

BLACK HOLES AND PARTICLES AROUND

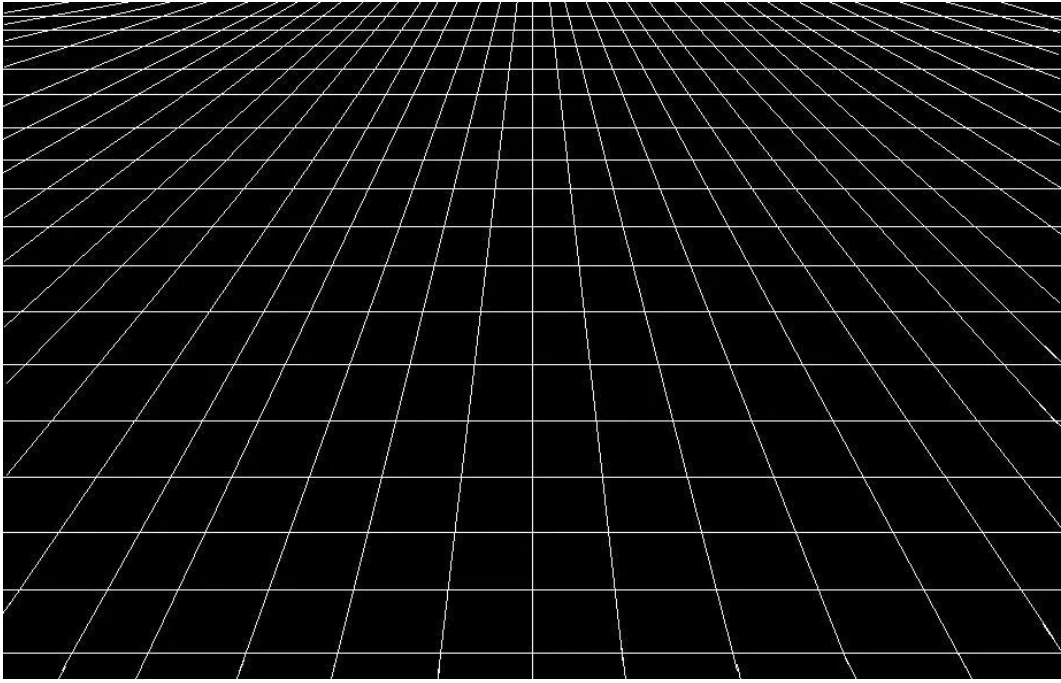
Durmuş Demir

Sabancı University

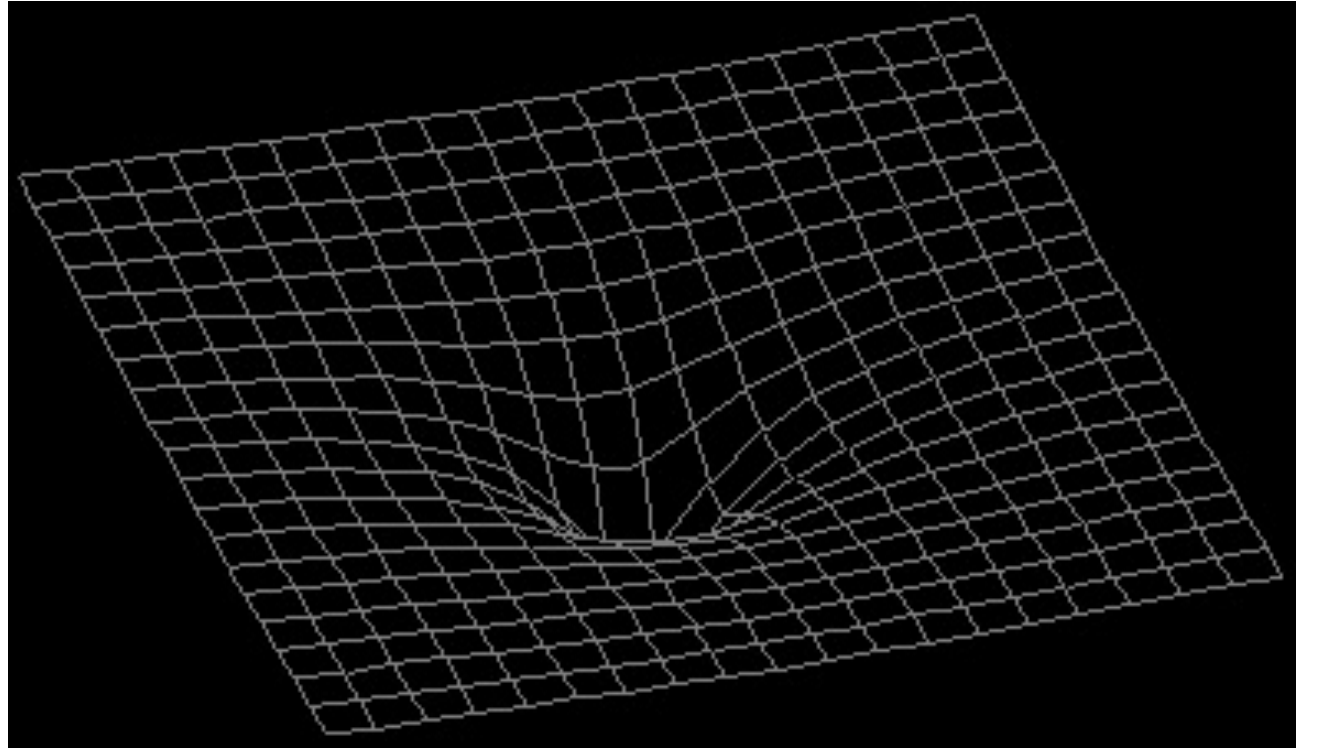
<http://myweb.sabanciuniv.edu/durmusedemir/>



Sabancı University SIAM Seminar (May 9, 2023)

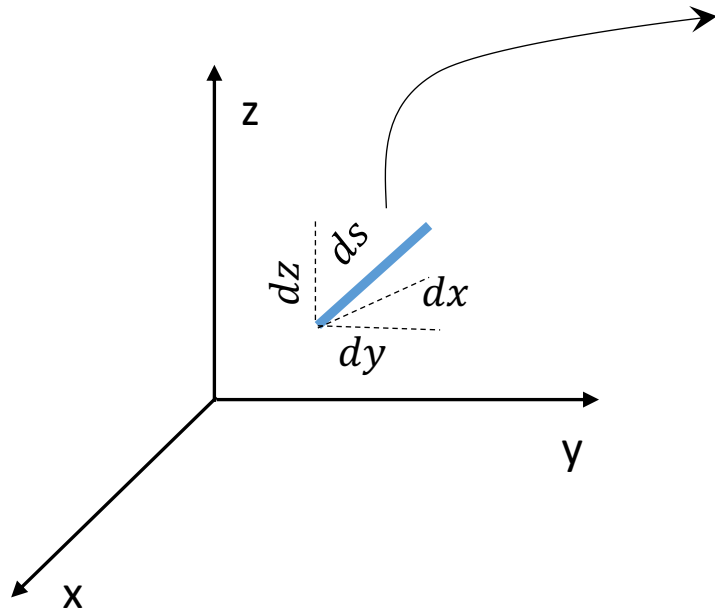


Flat space



Curved space

FLAT SPACE

3D

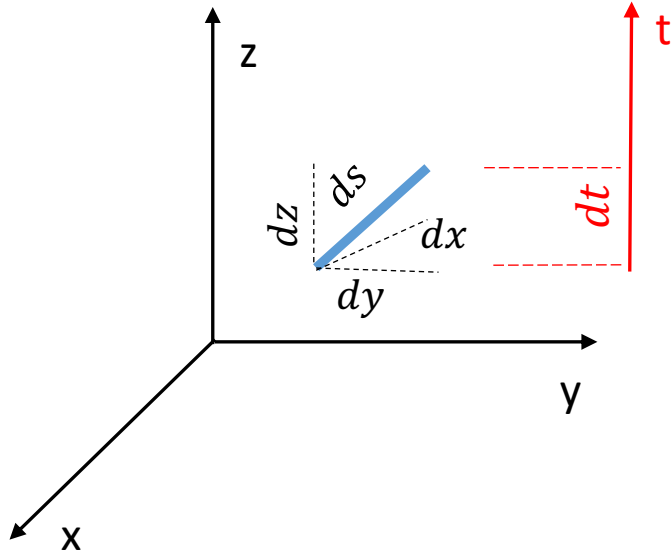
two infinitesimally close events
separated by a distance ds

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$$

space is flat

two events can have zero separation ($ds = 0$)
if they coincide ($dx = 0, dy = 0, dz = 0$)

FLAT SPACETIME

4D

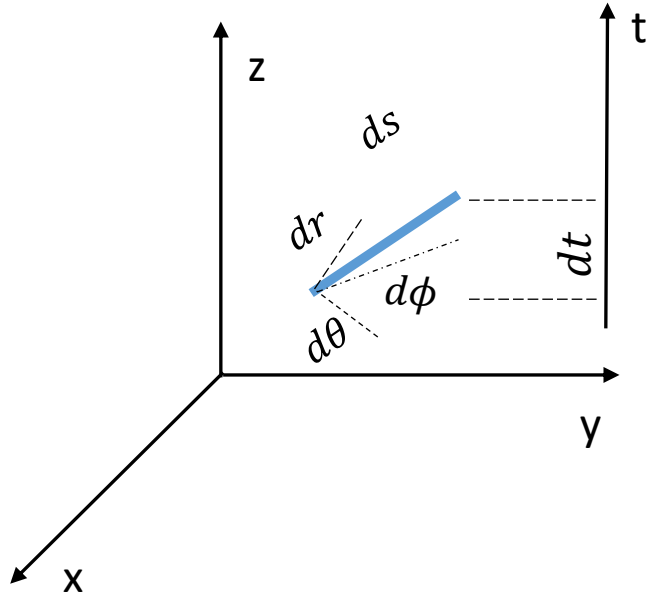
$$(ds)^2 = -(c dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

“minus” sign is needed
for light to propagate

spacetime is flat

two events can have zero separation ($ds = 0$)
even if they do not coincide ($dt \neq 0, dx \neq 0,$
 $dy \neq 0, dz \neq 0$)

FLAT SPACETIME

4D

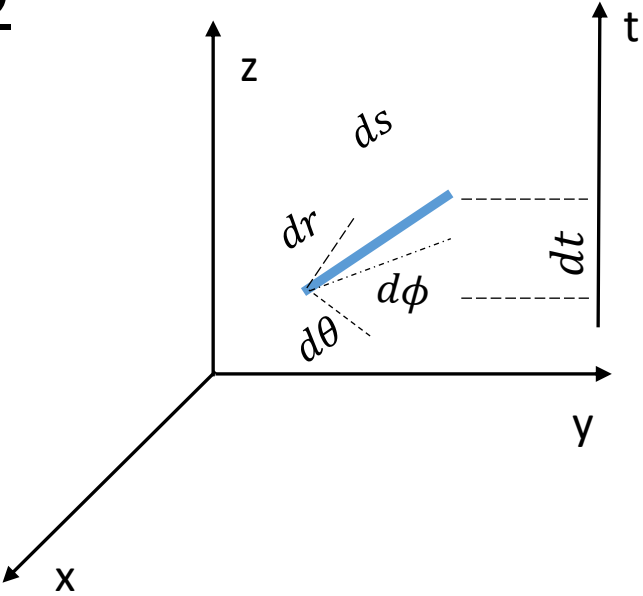
in spherical coordinates:

$$(ds)^2 = -(cdt)^2 + (dr)^2 + r^2((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

spacetime is flat
(coordinate change has no effect)

FLAT DILATED SPACETIME

4D



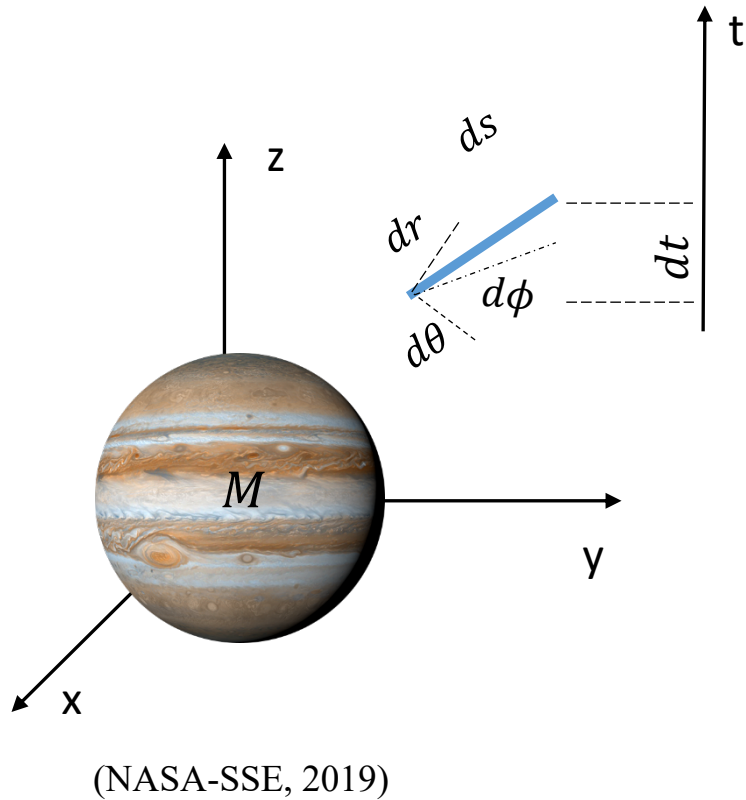
in an environment with intrinsic speed v_{int} :

$$(ds)^2 = - \left(1 - \frac{v_{int}^2}{c^2} \right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{int}^2}{c^2}} + r^2((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

dilated time

contracted distance

CURVED SPACETIME



in the presence of a body (Jupiter) of mass M , the escape speed v_{es} turns out to be the most natural intrinsic speed ($v_{int} = v_{es}$):

