Symmergent Gravity Black Holes with Observational Constraints

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A body of mass $M$ creates the gravitational potential:

$$V(r) = -\frac{GM}{r}$$

Spacetime is flat:

$$(ds)^2 = -(c dt)^2 + (dr)^2 + r^2 ((d \theta)^2 + \sin^2 \theta (d\phi)^2)$$

Gravity is a force like electricity.

Newtonian gravity is not able to account for the perihelion advancement in Mercury’s orbit.
¬ Gravity: Einstein’s View

- Escape speed from a body of mass $M$:

$$\frac{v_{es}^2}{c^2} = \frac{2GM}{c^2r} \Rightarrow r_s = \frac{2GM}{c^2}$$

- $r_s$ (Sun) = 3 km
- $r_s$ (Jupiter) = 3 m
- $r_s$ (Earth) = 10 mm

- Spacetime is curved:

$$ (ds)^2 = -\left(1 - \frac{r_s^2}{r^2}\right)(c dt)^2 + \left(\frac{dr}{1 - \frac{r_s^2}{r^2}}\right)^2 + r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2) $$

- Gravity is curving of spacetime.

- Einstein’s approach is able to account for perihelion advancement in Mercury’s orbit.
EINSTEIN’S GRAVITY

<table>
<thead>
<tr>
<th>gravity theory</th>
<th>solution of Einstein equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Einstein gravity action:</td>
<td>Schwarzschild solution (zero vacuum energy $V_0 = 0$) :</td>
</tr>
<tr>
<td>$\int d(Vol)_4 \left( \frac{R}{16\pi G_N} - V_0 \right)$</td>
<td>$(ds)^2 = -\left(1 - \frac{r_s}{r}\right)(cdt)^2 + \frac{(dr)^2}{1 - \frac{r_s}{r}} + r^2 \left((d\theta)^2 + \sin^2 \theta (d\phi)^2\right)$</td>
</tr>
</tbody>
</table>

- true measure of curving is the Riemann curvature:

$$(\text{Riemann curvature})^2 = \frac{12 r_s^2}{r^6}$$
if the body $M$ is too massive to require an escape speed bigger than speed of light:

$\nu_{es} > c$ or $r < r_s$

a black hole forms.
PHOTON MOTION AROUND BLACK HOLE

- Photon: \((ds)^2 = 0 \Rightarrow \left(\frac{ds}{d\tau}\right)^2 = 0\)

- Azimuthal plane \((\theta = \frac{\pi}{2})\): 
  \[-\left(1 - \frac{r_s}{r}\right) c^2 \dot{t}^2 + \frac{\dot{r}^2}{1 - \frac{r_s}{r}} + r^2 \dot{\phi}^2 = 0\]

- Energy is conserved: 
  \[\dot{t} = \frac{E}{\frac{r_s}{r}}\]

- Ang. Mom. is conserved: 
  \[\dot{\phi} = \frac{\ell}{r^2}\]

- Photon is a unit-mass particle: 
  \[\frac{\dot{r}^2}{2} + \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{E}{2}\]

- \(V_{eff}(r) = \frac{\ell^2}{2r^2} \left(1 - \frac{r_s}{r}\right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3}\)
Photons orbiting in xy-plane ($r = r_{ph} = \text{constant}, \ \theta = \frac{\pi}{2}$):

$$ds = 0 \Rightarrow \frac{d\phi}{dt} = \frac{c}{r_y} \left(1 - \frac{r_s}{r_y}\right)^{1/2}$$

EoM $\Rightarrow$ $\frac{d\phi}{dt} = \frac{c}{r_y} \left(\frac{r_s}{2r_y}\right)^{1/2}$

- Photon radius $r = r_{ph}$ is the last stable orbit.
- Photon radius $r = r_{ph}$ depends on the underlying gravity theory.
### Photon Sphere

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Mass (Solar Masses)</td>
<td>6.54</td>
</tr>
<tr>
<td></td>
<td>billion</td>
</tr>
<tr>
<td>Event Horizon diameter (AU)</td>
<td>258</td>
</tr>
<tr>
<td>Distance (Light Years)</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>million</td>
</tr>
</tbody>
</table>

(galaxy Messier 87 in the constellation Virgo)
BLACK HOLE SHADOW

- photons falling within the photon sphere fall into black hole – a large shadow!

