BLACK HOLES AND PARTICLES AROUND

Durmuş Demir

Sabancı University

http://myweb.sabanciuniv.edu/durmusdemir/
Flat space

Curved space

E. Siegel, Forbes, 2018
two infinitesimally close events separated by a distance $ds$

$$ (ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 $$

two events can have zero separation ($ds = 0$) if they coincide ($dx = 0, dy = 0, dz = 0$)

space is flat
FLAT SPACETIME

$\Delta s^2 = -(c \Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$

“minus” sign is needed for light to propagate

two events can have zero separation ($\Delta s = 0$) even if they do not coincide ($\Delta t \neq 0, \Delta x \neq 0, \Delta y \neq 0, \Delta z \neq 0$)

spacetime is flat
4D

in spherical coordinates:

$$(ds)^2 = -(cdt)^2 + (dr)^2 + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

spacetime is flat
(coordinate change has no effect)
in an environment with intrinsic speed $v_{int}$:

$$(ds)^2 = -\left(1 - \frac{v_{int}^2}{c^2}\right)(cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{int}^2}{c^2}} + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)$$

**FLAT DILATED SPACETIME**
in the presence of a body (Jupiter) of mass $M$, the escape speed $v_{es}$ turns out to be the most natural intrinsic speed ($v_{int} = v_{es}$):

- escape speed from the body:
  \[
  \frac{1}{2} m v_{es}^2 = \frac{G N M m}{r} \implies v_{es}^2 = \frac{2 G N M}{r}
  \]

- distance between two points outside the body:
  \[
  (ds)^2 = -\left(1 - \frac{v_{es}^2}{c^2}\right)(cdt)^2 + \frac{(dr)^2}{1 - \frac{v_{es}^2}{c^2}} + r^2((d \theta)^2 + \sin^2 \theta (d \phi)^2)
  \]

gravitational time dilation

gravitational space contraction
CURVED SPACETIME

- Shwarzschild radius:

\[
\frac{v_{es}^2}{c^2} = \frac{2G_N M}{rc^2} \equiv \frac{r_s}{r} \implies r_s = \frac{2G_N M}{c^2}
\]

- \( ds \) can be expressed in terms of Schwarzschild radius \( r_s \):

\[
(ds)^2 = -\left(1 - \frac{r_s}{r}\right)(c dt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 \left((d \theta)^2 + \sin^2 \theta (d \phi)^2\right)
\]

- \( r_s \) (Sun) = 3 km
- \( r_s \) (Jupiter) = 3 m
- \( r_s \) (Earth) = 10 mm
- distance $ds$ for the escape speed turns out to be the solution of the Einstein equations for gravity of a compact body of mass $M$:

<table>
<thead>
<tr>
<th>gravity theory</th>
<th>solution of Einstein equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Einstein gravity action:</td>
<td>Schwarzschchild solution (zero vacuum energy $V_0 = 0$):</td>
</tr>
<tr>
<td>$\int d(Vol)_4 \left( \frac{R}{16\pi G_N} - V_0 \right)$</td>
<td>$(ds)^2 = - \left( 1 - \frac{r_s}{r} \right) (c dt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 (d\theta)^2 + \sin^2 \theta (d\phi)^2$</td>
</tr>
</tbody>
</table>

- true measure of curving is the Riemann curvature:

$$(\text{Riemann curvature})^2 = \frac{12 r_s^2}{r^6}$$

BLACK HOLE

- if the body $M$ is too massive to require an escape speed bigger than the speed of light:

$$v_{es} > c$$

(or $r < r_s$) then a black hole forms:

- $r = r_s$ ⇒ event horizon (the point of no return)
- $r = r_s$ ⇒ event horizon (infinite time dilation)
- $r < r_s$ ⇒ black hole interior (cannot be probed from outside)
- $r < r_s$ ⇒ black hole interior (effectively: time $\Leftrightarrow$ radial distance)
PHOTON MOTION AROUND BLACK HOLE

- Photons: \((ds)^2 = 0 \implies \left(\frac{ds}{d\tau}\right)^2 = 0\)

- Azimuthal plane \((\theta = \frac{\pi}{2})\):
  
  
  \[-(1 - \frac{r_s}{r})c^2 \dot{t}^2 + \frac{\dot{r}^2}{1 - \frac{r_s}{r}} + r^2 \dot{\phi}^2 = 0\]

- Energy is conserved: \(\dot{t} = \frac{\epsilon}{1 - \frac{r_s}{r}}\)

- Angular Momentum is conserved: \(\dot{\phi} = \frac{\ell}{r^2}\)

- Photon is a unit-mass particle:
  
  \[\frac{\dot{r}^2}{2} + \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\epsilon}{2}\]

- \(V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}\)

![Energy barrier](energy_barrier.png)
photon radius $r = r_{ph}$ is the last stable orbit.