➤ escape speed from the body:

$$\frac{1}{2} m v_{es}^2 = \frac{G_N M m}{r} \implies v_{es}^2 = \frac{2 G_N M}{r}$$

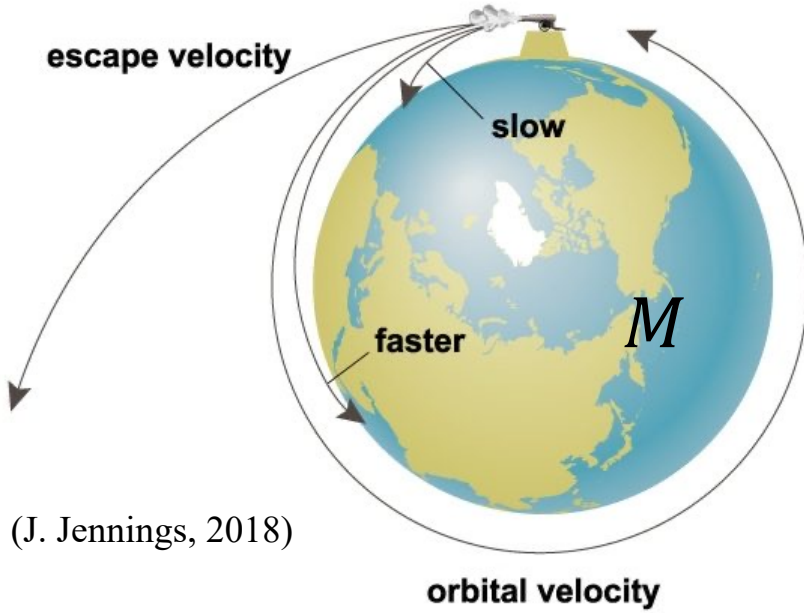
➤ distance between two points outside the body:

$$(ds)^2 = - \left(1 - \frac{v_{es}^2}{c^2} \right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{es}^2}{c^2}} + r^2 ((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

gravitational
time dilation

gravitational
space contraction

CURVED SPACETIME



(J. Jennings, 2018)

➤ Schwarzschild radius:

$$\frac{v_{es}^2}{c^2} = \frac{2G_N M}{rc^2} \equiv \frac{r_s}{r} \implies r_s = \frac{2G_N M}{c^2}$$

➤ ds can be expressed in terms of Schwarzschild radius r_s :

$$(ds)^2 = - \left(1 - \frac{r_s}{r}\right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 \left((d\theta)^2 + \sin^2 \theta (d\phi)^2 \right)$$

- $r_s(\text{Sun}) = 3 \text{ km}$
- $r_s(\text{Jupiter}) = 3 \text{ m}$
- $r_s(\text{Earth}) = 10 \text{ mm}$

EINSTEIN GRAVITY

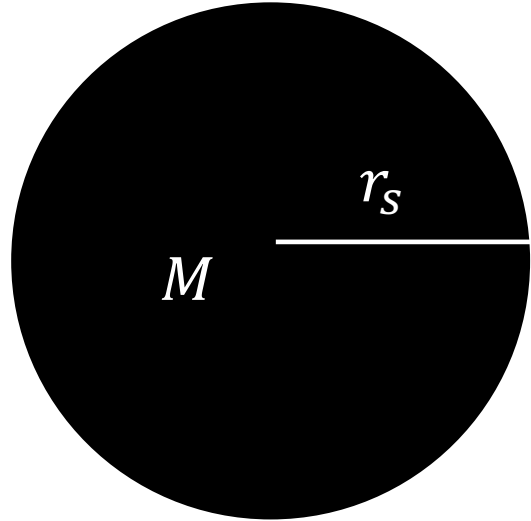
- distance ds for the escape speed turns out to be the solution of the Einstein equations for gravity of a compact body of mass M :

gravity theory	solution of Einstein equations
Einstein gravity action: $\int d(\text{Vol})_4 \left(\frac{R}{16\pi G_N} - V_0 \right)$	Schwarzschild solution (zero vacuum energy $V_0 = 0$) : $(ds)^2 = - \left(1 - \frac{r_s}{r} \right) (cdt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 ((d\theta)^2 + \sin^2 \theta (d\phi)^2)$

K. Schwarzschild, 1916 arXiv:physics/9905030

- true measure of curving is the Riemann curvature:

$$(\text{Riemann curvature})^2 = \frac{12 r_s^2}{r^6}$$

BLACK HOLE

- if the body M is too massive to require an escape speed bigger than the speed of light:

$$v_{es} > c$$

(or $r < r_s$) then a **black hole** forms:

- $r = r_s \Rightarrow$ event horizon (the point of no return)
- $r = r_s \Rightarrow$ event horizon (infinite time dilation)
- $r < r_s \Rightarrow$ black hole interior (cannot be probed from outside)
- $r < r_s \Rightarrow$ black hole interior (effectively: time \Leftrightarrow radial distance)

PHOTON MOTION AROUND BLACK HOLE

➤ Photon: $(ds)^2 = 0 \Rightarrow \left(\frac{ds}{d\tau}\right)^2 = 0$

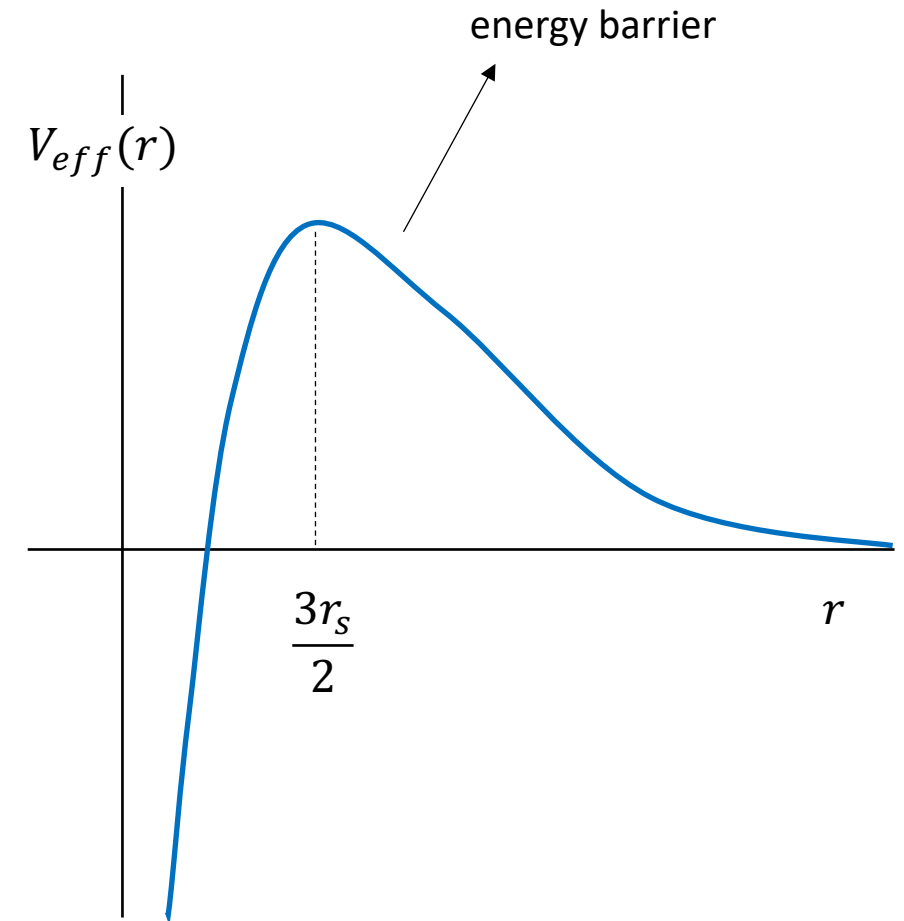
➤ Azimuthal plane ($\theta = \frac{\pi}{2}$): $-\left(1 - \frac{r_s}{r}\right) c^2 \dot{t}^2 + \frac{\dot{r}^2}{1 - \frac{r_s}{r}} + r^2 \dot{\phi}^2 = 0$

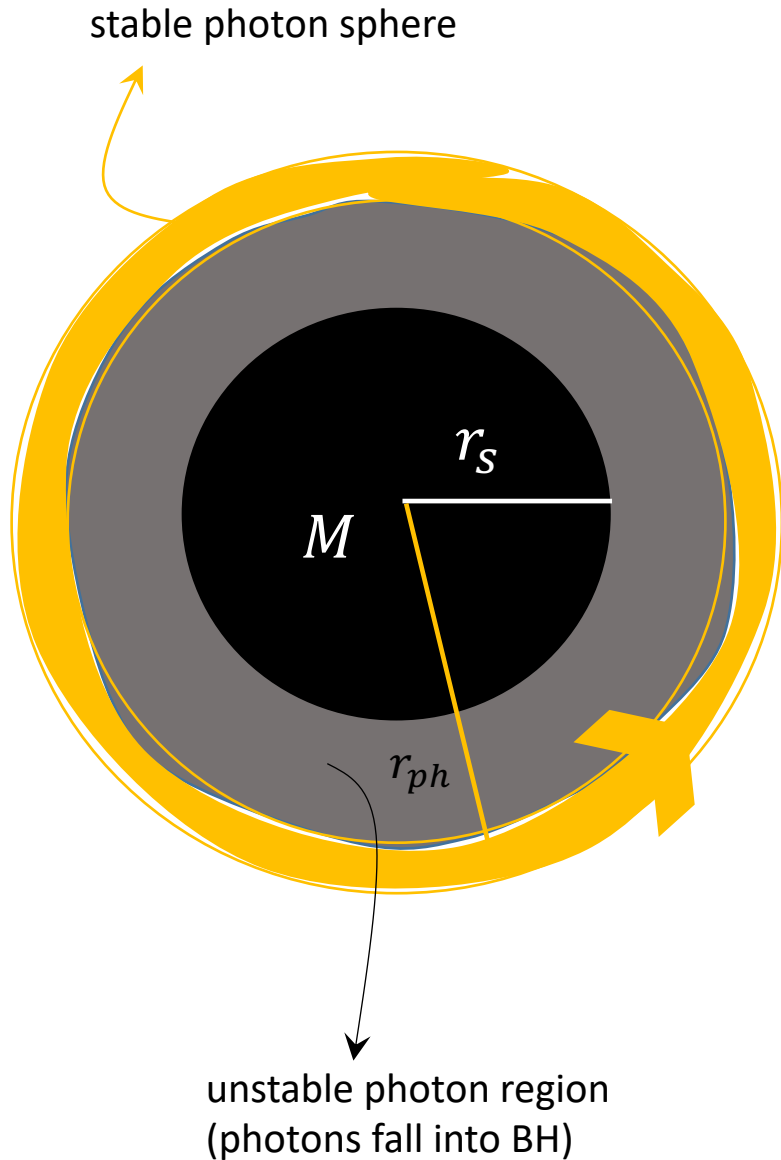
➤ Energy is conserved: $\dot{t} = \frac{\epsilon}{1 - \frac{r_s}{r}}$

➤ Ang. Mom. is conserved: $\dot{\phi} = \frac{\ell}{r^2}$

➤ Photon is a unit-mass particle: $\frac{\dot{r}^2}{2} + \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\epsilon^2}{2}$

➤ $V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}$



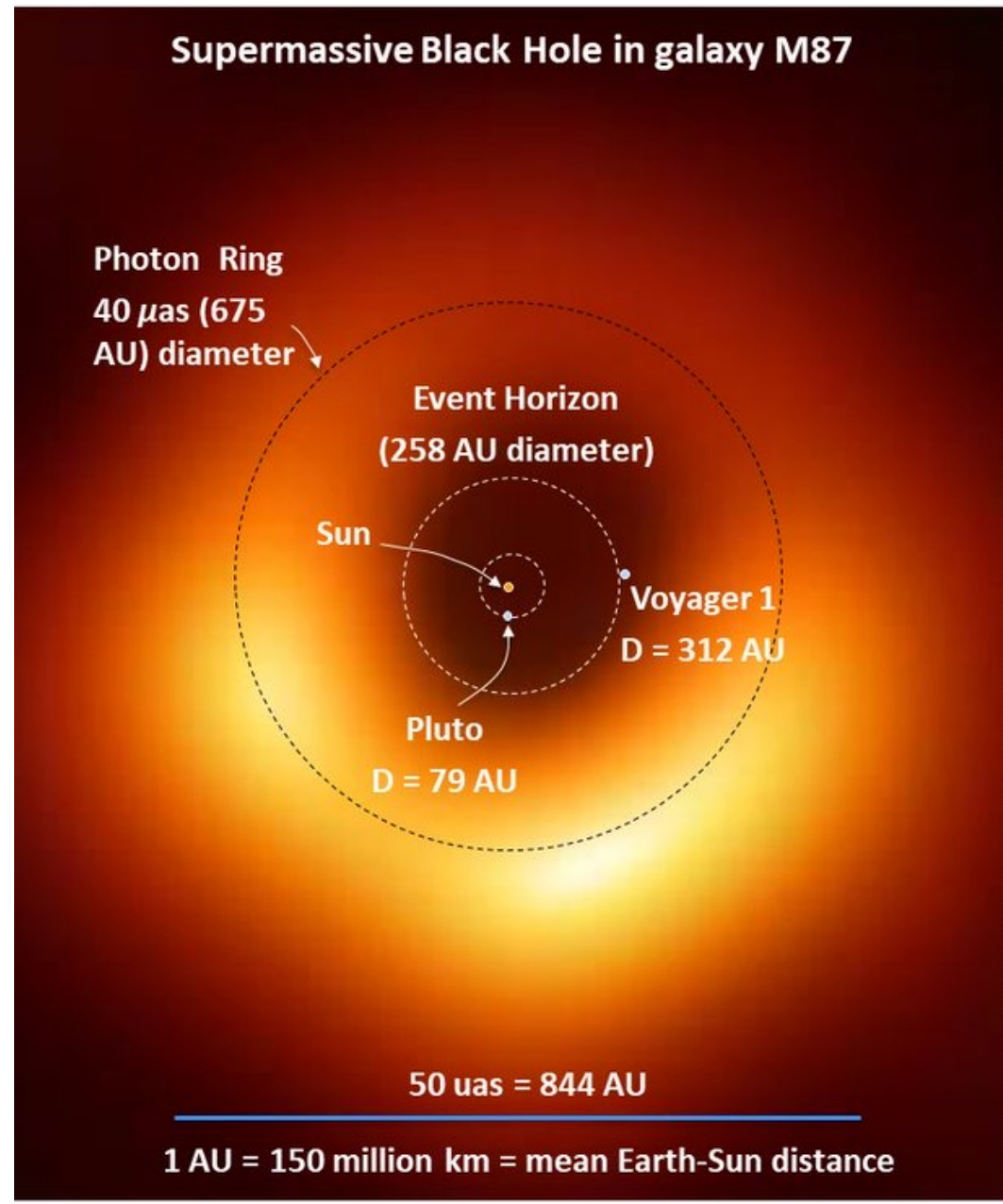
PHOTON SPHERE

- photons orbiting in xy -plane ($r = r_{ph} = \text{constant}$, $\theta = \frac{\pi}{2}$):

