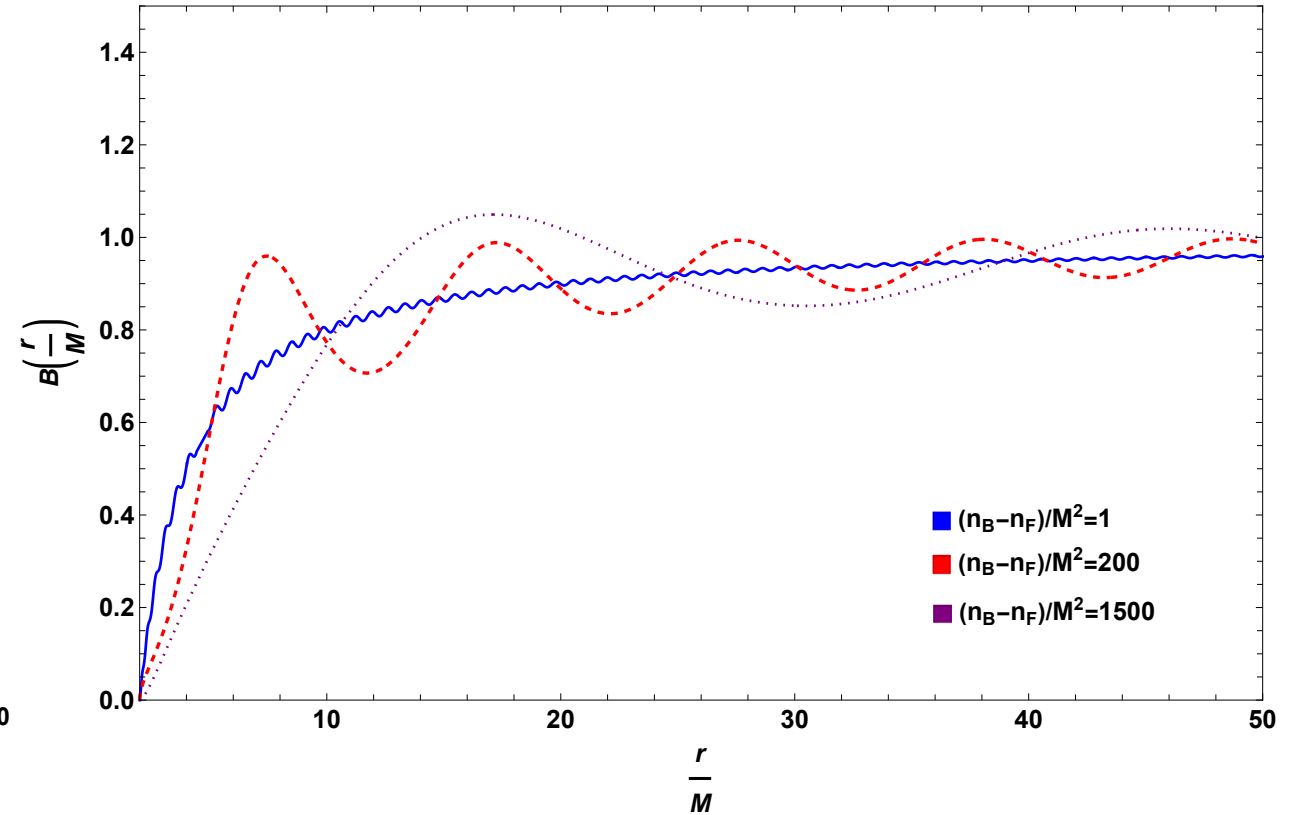
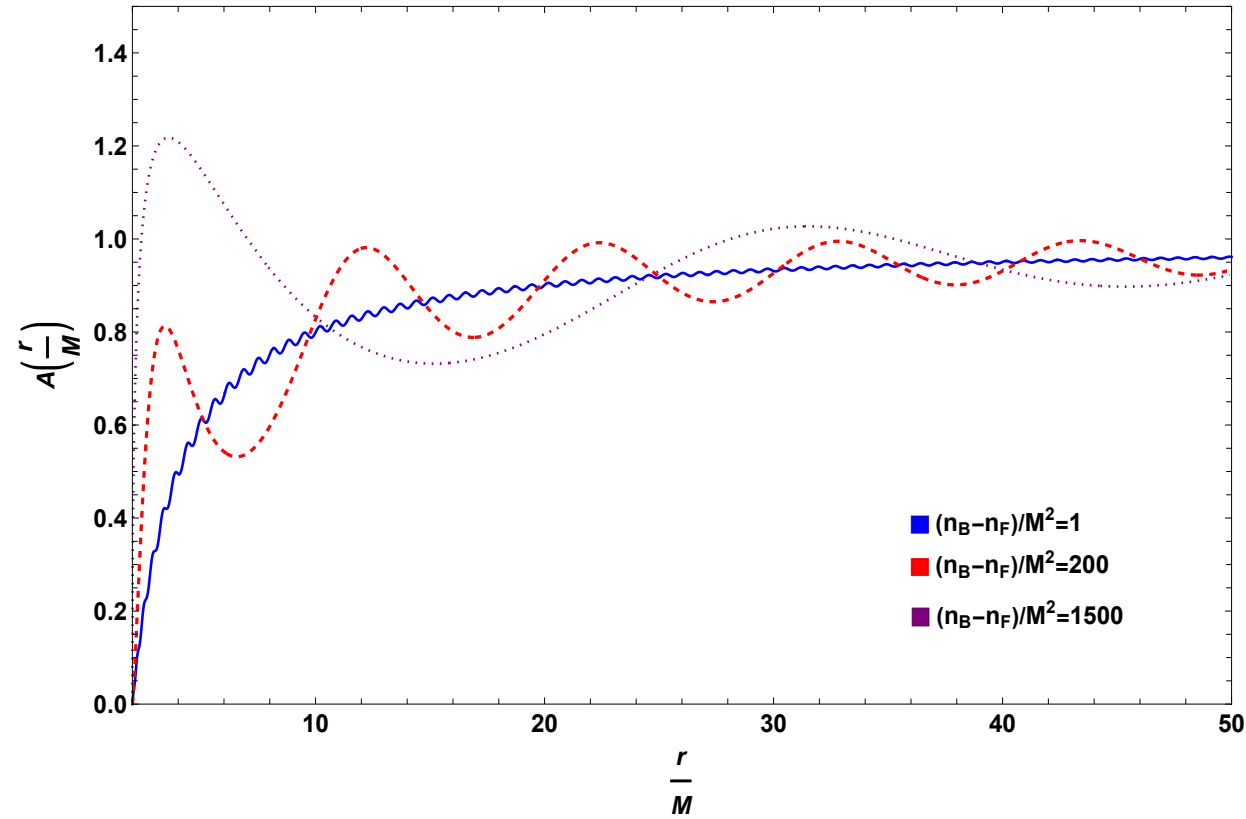
$$B(r) = e^{-\varphi(r)} \left(1 - \frac{r_s}{r}\right)$$

ASYMPTOTICALLY-FLAT SYMMERGENT BLACK HOLE



- The metric potentials $A(r)$ and $B(r)$ approach to the flat spacetime limit of $A(r) = B(r) = 1$ at large r for $n_B - n_F < 0$. The approach is exponential and different $n_B - n_F$ values are hard to distinguish observationally.