- consider a general spacetime:

\[
(ds)^2 = -A(r)(c \, dt)^2 + B(r)(dr)^2 + D(r)((d \theta)^2 + \sin^2 \theta \, (d\phi)^2)
\]

- shadow is characterized by “gravitational capture angle” ≡ shadow angle \( \alpha_{sh} \):

\[
\sin^2 \alpha_{sh} = \frac{D(r_{ph})A(r_o)}{A(r_{ph})D(r_o)} \text{ Schwarzschild } \frac{r_{ph}^2}{1 - \frac{r_s}{r_{ph}}} \times \left(1 - \frac{r_s}{r_o}\right) \frac{r_o^2}{r_{ph}^2}
\]

Distant observer: \[ \frac{27}{4} \frac{r_s^2}{r_o^2} \]
Shadow of a black hole

(Thomas Bronzwaer and Heino Falcke 2021 ApJ 920 155)

Shadow of a black hole

(EHT observation of Sgr. A* in 2019)
Shadow of M87* (2019)  
(EHT, 2019)  

Shadow of M87* (AI)  
(PRIMO, 2023)  

(Lia Medeiros et al 2023 ApJL 947 L7)
LIGHT BENDING BY BLACK HOLE

\[ \frac{dr}{d\tau} = \sqrt{\frac{1}{b^2} - \frac{1}{r^2} \left( 1 - \frac{r_s}{r} \right)} \]

\[ \frac{d\phi}{d\tau} = \frac{1}{r^2} \]

\[ \varphi_D = 2 \int_{R_0}^{\infty} \frac{dr}{r^2 \left( \frac{1}{b^2} - \frac{1}{r^2} \left( 1 - \frac{r_s}{r} \right) \right)^{\frac{1}{2}}} \quad \text{small } r_s \rightarrow 2r_s \]

(Burger, D. et al 2018 Gen Relativ Gravit 50, 156)
Light bending by black hole

Bending of light leads to multiple images for objects behind (lensing effect)

(P. Laursen, 2021)
QUANTUM TUNNELING AND HAWKING RADIATION

- Effective potential seen by massless particle (photon):
  \[ V_{eff}(r) = \frac{\ell^2}{2r^2} \left( 1 - \frac{r_s}{r} \right) = \frac{\ell^2}{2r^2} - \frac{\ell r_s}{2r^3} \]

- Potential energy barrier for massless particles is formed by angular momentum \(1/r^2\) and Schwarzschild radius \(r_s/r^3\)

- Potential energy barrier for massive particles involves in addition the Newtonian contribution \(1/r\)

- Quantum particles that fell into the black hole can tunnel out through the barrier.

- Tunneled particles appear as radiation – the Hawking radiation.
QUANTUM TUNNELING AND HAWKING RADIATION

- Time it takes to traverse the barrier region:

\[
\Delta t = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - 2V_{eff}(r)}} = \pm \int_{r_{in}}^{r_{out}} \frac{dr}{\sqrt{E - \ell^2/r^2 + \ell r_s/r^3}} = \pm \frac{r_s^2}{\sqrt{2\ell^2 - 3\ell}} \int_{r_{in}}^{r_{out}} \frac{d\hat{r}}{\sqrt{\hat{r} - 1}}
\]

(expand at \(r = r_s\))

\(E = \ell(\ell - 1)/r_s^2\)

- Wavefunction:

\[\psi \propto e^{iE\Delta t} \approx e^{\pm i\pi r_s E}\]

- Black body spectrum (a rough prediction):

\[T \simeq \frac{1}{4\pi r_s} \equiv \frac{\hbar c^3}{8\pi k_B G_N M}\]

Hawking temperature

\[
\rho_{em} \approx e^{-2\pi r_s E}
\]

\[
\rho_{ab} \approx e^{+2\pi r_s E}
\]
Emitted power (photons only):

\[ P = \frac{\hbar c^6}{15360\pi G_N^2 M^2} \]

Evaporation time (photons only):

\[ t_{eva} = \frac{5120 \pi G_N^2 M^3}{\hbar c^4} \approx 10^{67} \text{ years} \times \left( \frac{M}{M_{\text{Sun}}} \right)^3 \]

(D. N. Page 1976 PRD 13, 198)
Figure 10.7: Quantum decay of a non-rotating black hole. The fractions of gravitons (g), photons (γ), neutrinos (ν) and other elementary particles are given in percent of the total number of particles emitted by black holes of different masses.