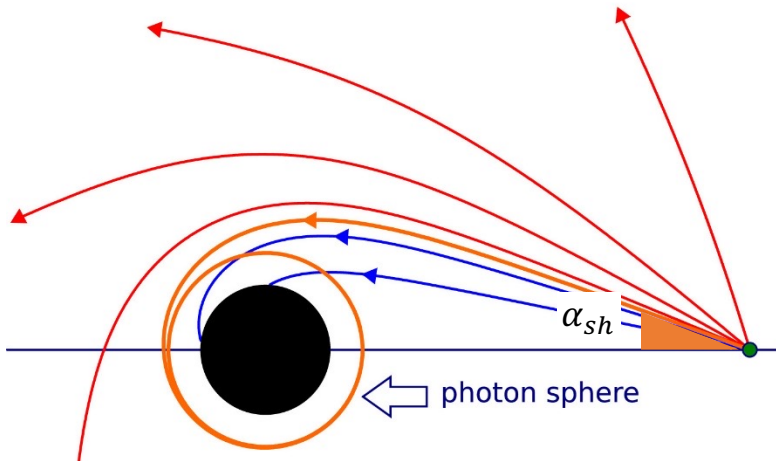
$$\begin{aligned}
 ds = 0 &\Rightarrow \frac{d\phi}{dt} = \frac{c}{r_\gamma} \left(1 - \frac{r_s}{r_\gamma}\right)^{1/2} \\
 \text{EoM} &\Rightarrow \frac{d\phi}{dt} = \frac{c}{r_\gamma} \left(\frac{r_s}{2r_\gamma}\right)^{1/2}
 \end{aligned}
 \left. \vphantom{\begin{aligned} ds = 0 \\ \text{EoM} \end{aligned}} \right\} r_{ph} = \frac{3}{2} r_s$$

- photon radius $r = r_{ph}$ is the last stable orbit.
- photon radius $r = r_{ph}$ depends on the underlying gravity theory.

Mass (Solar Masses)	6.54 billion
Event Horizon diameter (AU)	258
Distance (Light Years)	55 million



BLACK HOLE SHADOW



Perlick and Tsupko 2022 *Phys Rep* 947 1

➤ photons falling within the photon sphere fall into black hole – a large shadow!

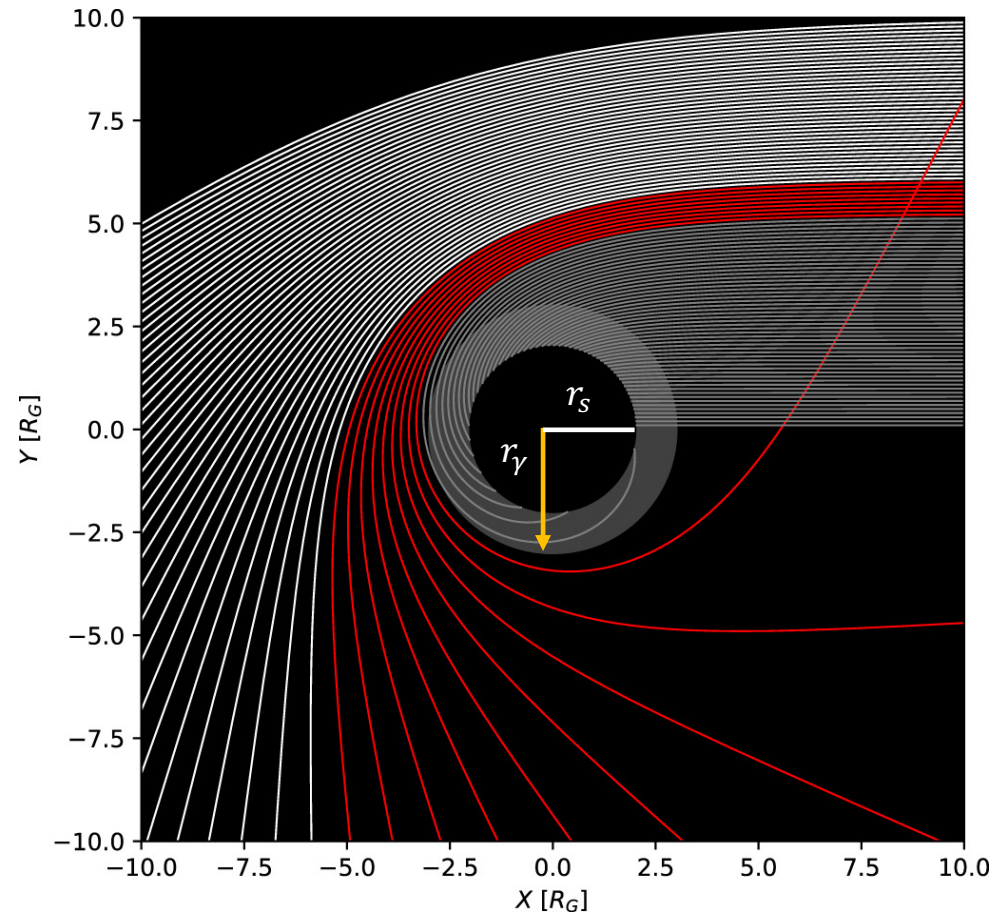
➤ consider a general spacetime:

$$(ds)^2 = -A(r)(c dt)^2 + B(r)(dr)^2 + D(r)((d\theta)^2 + \sin^2\theta (d\phi)^2)$$

➤ shadow is characterized by “gravitational capture angle” \equiv shadow angle α_{sh} :

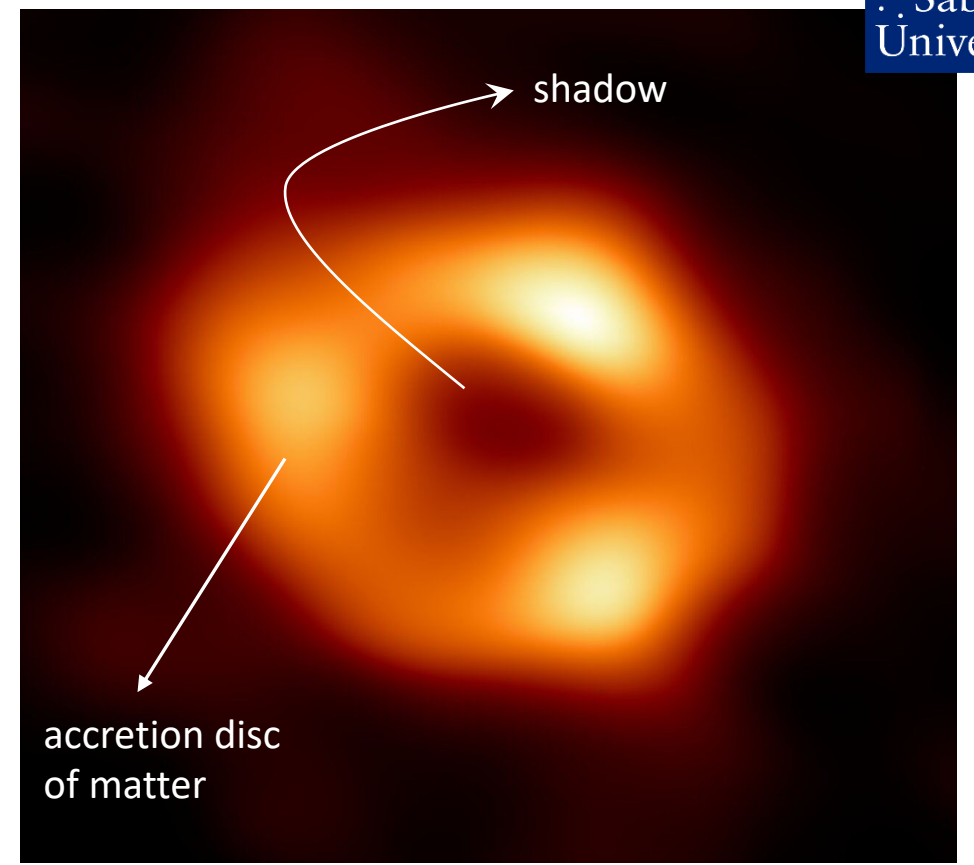
$$\sin^2 \alpha_{sh} = \frac{D(r_{ph}) A(r_o)}{A(r_{ph}) D(r_o)} \xrightarrow{\text{Schwarzschild}} \frac{r_{ph}^2}{1 - \frac{r_s}{r_{ph}}} \times \frac{\left(1 - \frac{r_s}{r_o}\right)}{r_o^2}$$

Distant observer: $\frac{27 r_s^2}{4 r_o^2}$



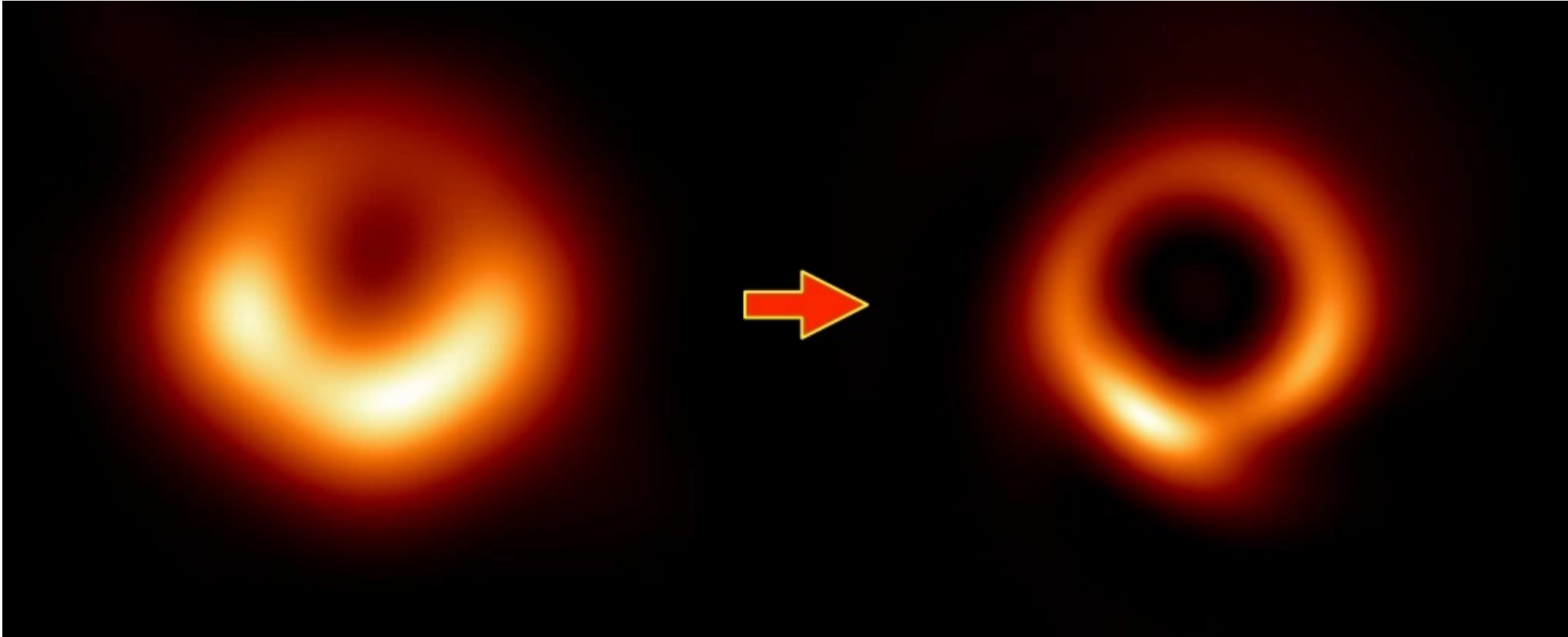
Shadow of a black hole

Thomas Bronzwaer and Heino Falcke 2021 *ApJ* **920** 155



Shadow of a black hole

EHT observation of Sgr. A* in 2019



Shadow of M87* (2019)