ASYMPTOTICALLY-FLAT SYMMERGENT BLACK HOLE



- The metric potentials $A(r)$ and $B(r)$ approach to the flat spacetime limit of $A(r) = B(r) = 1$ at large r for $n_B - n_F > 0$. The approach is power-law/periodic and different $n_B - n_F$ values are easier to distinguish observationally.

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

gravity theory	solution of Einstein equations
Symmergent gravity action: $\int d(\text{Vol})_4 \left(\frac{R}{16\pi G_N} - V_0 + \frac{c_O}{16} R^2 - \frac{1-a}{(8\pi G_N)^2 c_O} \right)$	Schwarzschild-dS/AdS solution ($V_0 = 0$): $(ds)^2 = h(r)(cdt)^2 + \frac{(dr)^2}{h(r)} + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)$

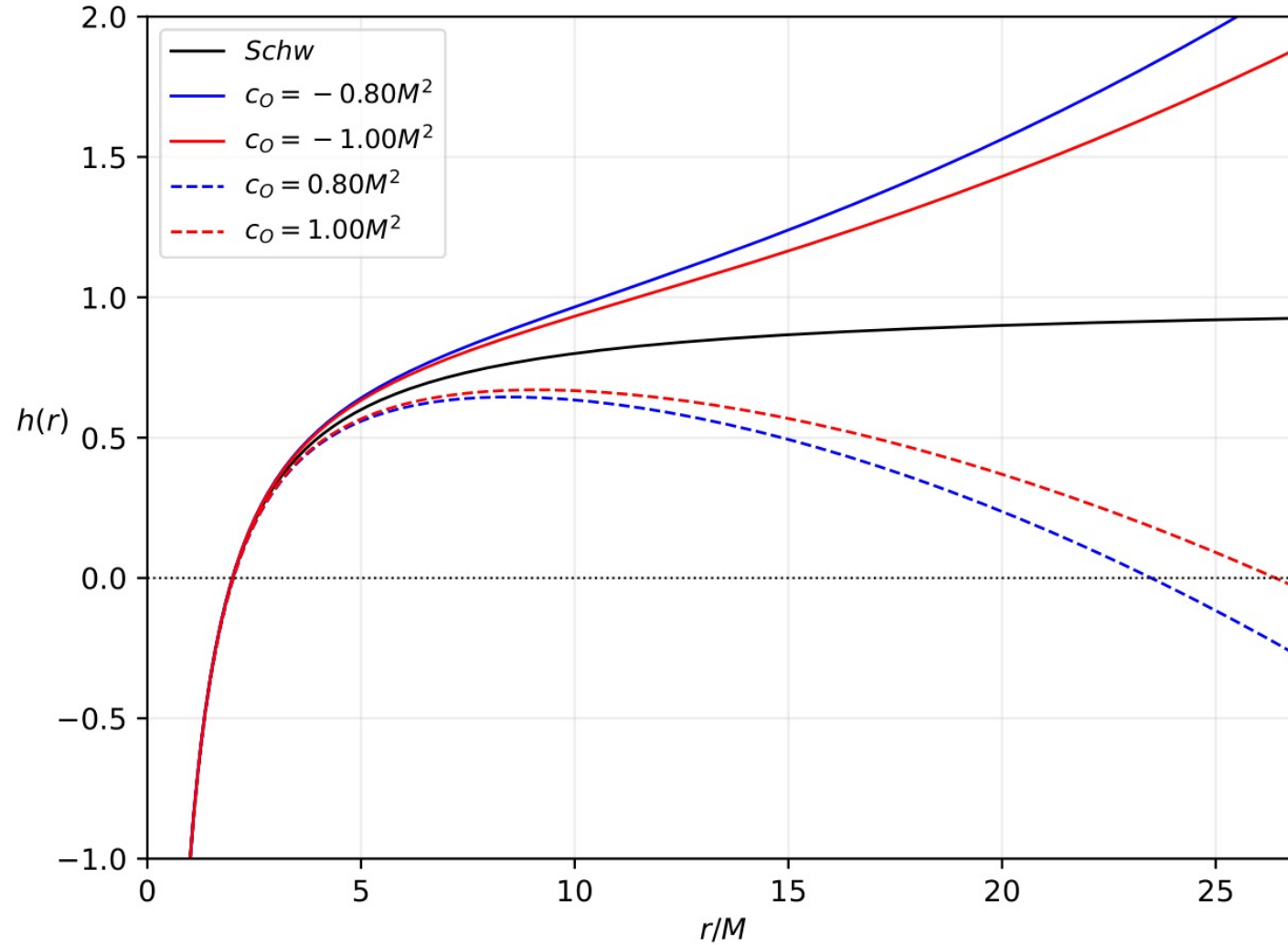
loop-induced quadratic curvature constant:

$$c_O = \frac{n_B - n_F}{248\pi^2}$$

a : an $O(1)$ constant parametrizing the symmergent vacuum energy

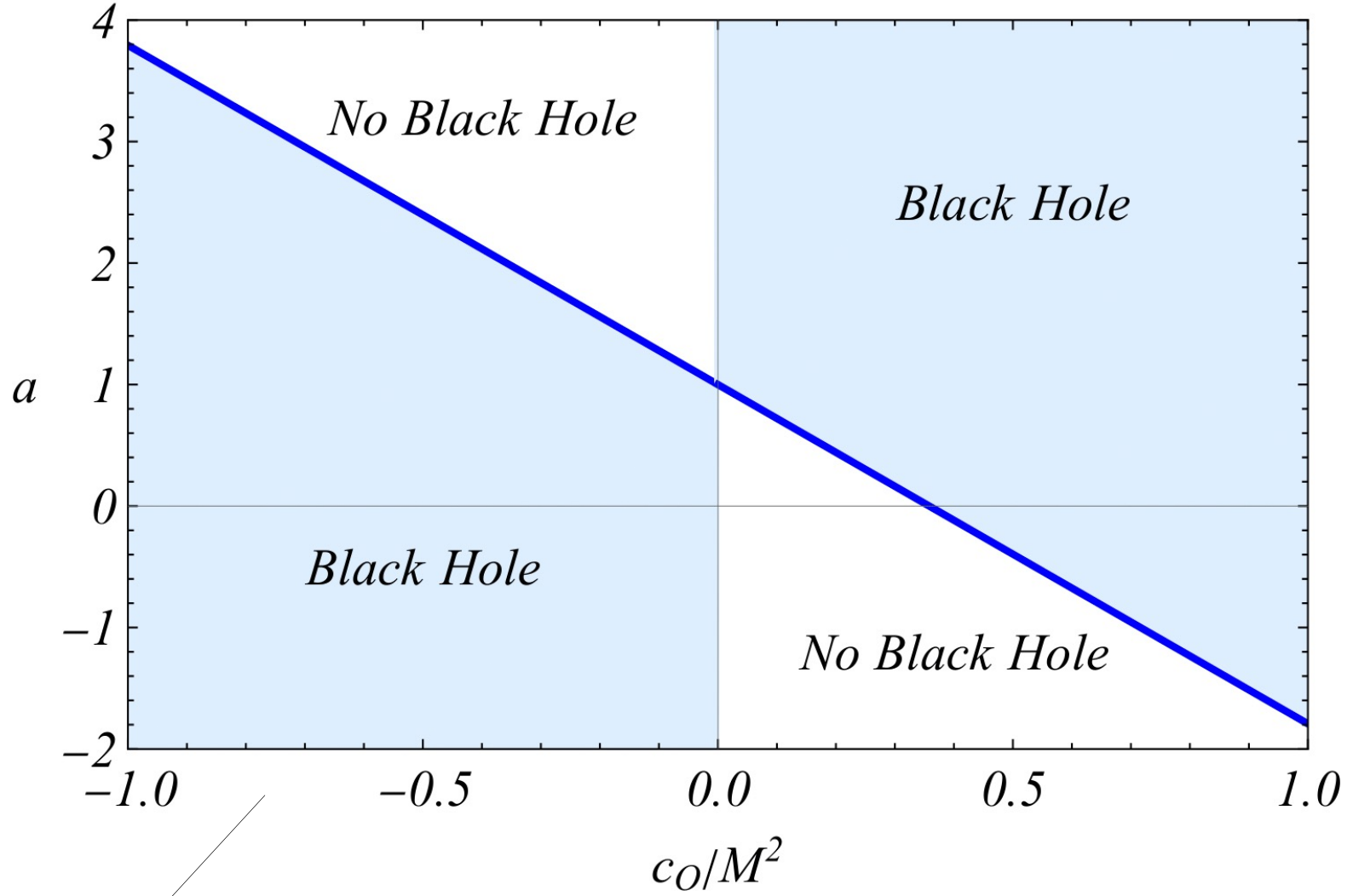
$$h(r) = - \left(1 - \frac{r_s}{r} - \frac{(1-a)r^2}{24\pi G_N c_O} \right)$$

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



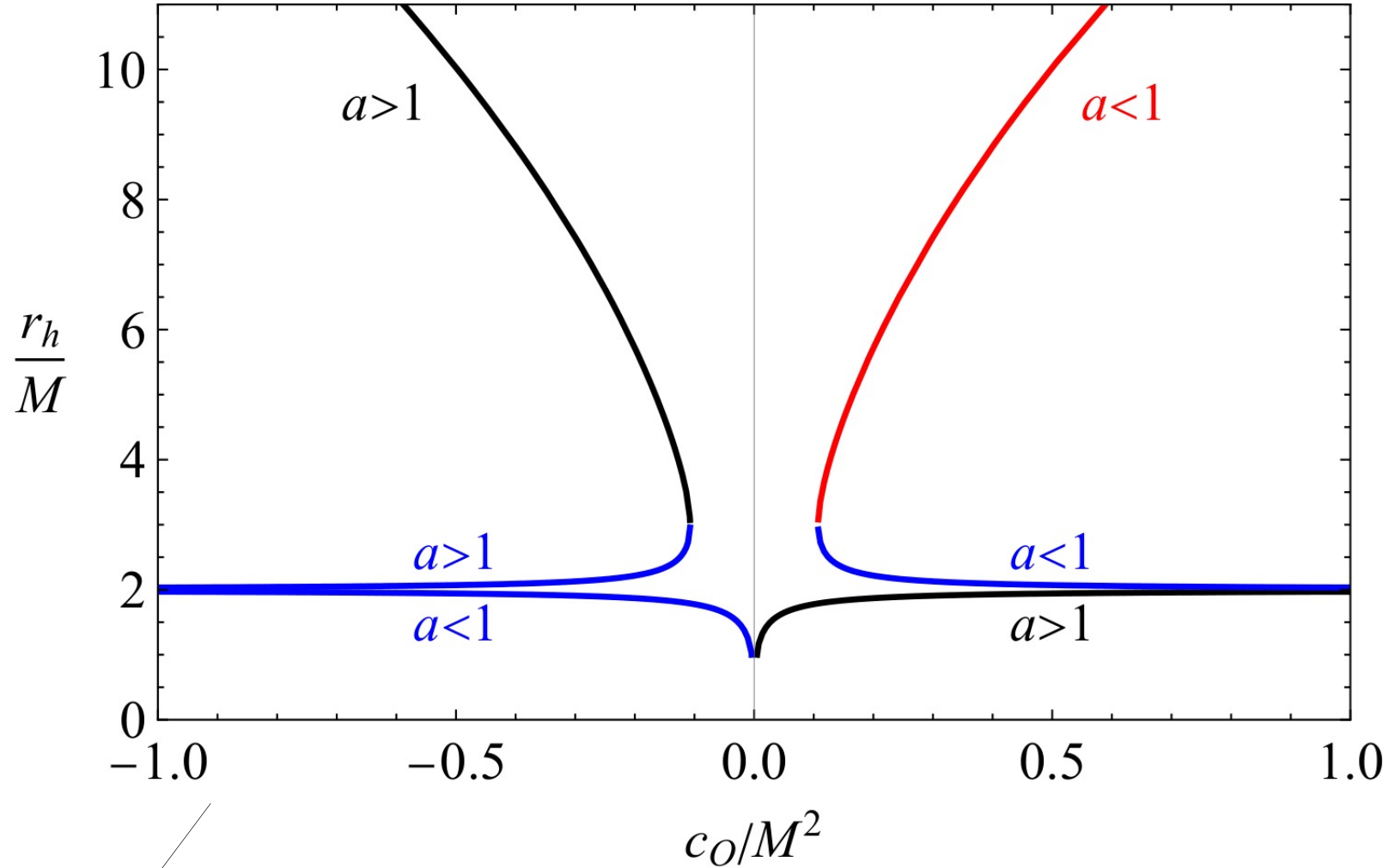
(J. Raimbayev et al 2206.06599 (Annals of Physics))

(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)



the regions in $a - c_0$ plane admitting BH solutions

(J. Raimbayev et al 2206.06599 (Annals of Physics))
 (I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)