- photon radius $r = r_{ph}$ depends on the underlying gravity theory.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (Solar Masses)</td>
<td>6.54 billion</td>
</tr>
<tr>
<td>Event Horizon diameter (AU)</td>
<td>258</td>
</tr>
<tr>
<td>Distance (Light Years)</td>
<td>55 million</td>
</tr>
</tbody>
</table>
black hole shadow

- photons falling within the photon sphere fall into black hole – a large shadow!

- consider a general spacetime:

\[(ds)^2 = -A(r)(c\,dt)^2 + B(r)(dr)^2 + D(r)((d\,\theta)^2 + \sin^2\theta\,(d\phi)^2)\]

- shadow is characterized by “gravitational capture angle” \(\equiv\) shadow angle \(\alpha_{sh}\):

\[
\sin^2\alpha_{sh} = \frac{D(r_{ph})A(r_o)}{A(r_{ph})D(r_o)} \text{ Schr"{o}dinger } \frac{r_{ph}^2}{1 - \frac{r_s}{r_{ph}}} \times \frac{\left(1 - \frac{r_s}{r_o}\right)}{r_o^2}
\]

Distant observer: \(\frac{27}{4}\frac{r_s^2}{r_o^2}\)
Shadow of a black hole

Thomas Bronzwaer and Heino Falcke 2021 *ApJ* 920 155

Shadow of a black hole

EHT observation of Sgr. A* in 2019
Shadow of M87* (2019)

EHT observation of M87* in 2019

Shadow of M87* (AI)

PRIMO, 2023

Lia Medeiros et al 2023 ApJL 947 L7
\[ \frac{dr}{d\tau} = \sqrt{\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)} \]

\[ \frac{d\phi}{d\tau} = \frac{1}{r^2} \]

\[ \varphi_D = 2 \int_{R_0}^{\infty} \frac{dr}{r^2 \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{r_s}{r}\right)\right)^{1/2}} \text{ small } r_s \rightarrow \frac{2r_s}{R_0} \]

Burger, D. et al 2018 Gen Relativ Gravit 50, 156
bending of light leads to multiple images for objects behind (lensing effect)

P. Laursen, 2021
effective potential seen by massless particle (photon):

\[ V_{\text{eff}}(r) = \frac{\ell^2}{2r^2} \left( 1 - \frac{r_s}{r} \right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3} \]

potential energy barrier for massless particles is formed by angular momentum \((1/r^2)\) and Schwarzschild radius \((r_s/r^3)\)

potential energy barrier for massive particles involves in addition the Newtonian contribution \((1/r)\)

quantum particles that fell into the black hole can tunnel out through the barrier.

tunneled particles appear as radiation – the Hawking radiation.
Time it takes to traverse the barrier region:

\[
\Delta t = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - 2V_{eff}(r)}} = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - \frac{\ell^2}{r^2} + \frac{\ell r_s}{r^3}}}
\]

\[
= \pm \frac{r_s^2}{\sqrt{2\ell^2 - 3\ell}} \int_{r_{in}}^{r_{out}} \frac{d\hat{r}}{\sqrt{\hat{r} - 1}}
\]

(expand at \(r = r_s\))

\(E = \ell(\ell - 1)/r_s^2\)

\[
= \pm \frac{r_s^2}{\sqrt{2\ell^2 - 3\ell}} \times \pi i \approx \pm \pi i r_s
\]

(residue theorem)

\(\ell \approx r_s\)

Wavefunction: \(\psi \propto e^{iE\Delta t} \approx e^{\pm \pi i r_s E}\)

\(\rho_{em} \approx e^{-2\pi r_s E}\)

\(\rho_{ab} \approx e^{+2\pi r_s E}\)

Net emission rate:

\[
\frac{\rho_{em}}{\rho_{ab}} = e^{-4\pi r_s E} \implies T_H \approx \frac{1}{4\pi k_B r_s} \equiv \frac{\hbar c^3}{8\pi k_B G_N M}
\]
Emitted power (photons only):

\[ P = \frac{\hbar c^6}{15360\pi G_N^2 M^2} \]

Evaporation time (photons only):

\[ t_{eva} = \frac{5120 \pi G_N^2 M^3}{\hbar c^4} \approx 10^{67} \text{ years} \times \left(\frac{M}{M_{\text{Sun}}}\right)^3 \]

D. N. Page, Phys. Rev. D 1976
Figure 10.7: Quantum decay of a non-rotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν) and other elementary particles are given in percent of the total number of particles emitted by black holes of different masses.