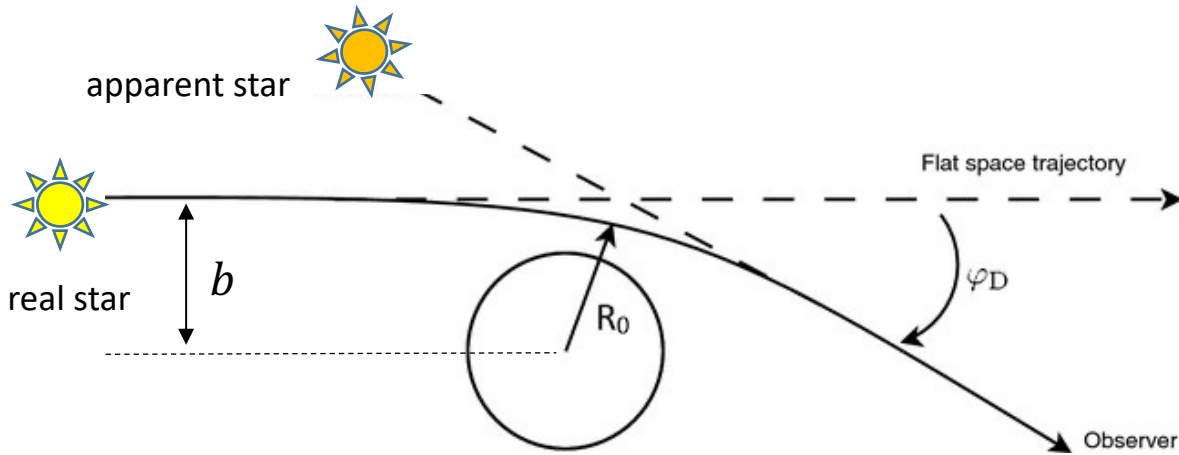
EHT observation of M87* in 2019

Shadow of M87* (AI)

PRIMO, 2023

Lia Medeiros et al 2023 ApJL 947 L7

LIGHT BENDING BY BLACK HOLE

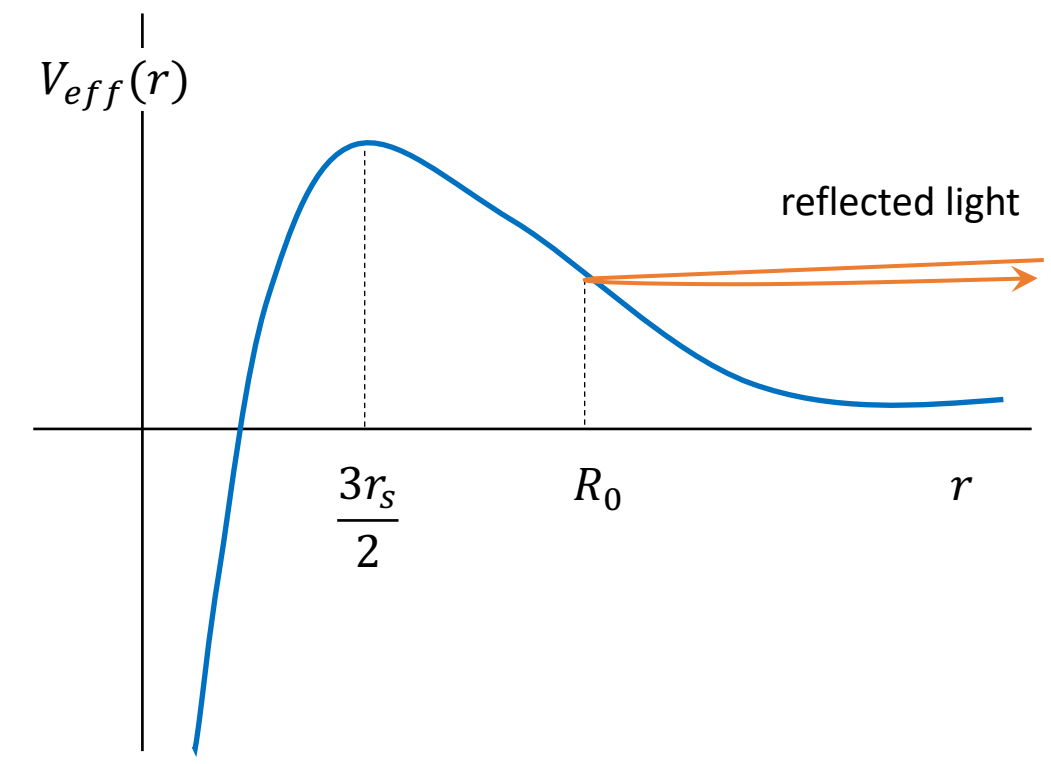


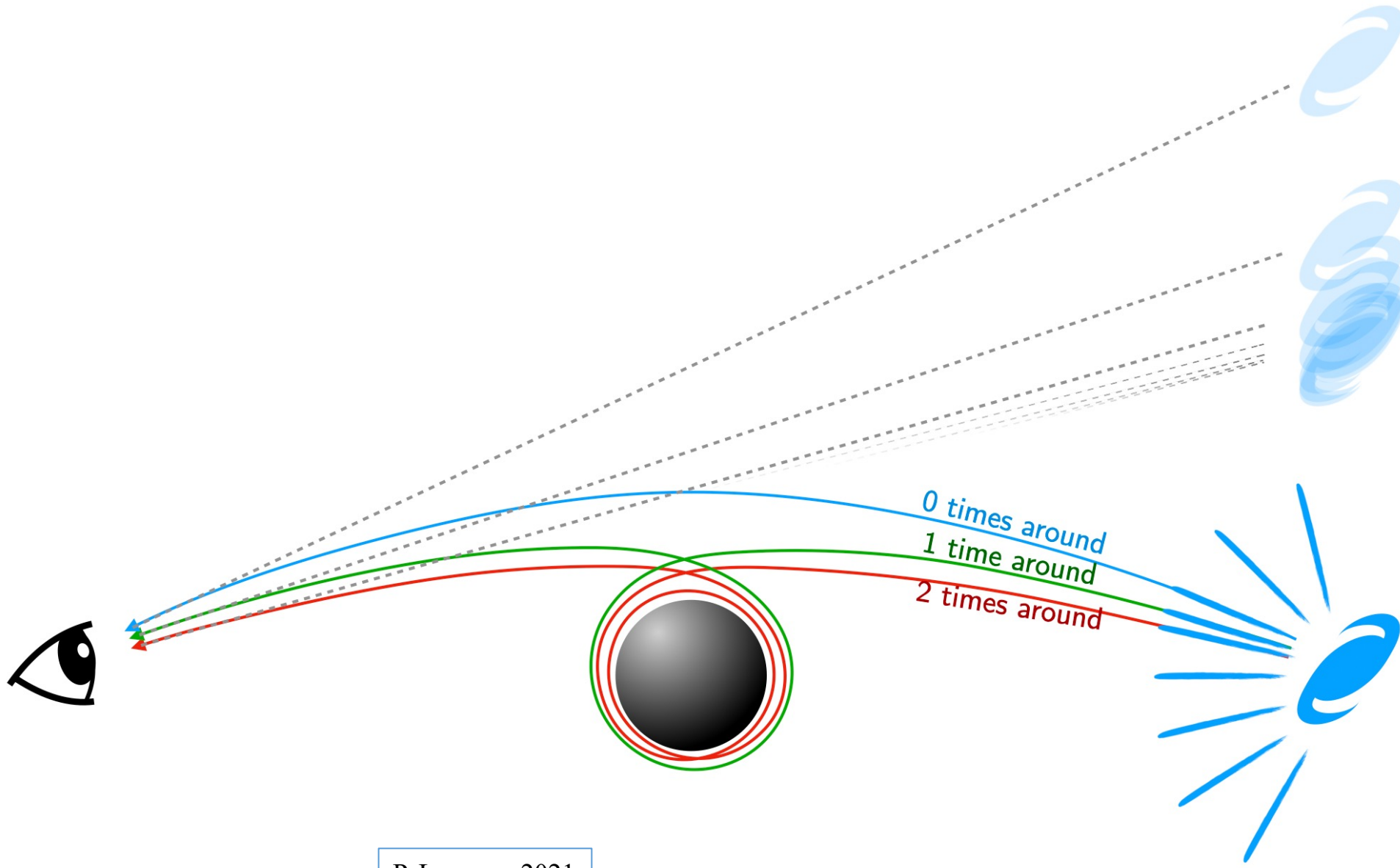
Burger, D. et al 2018 Gen Relativ Gravit 50, 156

$$\rightarrow \frac{dr}{d\tau} = \sqrt{\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)}$$

$$\rightarrow \frac{d\phi}{d\tau} = \frac{1}{r^2}$$

$$\rightarrow \varphi_D = 2 \int_{R_0}^{\infty} \frac{dr}{r^2 \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)\right)^{\frac{1}{2}}} \xrightarrow{\text{small } r_s} \frac{2r_s}{R_0}$$





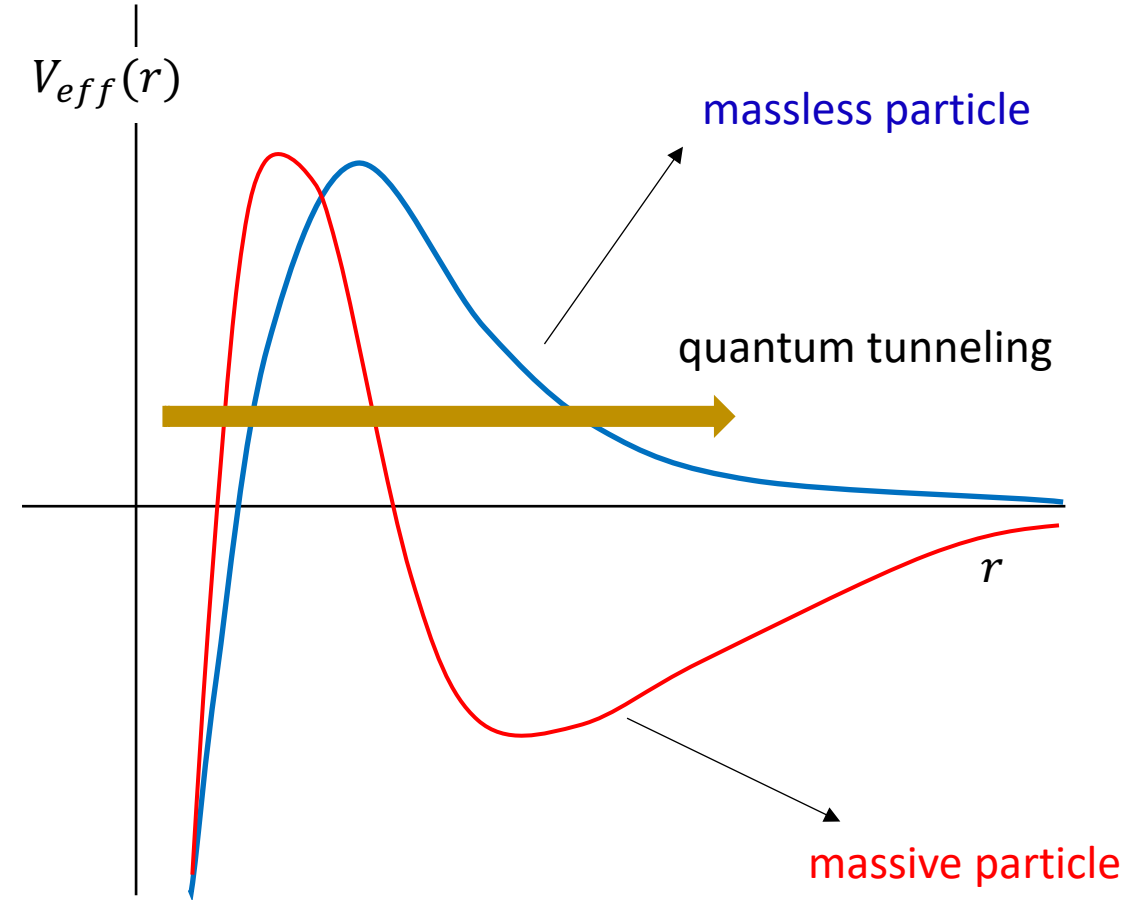
P. Laursen, 2021

bending of light leads to
multiple images for objects
behind (lensing effect)

- effective potential seen by massless particle (photon):

$$V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}$$

- potential energy barrier for massless particles is formed by angular momentum ($1/r^2$) and Schwarzschild radius (r_s/r^3)
- potential energy barrier for massive particles involves in addition the Newtonian contribution ($1/r$)
- quantum particles that fell into the black hole can tunnel out through the barrier.
- tunneled particles appear as radiation – the Hawking radiation.

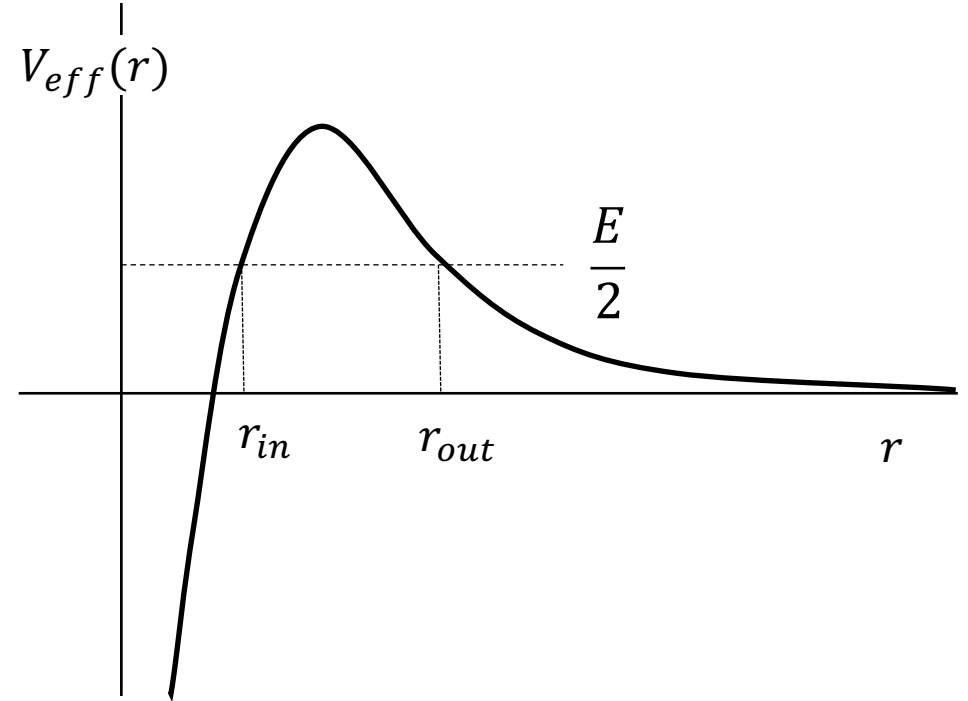


➤ Time it takes to traverse the barrier region:

$$\Delta t = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - 2V_{eff}(r)}} = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - \frac{\ell^2}{r^2} + \frac{\ell r_s}{r^3}}} =$$

$$= \pm \frac{r_s^2}{\sqrt{2\ell^2 - 3\ell}} \int_{r_{in}}^{r_{out}} \frac{d\hat{r}}{\sqrt{\hat{r} - 1}} \quad \begin{matrix} \text{(expand at } r = r_s) \\ (E = \ell(\ell - 1)/r_s^2) \end{matrix}$$

$$= \pm \frac{r_s^2}{\sqrt{2\ell^2 - 3\ell}} \times \pi i \simeq \pm \pi i r_s \quad \begin{matrix} \text{(residue theorem)} \\ (\ell \simeq r_s) \end{matrix}$$