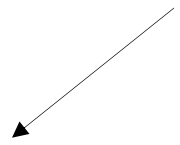
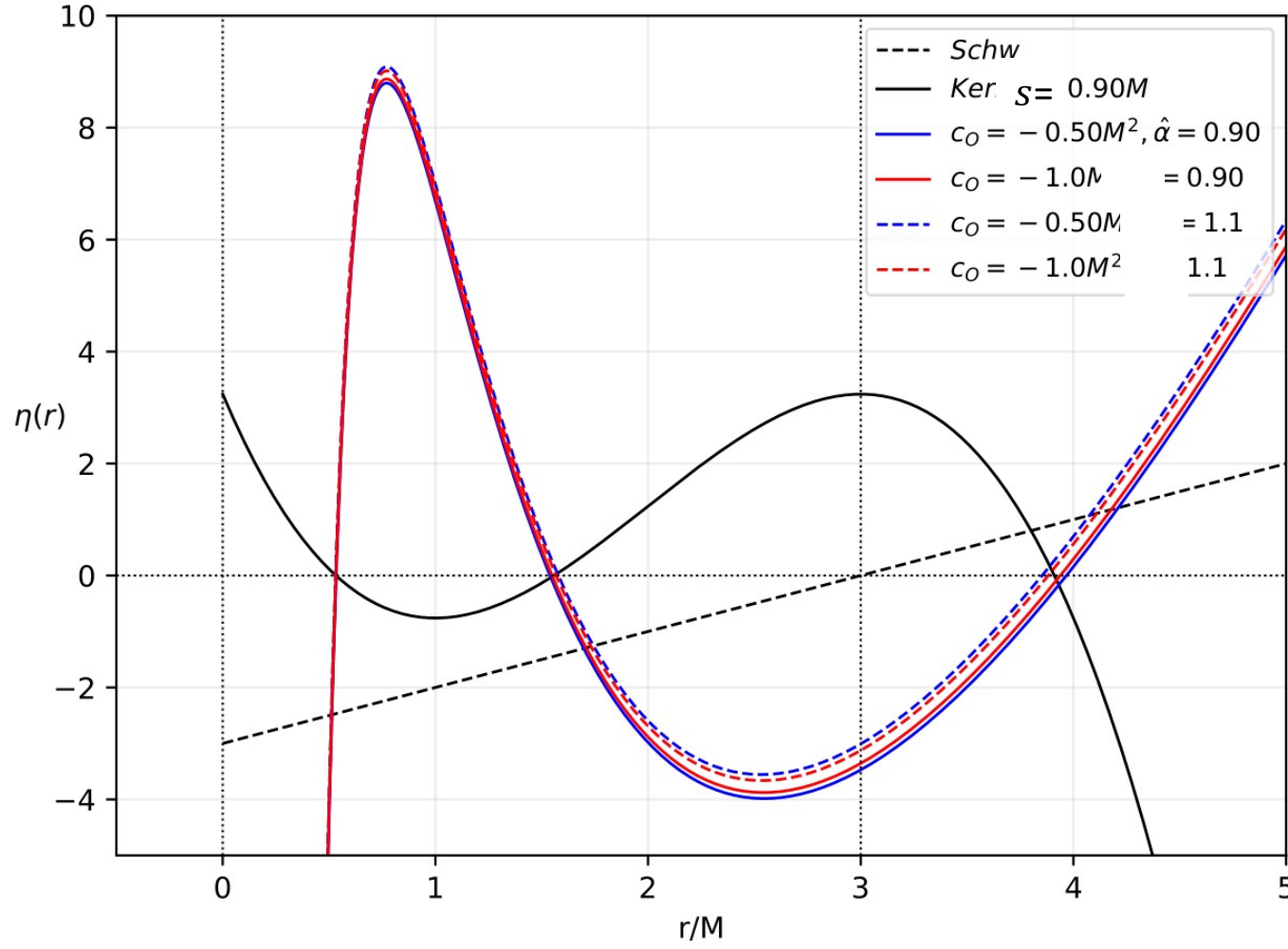


event horizon: $r_h = 2 M$ is the Schwarzschild radius

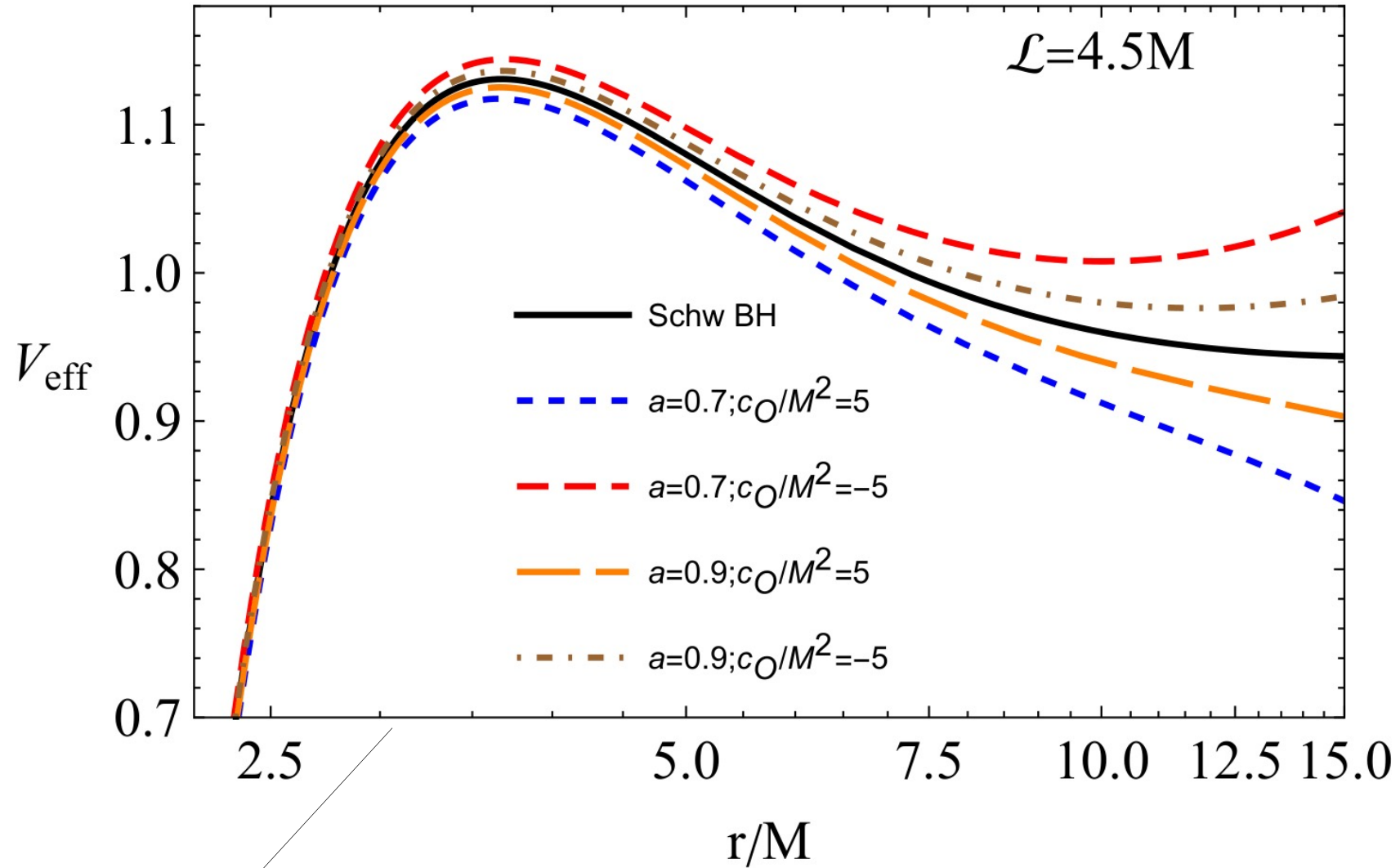
(J. Raimbayev et al 2206.06599 (Annals of Physics))