Frolov and Novikov 1998 “Black hole physics” (Springer)
In classical physics: Black holes must have zero temperature and zero entropy!

In quantum physics: Black holes must have nonzero temperature and nonzero entropy!

First law of thermodynamics: \( \Delta E = \Delta Q = T_{BH} \Delta S_{BH} \)

\[
T_{BH} = T_H = \frac{\hbar c^3}{8\pi k_B G_N M}
\]

\[
S_{BH} = \frac{k_B c^3 A}{4\hbar G_N}
\]

To ensure conservation of electric charge, photon interactions remain unchanged under the shift ($S$ is an arbitrary scalar): $\gamma \rightarrow \gamma + \partial S$

Uranium disassociates via the decay of neutrons into protons, electrons and neutrinos.

The decay is mediated by a massive and charged photon $V$.

The mass breaks $V \rightarrow V + \partial S$ symmetry and charge conservation breaks down.

To restore charge conservation one replaces mass by some scalar field: $M_V^2 \rightarrow \phi^2$

the god particle (Higgs particle)

Quantum fields can be understood only by a UV cutoff $\Lambda_{UV}$.

The cutoff $\Lambda_{UV}$ breaks translation symmetry.

Quantum loops generate masses in proportion to $\Lambda_{UV}$.

Photon acquires mass in proportion to $\Lambda_{UV}$.

Photon mass breaks $\gamma \rightarrow \gamma + \partial S$ symmetry and electric charge conservation breaks down.

To restore electric charge conservation one replaces the cutoff by curvature: $\Lambda_{UV}^2 \rightarrow R$

$\because$ curvature breaks translation symmetry

symmetry-restoring emergent gravity: symmergent gravity
Symmergent gravity is new topic.

- It can be tested at particle colliders like CERN-LHC.
- It can bring explanations for elusive particles like dark matter.
- It can be tested at strong gravity domains like black holes and early Universe.

From the referee report:

“... This paper is still difficult because its point of view is novel. However, that is to be expected for novel ideas.”
gravity theory | solution of Einstein equations
---|---
Symmergent gravity action: \[
\int d(\text{Vol})_4 \left( \frac{R}{16\pi G_N} - V_0 + \frac{c_0}{16} R^2 - \frac{1-a}{(8\pi G_N)^2 c_0} \right)
\] | Schwarzschild-dS/AdS solution \((V_0 = 0)\): \[
(ds)^2 = h(r)(c dt)^2 + \frac{(dr)^2}{h(r)} + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)
\]

loop-induced quadratic curvature constant: \(c_0 = \frac{n_B - n_F}{248\pi^2}\)

\(h(r) = -\left(1 - \frac{r_s}{r} - \frac{(1-a)r^2}{24\pi G_N c_0}\right)\)

\(a: \) an \(O(1)\) constant parametrizing the symmergent vacuum energy

İ. Çimdiker, A. Övgün, DD, Phys. Dark Univ., 2021
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

\[ h(r) \]

- \( Schw \)
- \( c_O = -0.80M^2 \)
- \( c_O = -1.00M^2 \)
- \( c_O = 0.80M^2 \)
- \( c_O = 1.00M^2 \)

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
the regions in $a - c_0$ plane admitting BH solutions

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
event horizon: $r_h = 2M$ is the Schwarzschild radius

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
Photon sphere radius for a rotating symmergent black hole of angular momentum number \( s \).
effective potential for $\ell = 4.5 \, M$
innermost (ISCO) and outermost (OSCO) stable circular orbits

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
bounds on model parameters form shadow radii of Sgr.A* (left) and M87* (right)

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
bending angle ($\mu\text{as}$) for a nearby source ($r_0/20$) and an observer with impact parameter $b$. 

J. Raimbayev et al, Annals of Physics, 2023
İ. Çimdiker, A. Övgün, DD, Phys.Dark Univ., 2021
Hawking temperature ($r_s = 1$) for a charged symmergent black hole ($c_0 = 0.9$ and $a = 0.5$).
Hawking radiation from photon, electron etc.:

Hawking radiation from new dark fields:

Black hole temperature and evaporation rate change if symmergent particles are included!

B. Puliçe, R. Pantig, A. Övgün, DD, work in progress, 2023
THANK YOU!