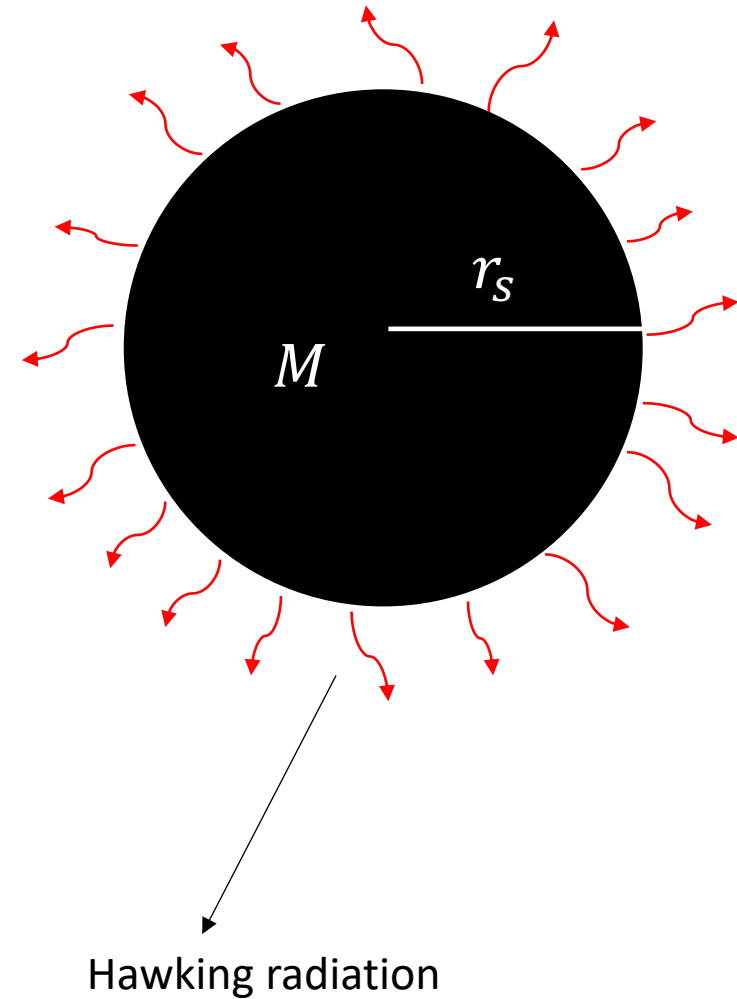


➤ Wavefunction: $\psi \propto e^{iE\Delta t} \simeq e^{\pm \pi r_s E}$

$$\begin{matrix} \nearrow \rho_{em} \simeq e^{-2\pi r_s E} \\ \searrow \rho_{ab} \simeq e^{+2\pi r_s E} \end{matrix}$$

Hawking temperature

➤ Net emission rate = $\frac{\rho_{em}}{\rho_{ab}} = e^{-4\pi r_s E} \Rightarrow T_H \simeq \frac{1}{4\pi k_B r_s} \equiv \frac{\hbar c^3}{8\pi k_B G_N M}$



Emitted power (photons only):

$$P = \frac{\hbar c^6}{15360\pi G_N^2 M^2}$$

Evaporation time (photons only):

$$t_{\text{eva}} = \frac{5120 \pi G_N^2 M^3}{\hbar c^4} \approx 10^{67} \text{ years} \times \left(\frac{M}{M_{\text{Sun}}}\right)^3$$

QUANTUM TUNNELING AND HAWKING RADIATION

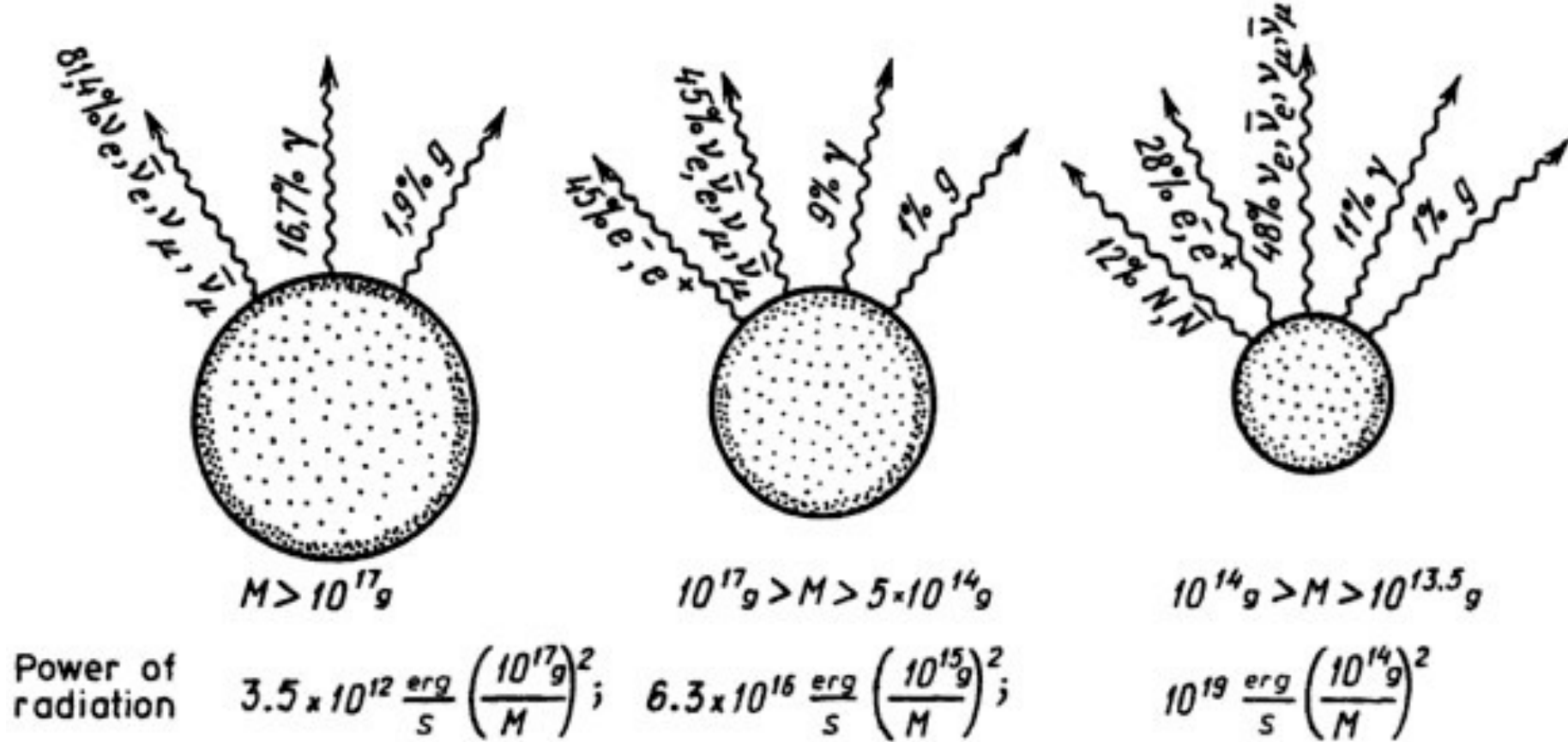


Figure 10.7: Quantum decay of a non-rotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν) and other elementary particles are given in percent of the total number of particles emitted by black holes of different masses.

QUANTUM TUNNELING AND HAWKING RADIATION

In classical physics: Black holes must have **zero temperature** and **zero entropy**!

In quantum physics: Black holes must have **nonzero temperature** and **nonzero entropy**!

First law of thermodynamics: $\Delta E = \Delta Q = T_{BH} \Delta S_{BH}$

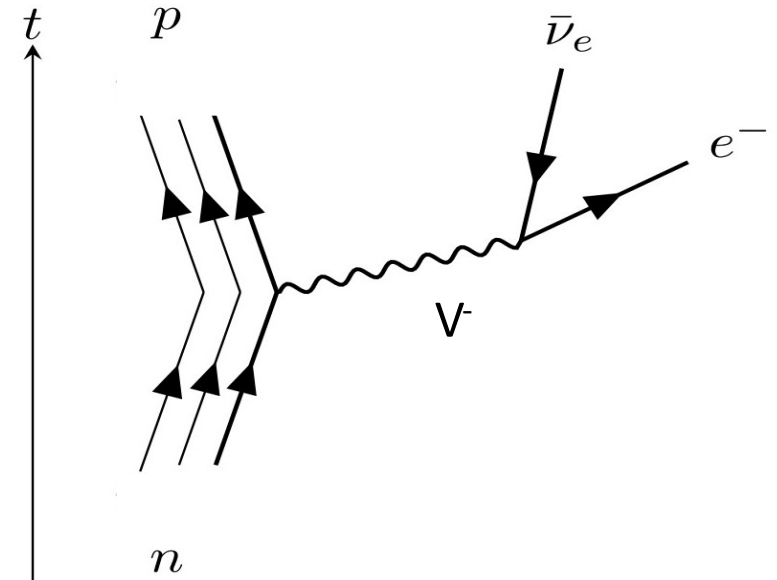
$$T_{BH} = T_H = \frac{\hbar c^3}{8\pi k_B G_N M}$$

$$S_{BH} = \frac{k_B c^3 A}{4\hbar G_N}$$

J. Bekenstein, Lett. Nuovo Cimento, 1972

- To ensure conservation of electric charge, **photon** interactions remain unchanged under the shift (S is an arbitrary scalar): $\gamma \rightarrow \gamma + \partial S$
- Uranium disassociates via the **decay of neutrons** into protons, electrons and neutrinos.
- The decay is mediated by a **massive and charged** photon V .
- The mass **breaks** $V \rightarrow V + \partial S$ symmetry and **charge conservation breaks down**.
- To restore charge conservation one replaces **mass by some scalar field**: $M_V^2 \rightarrow \phi^2$

the god particle
(Higgs particle)

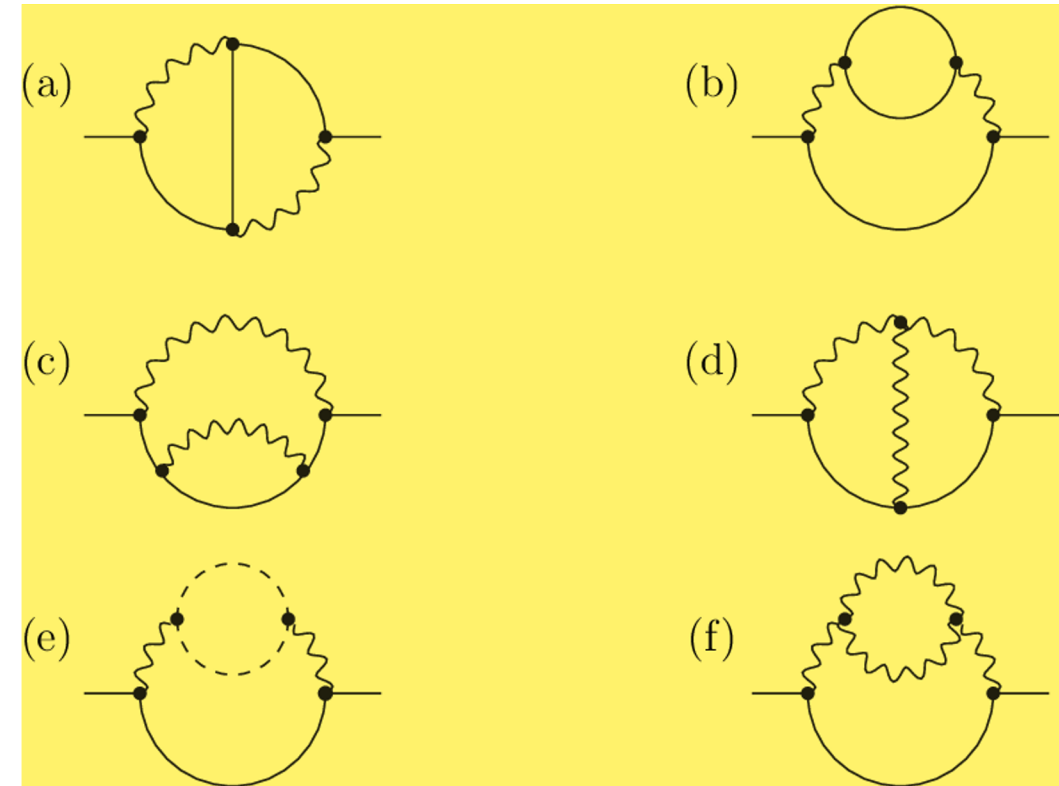


CHARGE CONSERVATION BY CURVATURE

- Quantum fields can be understood only by a **UV cutoff** Λ_{UV} .
- The cutoff Λ_{UV} **breaks translation symmetry**.
- Quantum loops **generate masses** in proportion to Λ_{UV} .
- **Photon acquires mass** in proportion to Λ_{UV} .
- Photon mass **breaks** $\gamma \rightarrow \gamma + \partial S$ symmetry and electric **charge conservation breaks down**.
- To restore electric charge conservation one replaces **the cutoff by curvature**: $\Lambda_{UV}^2 \rightarrow R$

∴ curvature breaks translation symmetry

symmetry-restoring emergent gravity: **symmergent gravity**



DD, Adv. High En. Phys., 2019
 DD, Gen. Relativity Gravit., 2021
 DD, C. Karahan, O. Sargin, 2023
 DD, Phys. Rev. D, 2023

- Symmergent gravity is new topic.
- It can be tested at particle colliders like CERN-LHC.
- It can bring explanations for elusive particles like dark matter.
- It can be tested at strong gravity domains like [black holes](#) and early Universe.