(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



Photon sphere radius for a rotating symmergent black hole of angular momentum number s

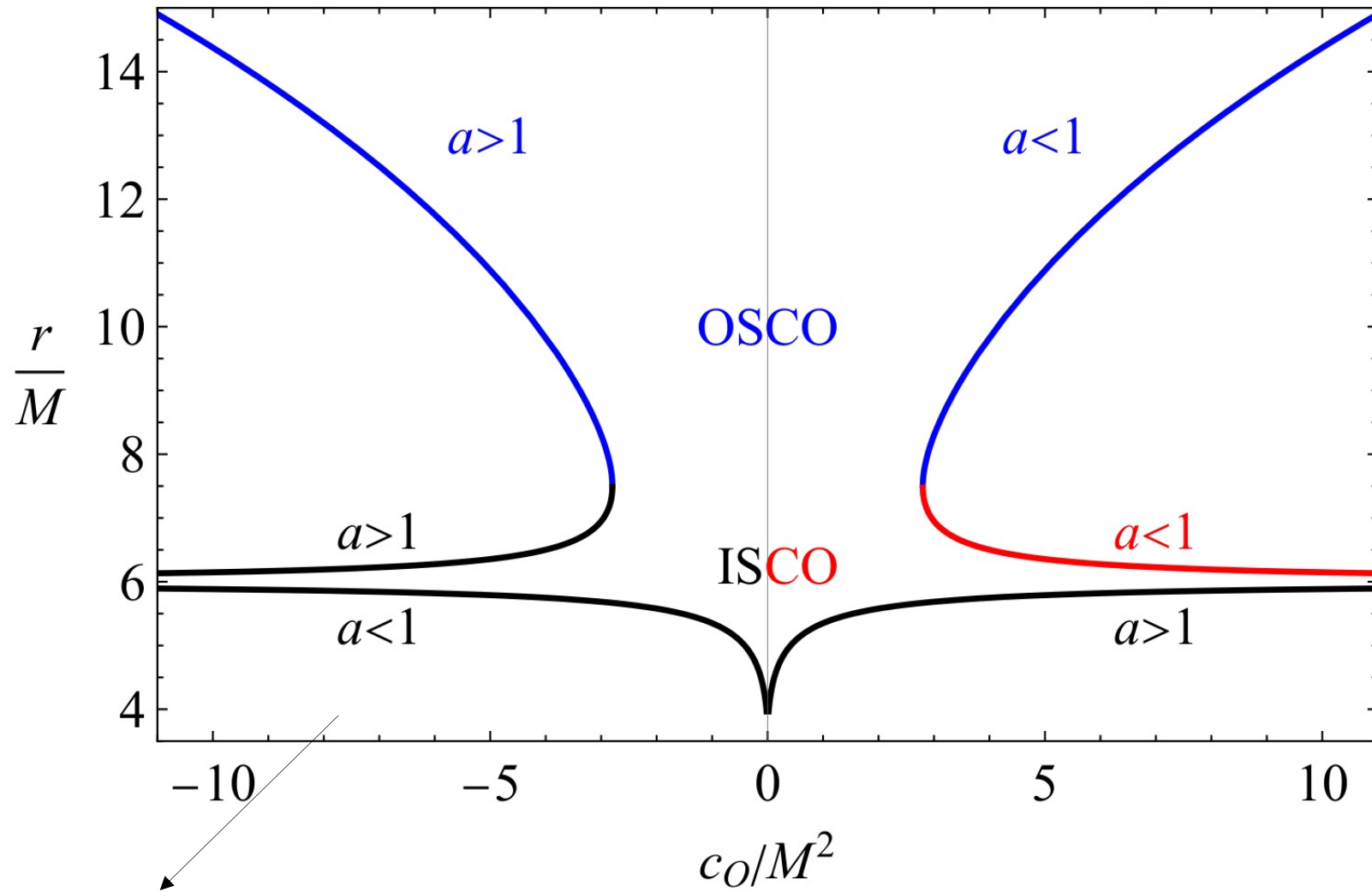


effective potential for $\ell = 4.5 M$

(J. Raimbayev et al 2206.06599 (Annals of Physics))

(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

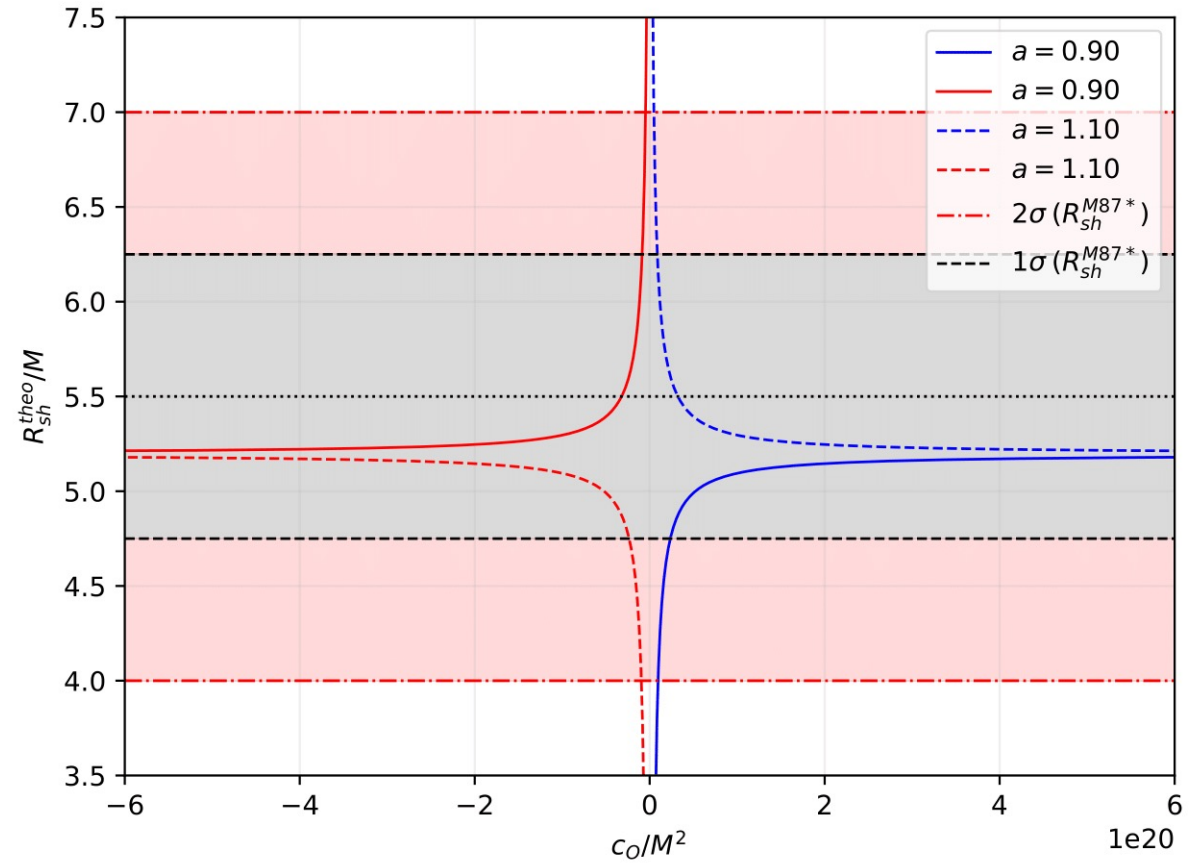
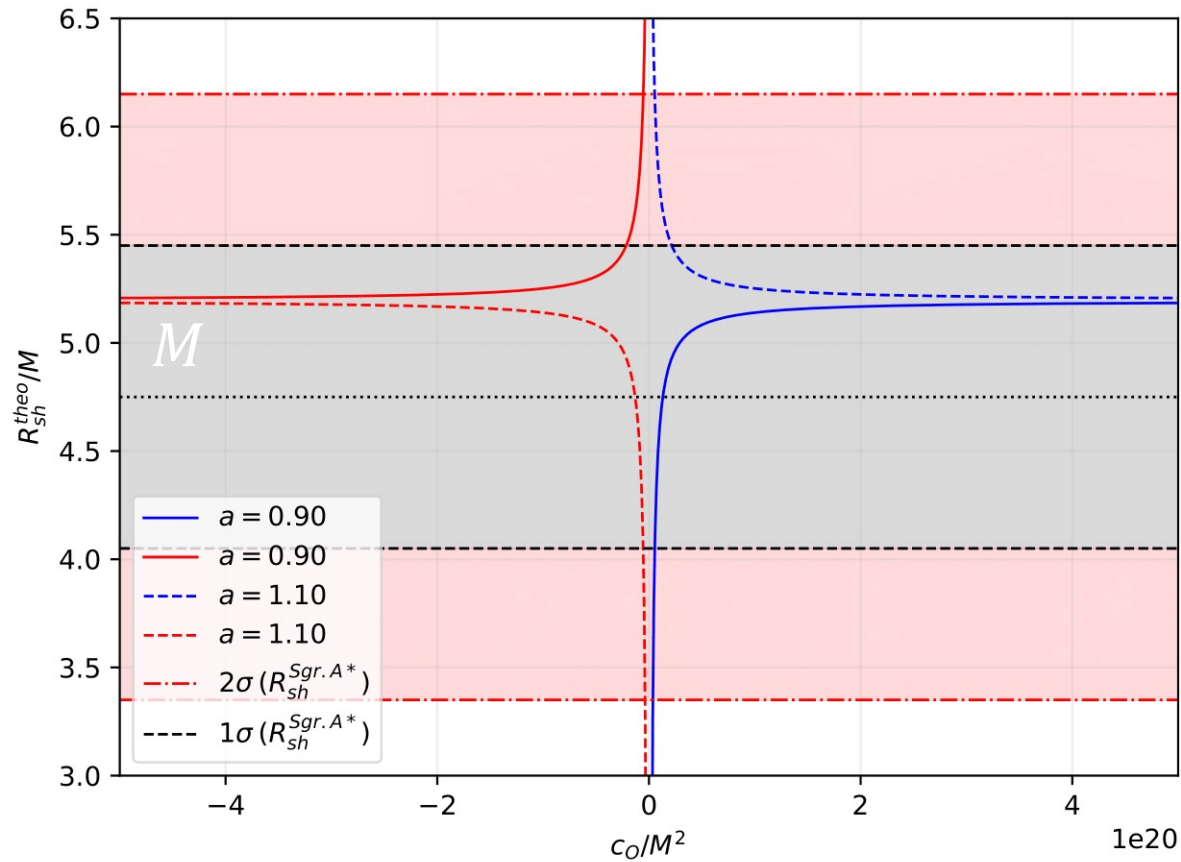


innermost (ISCO) and outermost (OSCO) stable circular orbits

(J. Raimbayev et al 2206.06599 (Annals of Physics))

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SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