From the referee report:

“... This paper is still difficult because its point of view is novel. However, that is to be expected for novel ideas.”

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

gravity theory	solution of Einstein equations
Symmergent gravity action: $\int d(\text{Vol})_4 \left(\frac{R}{16\pi G_N} - V_0 + \frac{c_0}{16} R^2 - \frac{1-a}{(8\pi G_N)^2 c_0} \right)$	Schwarzschild-dS/AdS solution ($V_0 = 0$): $(ds)^2 = h(r)(cdt)^2 + \frac{(dr)^2}{h(r)} + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)$

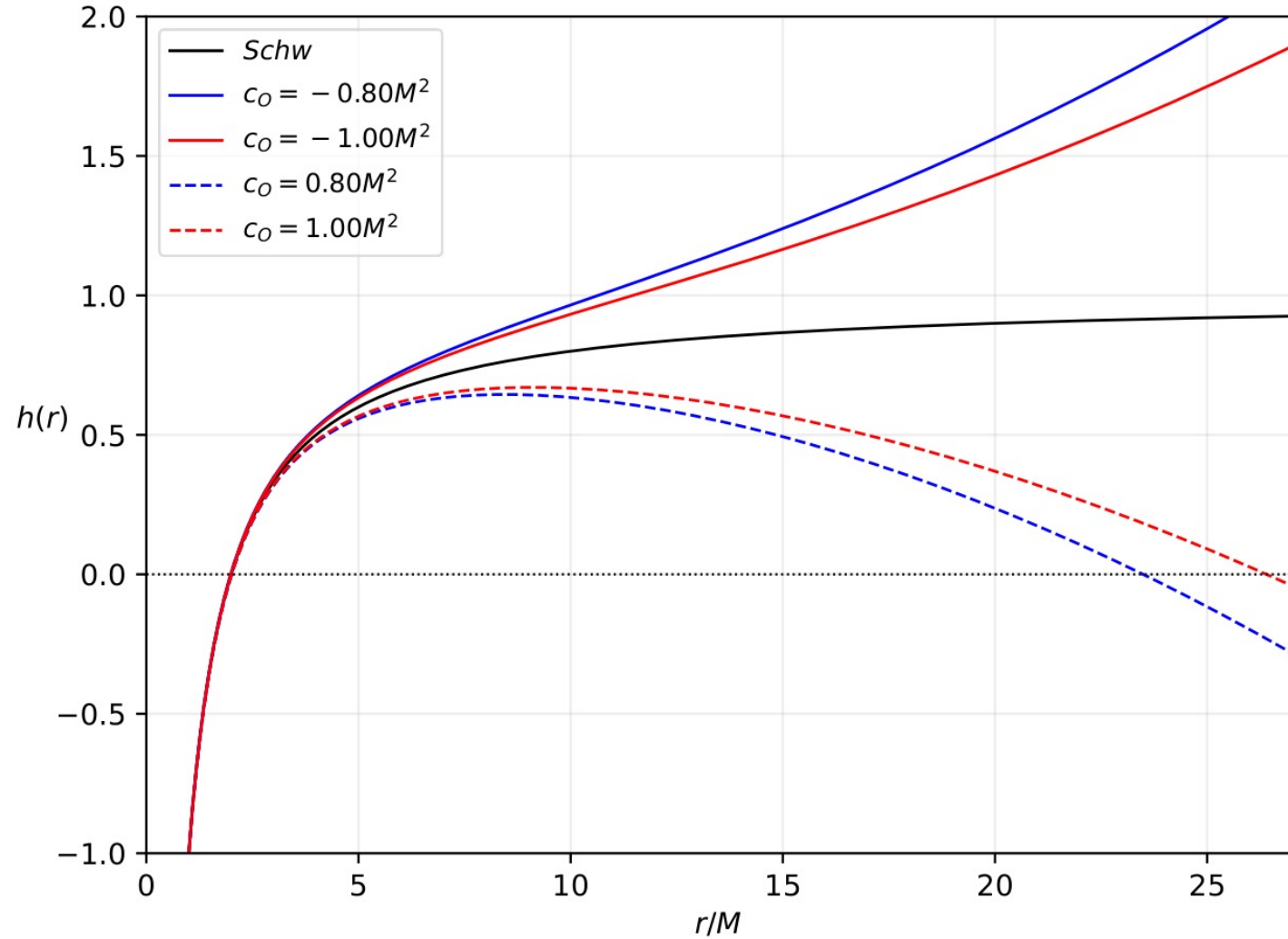
loop-induced quadratic curvature constant:

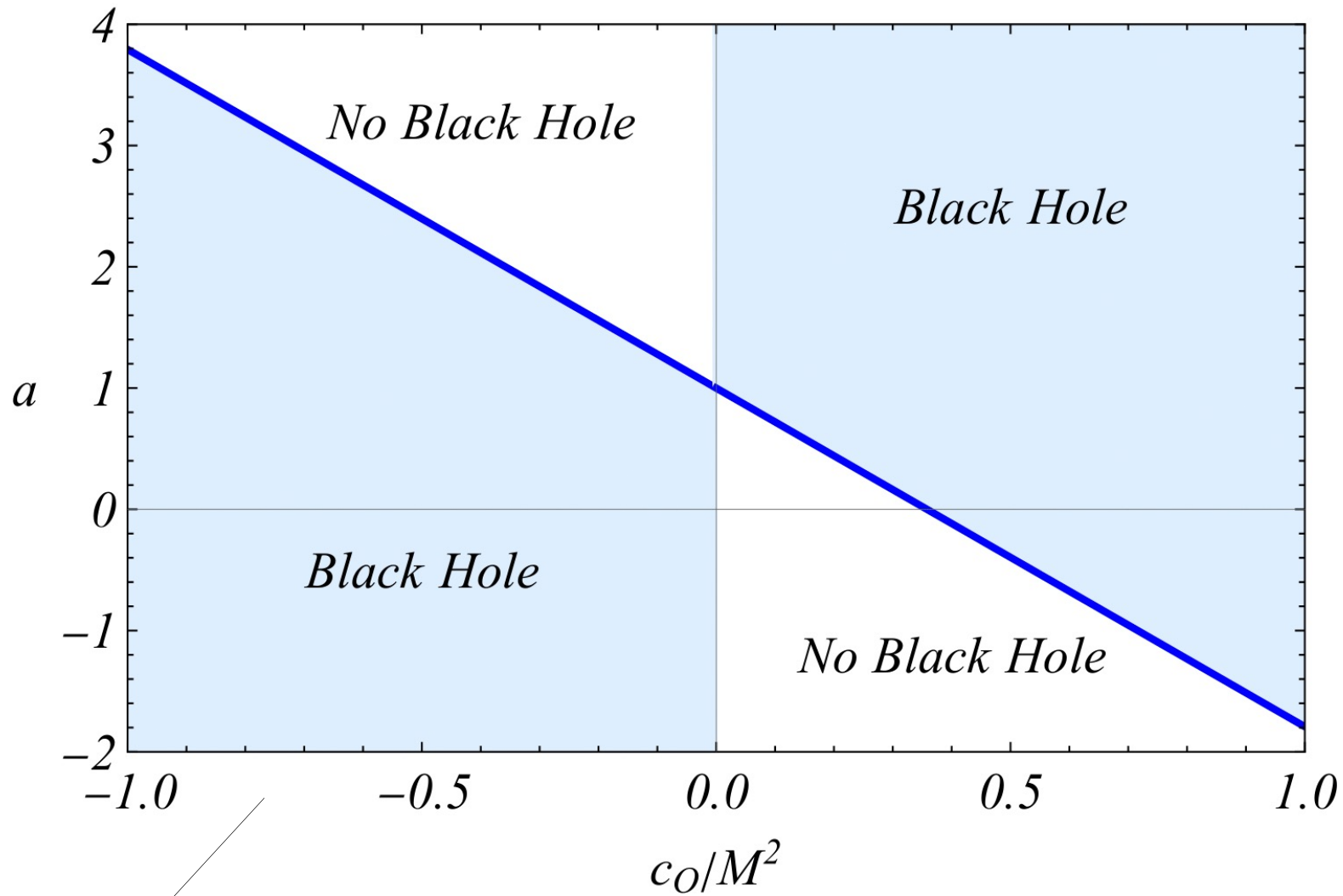
$$c_0 = \frac{n_B - n_F}{248\pi^2}$$

a : an $O(1)$ constant parametrizing the symmergent vacuum energy

$$h(r) = - \left(1 - \frac{r_s}{r} - \frac{(1-a)r^2}{24\pi G_N c_0} \right)$$

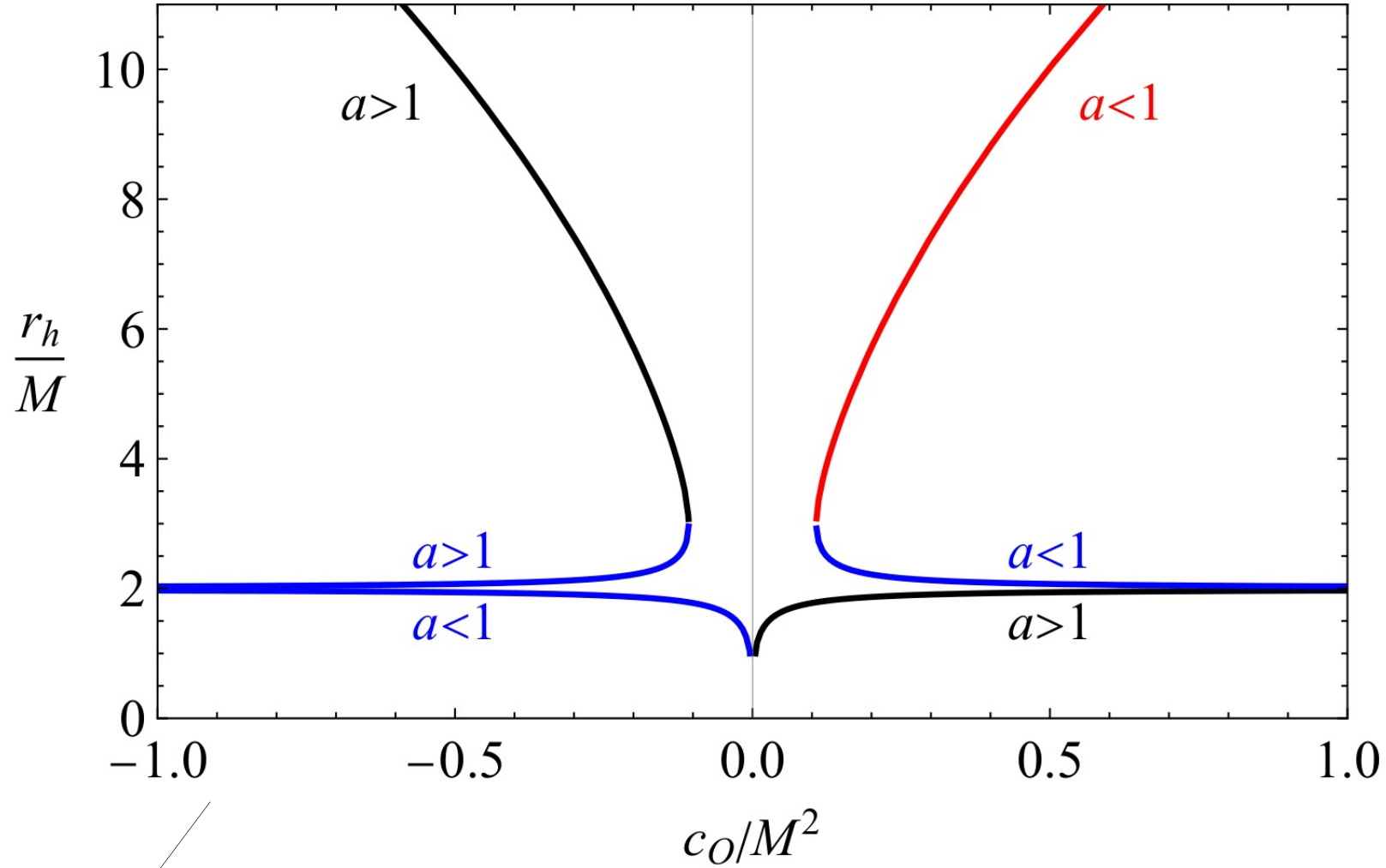
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE