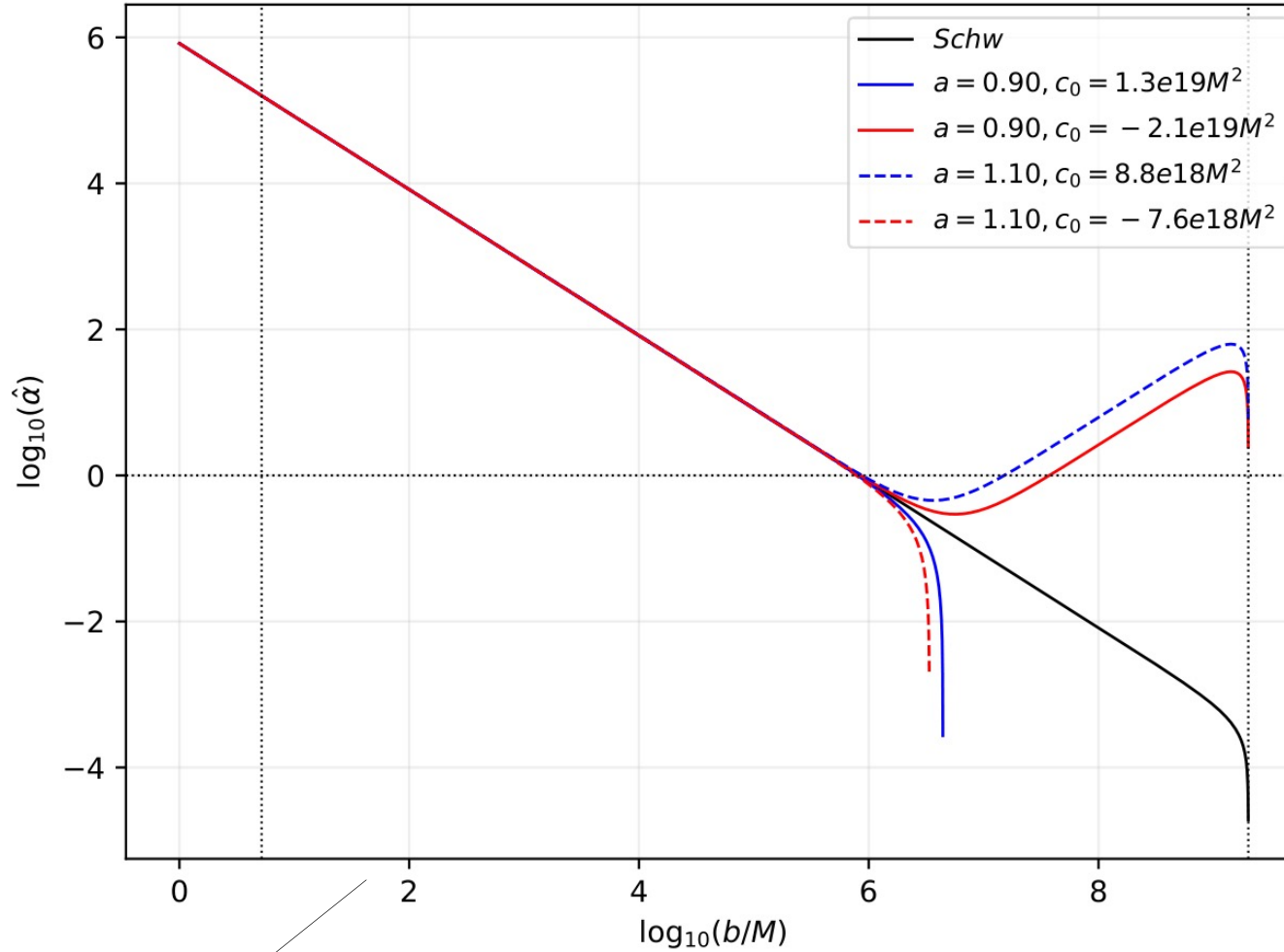


bounds on model parameters form shadow radii of Sgr.A* (left) and M87* (right)

(J. Raimbayev et al 2206.06599 (Annals of Physics))

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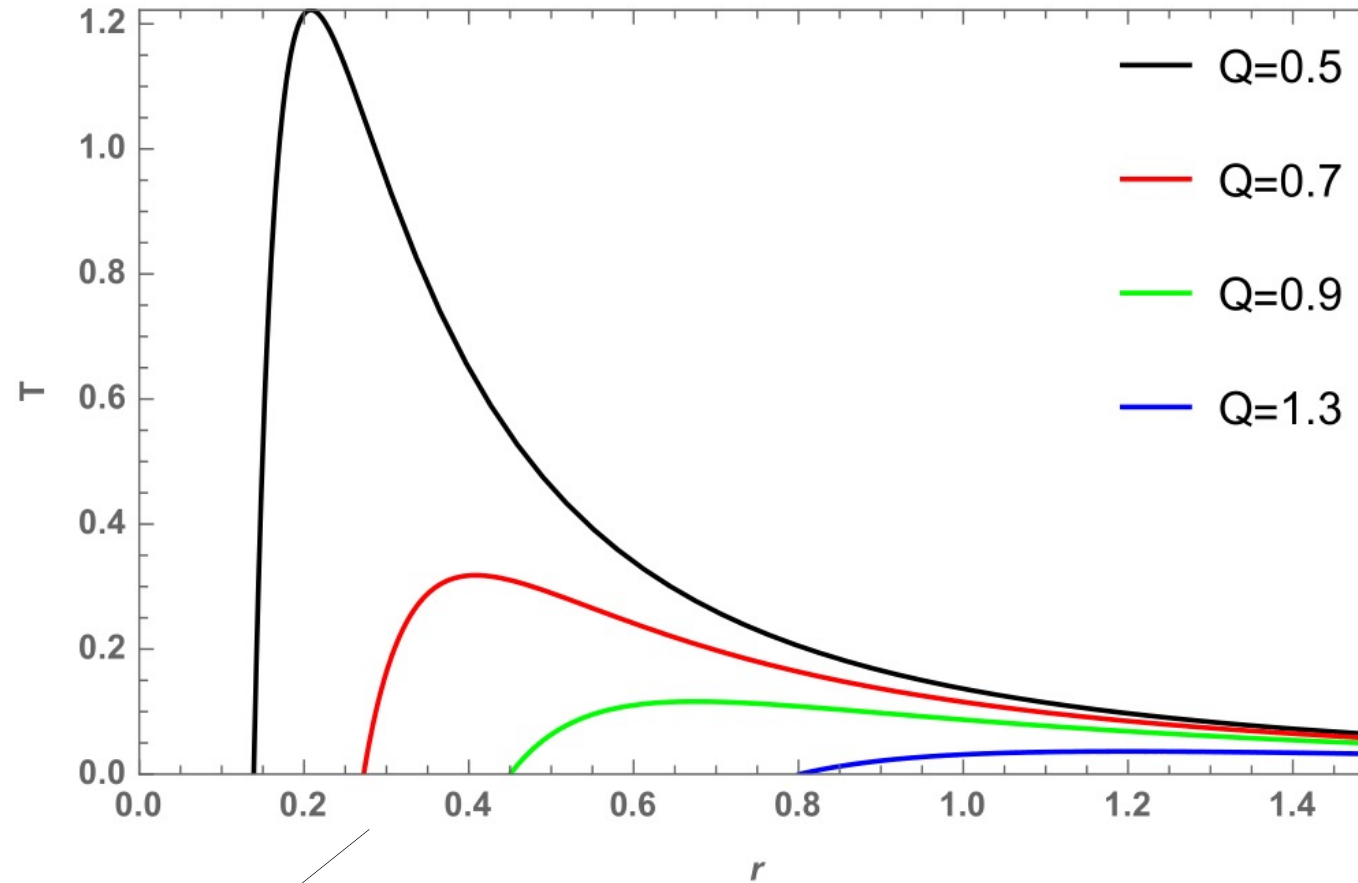
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



bending angle (μas) for a nearby source ($r_o/20$) and an observer with impact parameter b .

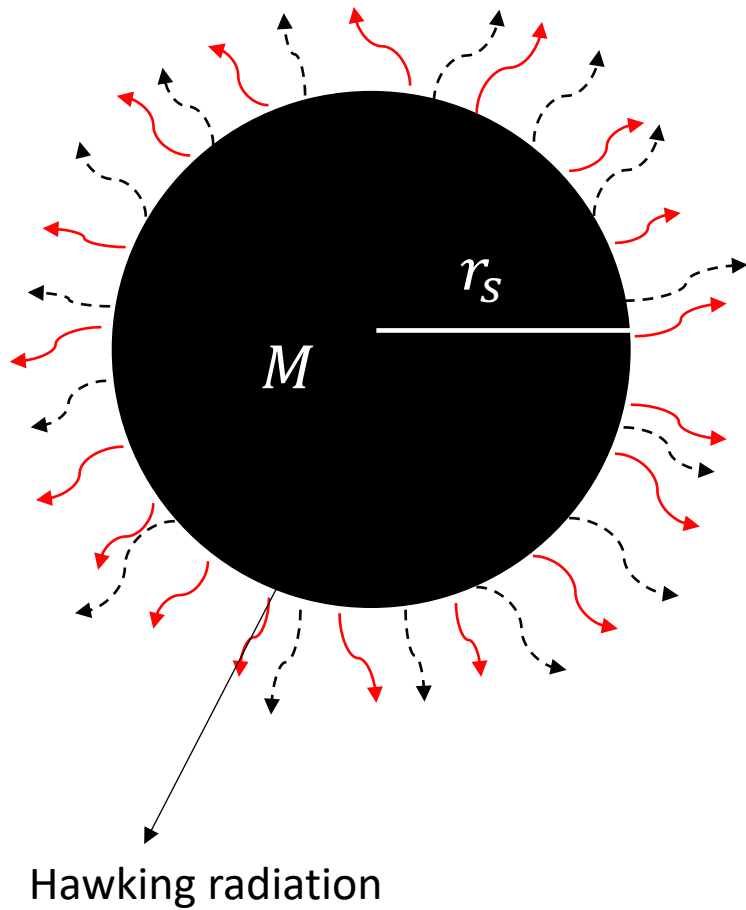
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Hawking temperature ($r_s = 1$) for a charged symmergent black hole ($c_0 = 0.9$ and $a = 0.5$).

SYMMERGENT BLACK HOLE



Hawking radiation from photon, electron etc. :

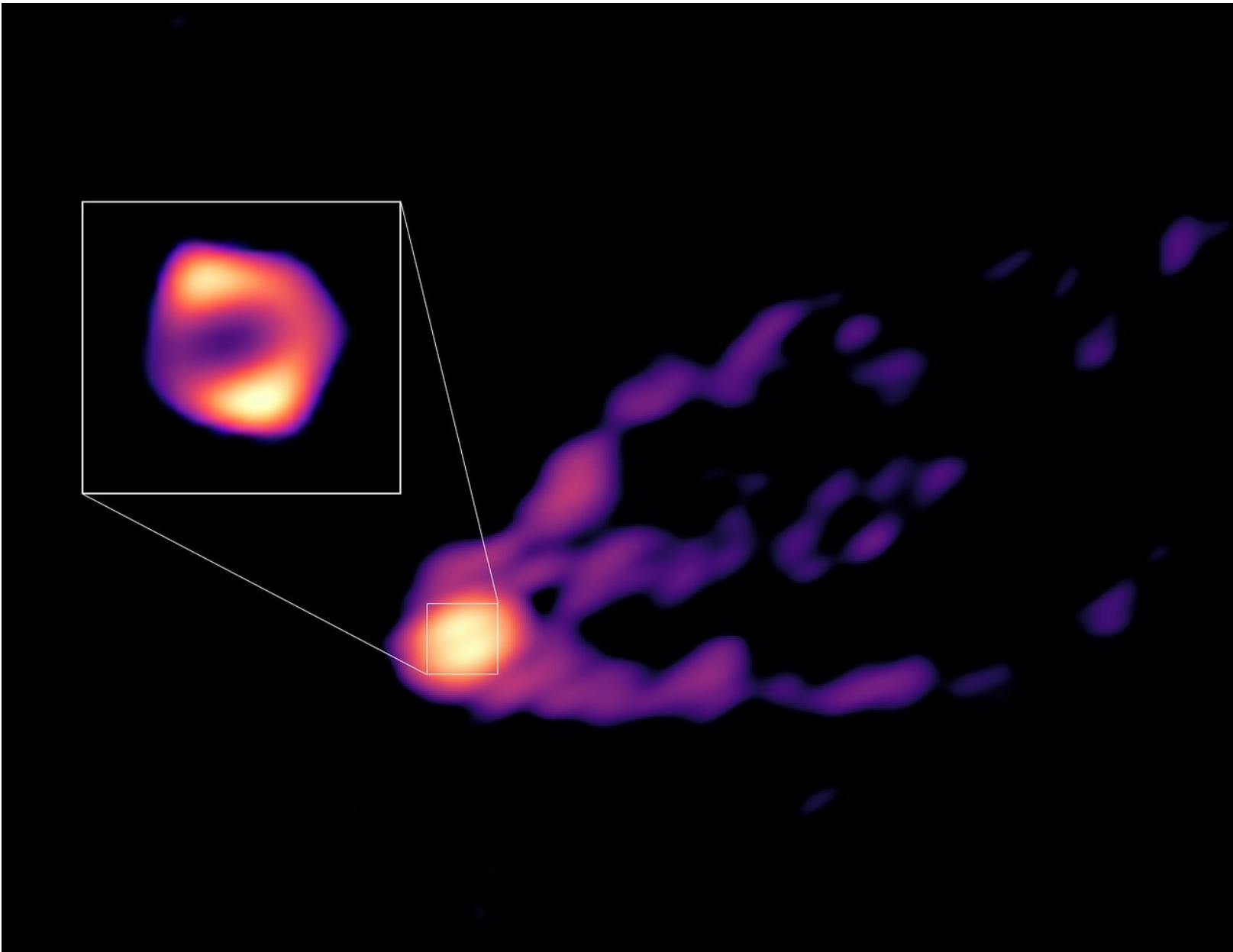


Hawking radiation from new dark fields:



Black hole temperature and evaporation rate change if symmergent particles are included!

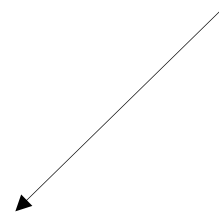
THANK YOU!



Accretion Disk around and Shadow of M87*

(R S Lu et al 2023 Nature 616, 686–690)

Sgr. A*	observed R_{sh}	1σ		2σ	
dS type	mean	lower	upper	lower	upper
$c_0 > 0, a = 0.90$	1.3×10^{19}	5.6×10^{18}	-	3.8×10^{18}	-
$c_0 < 0, a = 1.10$	-1.3×10^{19}	-5.6×10^{18}	-	-3.8×10^{18}	-
AdS type	mean	lower	upper	lower	upper
$c_0 > 0, a = 1.10$	-	-	2.1×10^{19}	-	5.4×10^{18}
$c_0 < 0, a = 0.90$	-	-	-2.1×10^{19}	-	-5.4×10^{18}
M87*	observed R_{sh}	1σ		2σ	
dS type	mean	lower	upper	lower	upper
$c_0 > 0, a = 0.90$	-	2.4×10^{19}	-	9.5×10^{18}	-
$c_0 < 0, a = 1.10$	-	-2.4×10^{19}	-	-9.5×10^{18}	-
AdS type	mean	lower	upper	lower	upper
$c_0 > 0, a = 1.10$	3.2×10^{19}	-	8.7×10^{18}	-	4.8×10^{18}
$c_0 < 0, a = 0.90$	-3.2×10^{19}	-	-8.7×10^{18}	-	-4.8×10^{18}



bounds on model parameters from shadow radii of M87* and Sgr.A*

(J. Raimbayev et al 2206.06599 (Annals of Physics))

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