the regions in $a - c_0$ plane admitting BH solutions

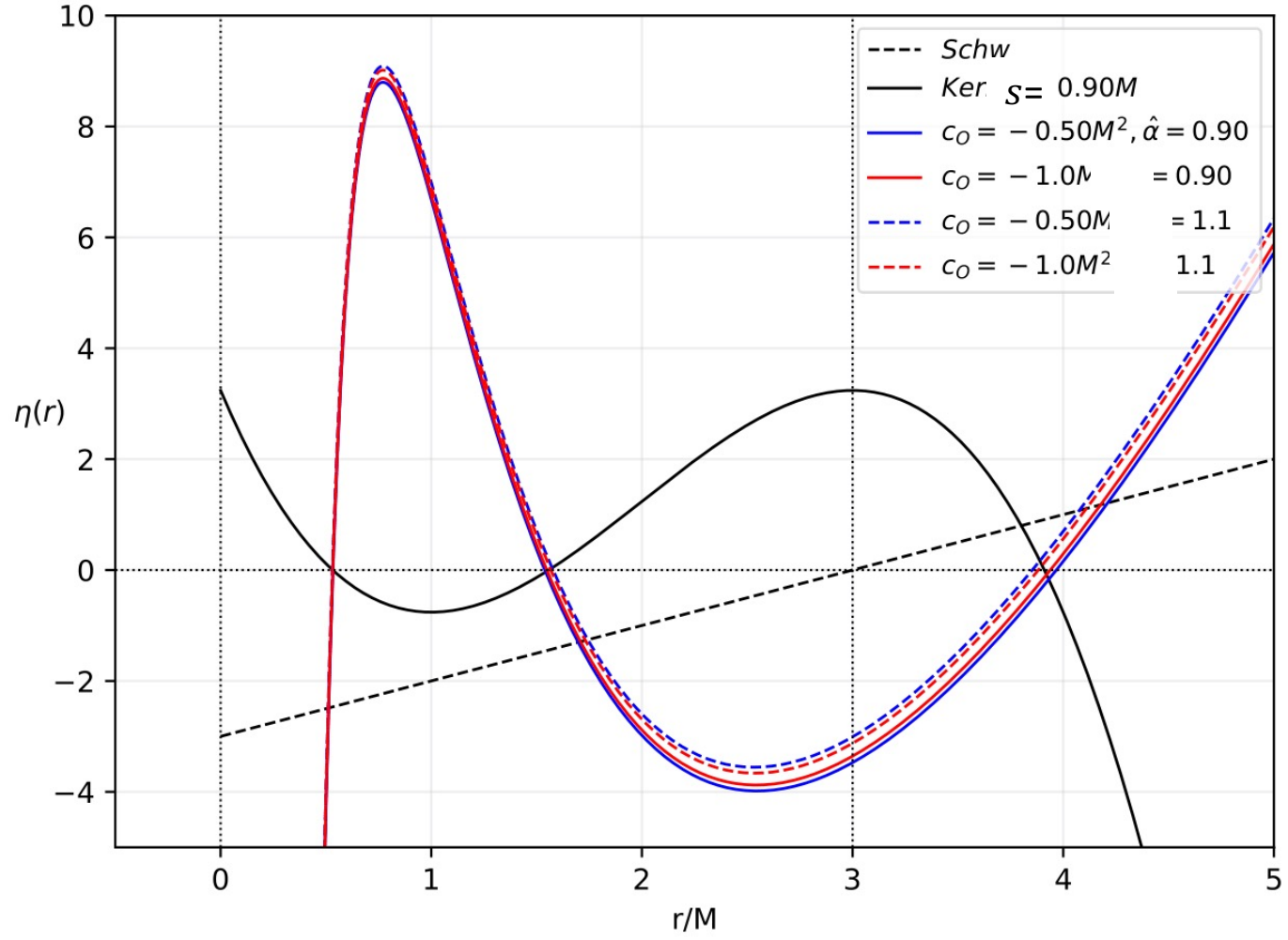
J. Raimbayev et al, Annals of Physics, 2023
 İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021



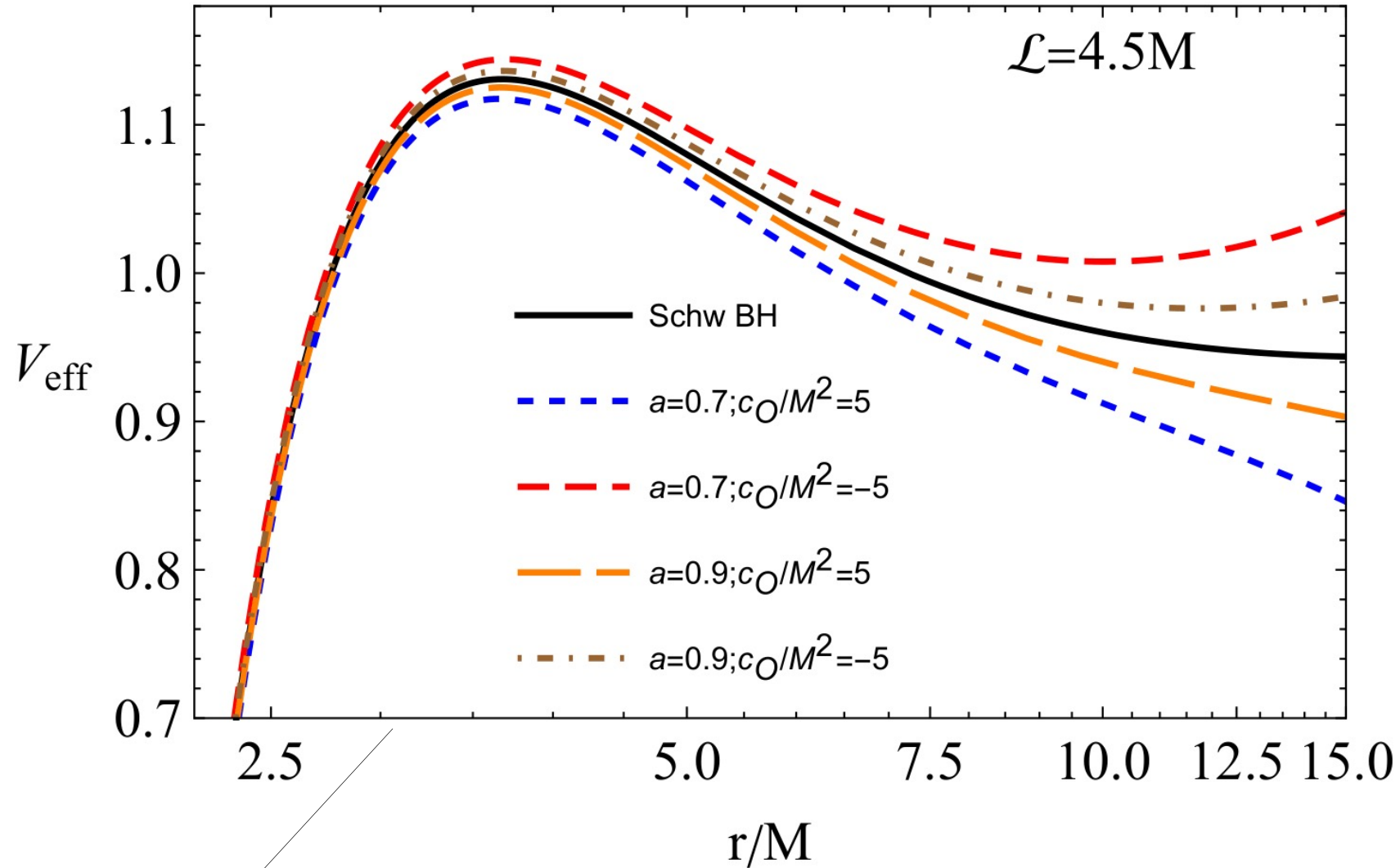
event horizon: $r_h = 2 M$ is the Schwarzschild radius

J. Raimbayev et al, Annals of Physics, 2023
 İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



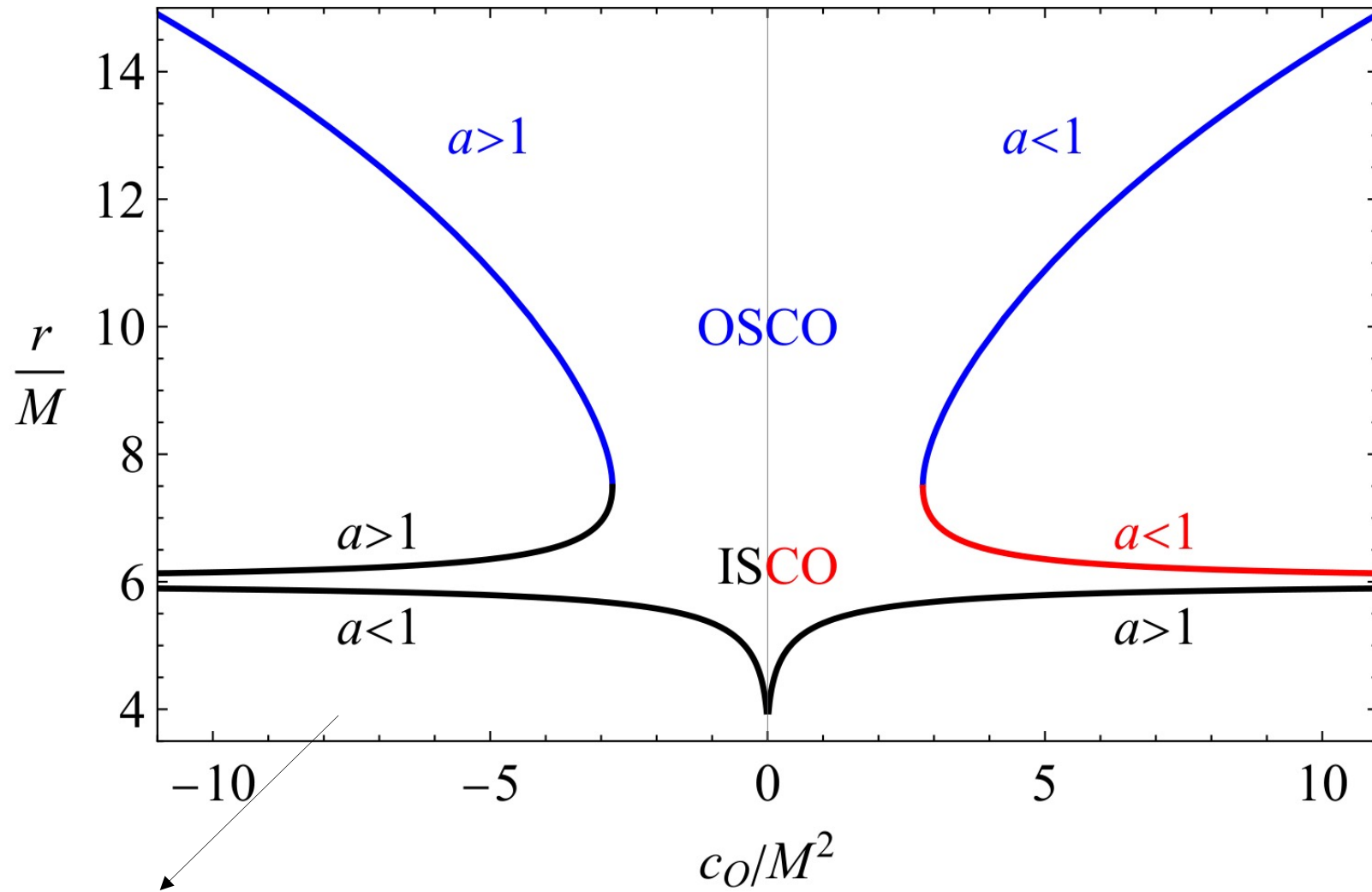
Photon sphere radius for a rotating symmergent black hole of angular momentum number s)



effective potential for $\ell = 4.5 M$

J. Raimbayev et al, Annals of Physics, 2023
 İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021

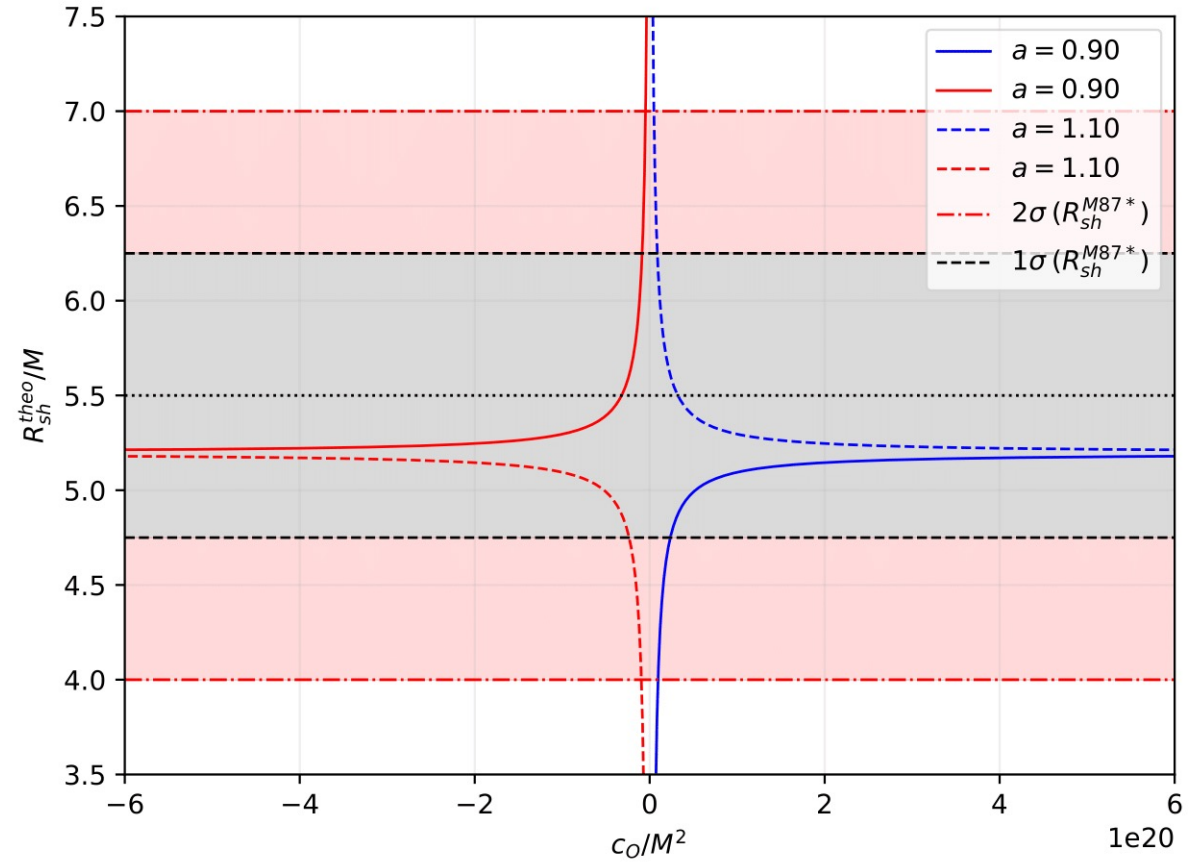
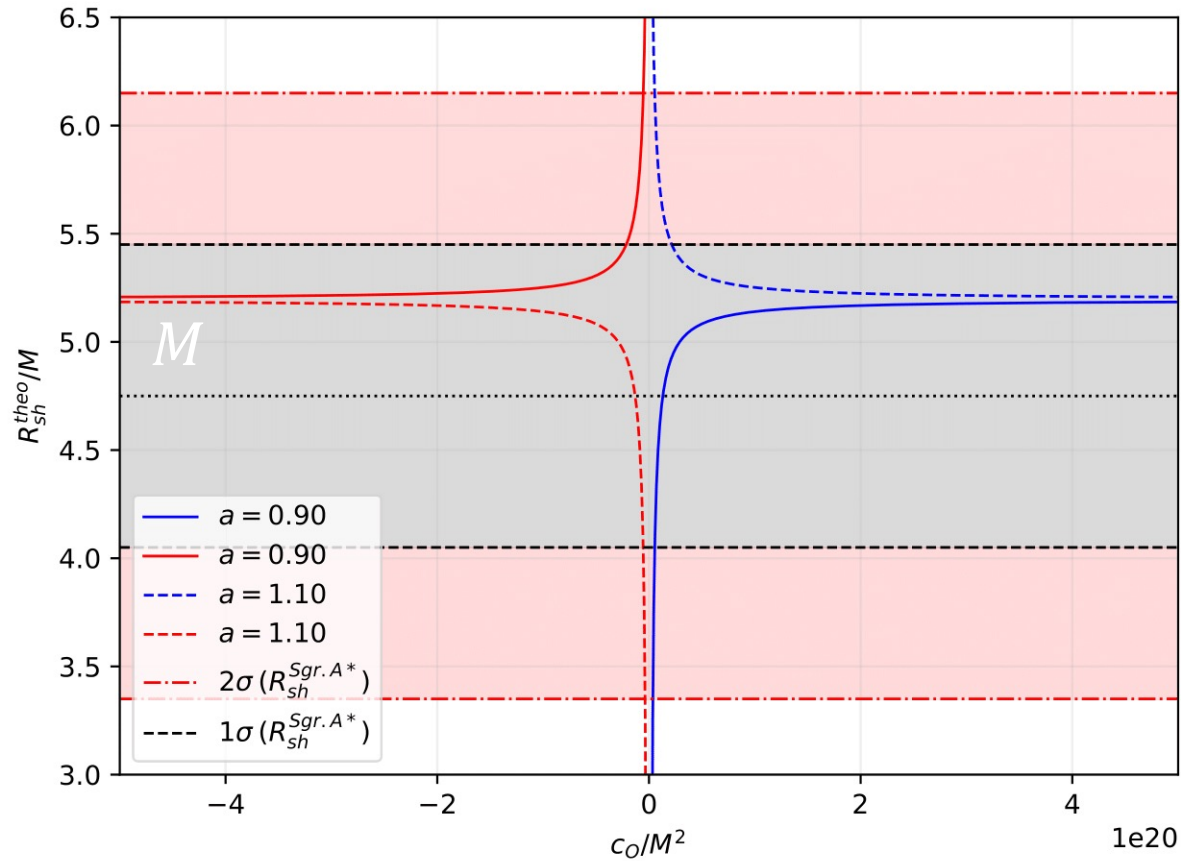
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



innermost (ISCO) and outermost (OSCO) stable circular orbits

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021

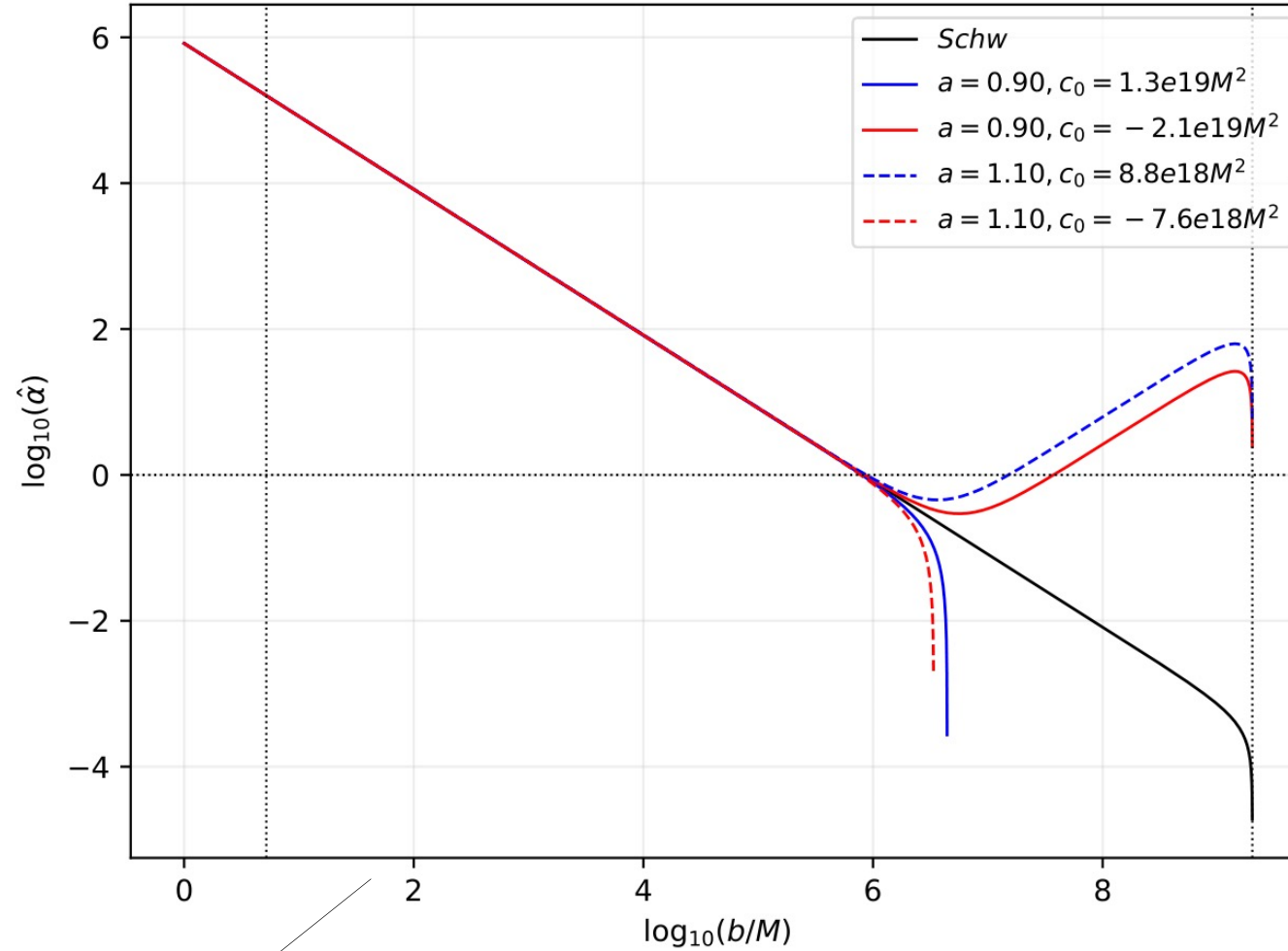
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



bounds on model parameters form shadow radii of Sgr.A* (left) and M87* (right)

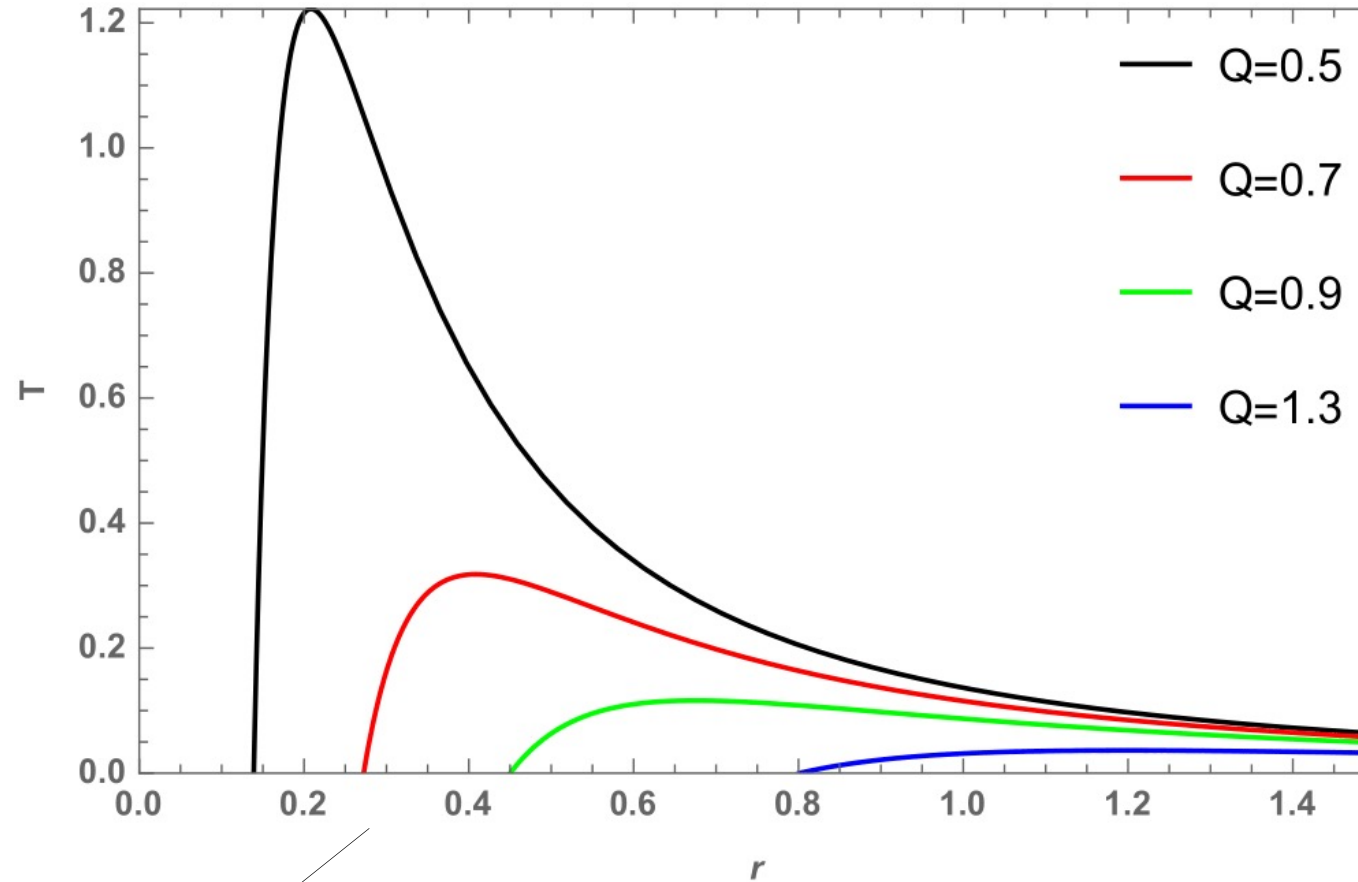
J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE



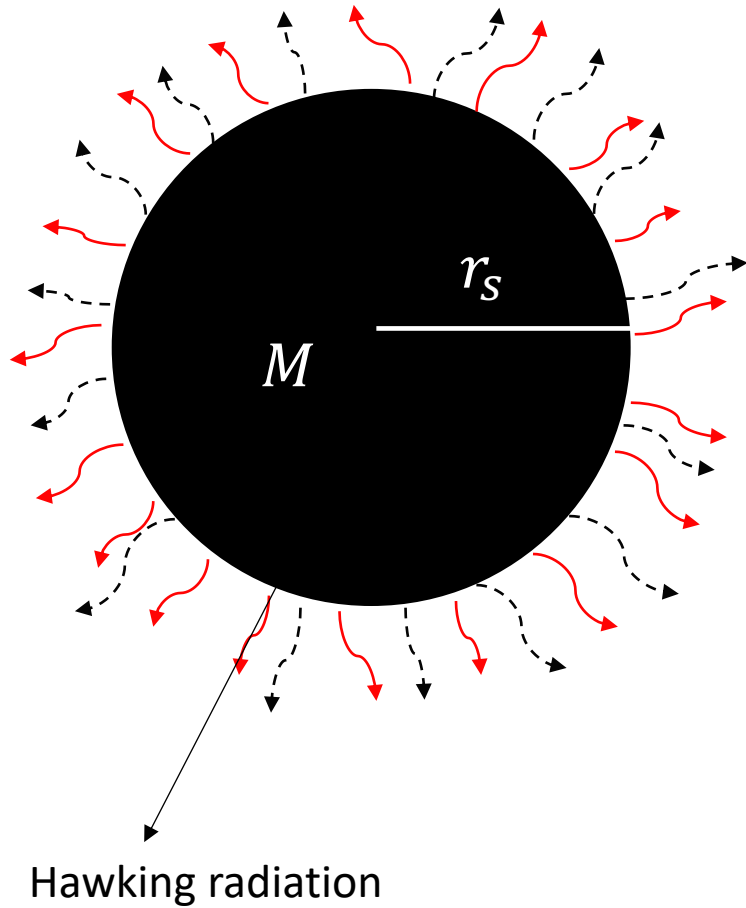
bending angle (μas) for a nearby source ($r_o/20$) and an observer with impact parameter b .

J. Raimbayev et al, Annals of Physics, 2023
 İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021



Hawking temperature ($r_s = 1$) for a charged symmergent black hole ($c_0 = 0.9$ and $a = 0.5$).

SYMMERGENT BLACK HOLE



Hawking radiation from photon, electron etc. :



Hawking radiation from new dark fields:



Black hole temperature and evaporation rate
change if symmergent particles are included!

THANK YOU!