(Frolov and Novikov 1998 “Black hole physics” (Springer))
### SYMMERGENT GRAVITY

**Poincare-conserving UV cutoff \( M_\phi \):**

Tree-level mass term: 
\[
M_\phi^2 \tr[V_\mu V^\mu] 
\]

**Spurion \( S \):**

\[
\tr[S^\dagger V_\mu V^\mu S] 
\]

**Spurion \( S \) → Higgs \( \phi \):**

\[
\tr[(D_\mu \phi)^\dagger D^\mu \phi] 
\]

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**Poincare-breaking UV cutoff \( \Lambda_\phi \):**

Loop-level mass term: 
\[
\Lambda_\phi^2 \tr[V_\mu V^\mu] 
\]

**Spurion \( \Sigma \):**

\[
\tr[V_\mu \Sigma_{\mu\nu} V^\nu] 
\]

**Spurion \( \Sigma \) → Affine curvature \( \mathcal{R} \):**

\[
\tr[V_\mu \mathcal{R}_{\mu\nu} V^\nu] 
\]

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**References:**

P. Higgs, Phys. Rev. Lett. 13, 508 (1964)

**DD:**

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)
SYMMERGENT GRAVITY

Poincare-conserving UV Cutoff (Higgs mechanism)

\[ M_\phi^2 \text{Tr}[V_\mu \eta^{\mu\nu} V_\nu] \quad \Phi^\dagger V_\mu \eta^{\mu\nu} V_\nu \Phi \quad \eta^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi) \]

- eq. (6)
- eq. (7)

Poincare-breaking UV Cutoff (Symmergence)

\[ \Lambda^2 g_\mu V_\mu \eta^{\mu\nu} V_\nu \quad V_\mu \mathbb{R}^{\mu\nu}(\Gamma) V_\nu \quad V_\mu (\mathbb{R}^{\mu\nu}(\Gamma) - R^{\mu\nu}(\theta \Gamma)) V_\nu \]

- eq. (24)
- eq. (35)

\[ \Phi \text{-potential [eq. (8)]} \]
\[ M_\phi \rightarrow \Phi \]
\[ \Phi = 0 \quad \Phi \]

\[ \text{gauge symmetry is restored [eq. (9)]} \]

\[ \Gamma \approx \theta \Gamma \quad \mathbb{R}(\Gamma) \approx R(\theta \Gamma) \]

\[ \text{gravity emerged [eq. (46)]} \]
\[ \text{gauge symmetry is restored [eq. (45)]} \]

DD, Phys. Rev. D 107, 105014 (2023)
\[ S_{\text{QFT+GR}} = S(g, \psi) + \delta S(g, \psi) + \int d^4x \sqrt{-g} \left\{ -\frac{R(g)}{16\pi G_N} - \frac{c_0}{16} R(g)^2 - \frac{c_\phi}{4} R(g) \phi^\dagger \phi + \mathcal{O}(G_N) \right\} \]

**QFT with**

- dimensional-regularization in curved background geometry,
- loop corrections computed in the flat spacetime QFT

**R + R^2 gravity with**

- non-minimal coupling to scalars,
- loop-induced coefficients originating from the flat spacetime QFT.

*symmetry-restoring emergent gravity = “symmergent gravity”*

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)
\[ S_{sgr} = \int d^4x \sqrt{-g} \left\{ -\frac{R(g)}{16\pi G_N} - \frac{c_0}{16} R(g)^2 - V_{tot} \right\} \]

\[ G_N^{-1} = 8\pi \, \text{str}[M^2] \]

\[ c_0 = \frac{n_b - n_f}{128\pi^2} \]

\[ V_{tot} = V_{tree} + \delta V \]

\[ \delta V = \frac{1}{64\pi^2} \, \text{str}[M^4] \]

\[ \delta V = \frac{m_0^4}{64\pi^2} (n_b - n_f) = \frac{1}{24\pi G_N^2 c_0} \]

(just one possible parametrization)

\[ \text{(if bosons and fermions had equal masses } m_0) \]

J. Rayimbaev et al., Annals of Physics 454, 169335 (2023)

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 2\pi G_N c_O \left( R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} + \Box g_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} \right) R - 8\pi G_N V_{tot} = 0 \]

- One class of solutions corresponds to constant scalar curvature \((R = R_0 = -8\pi G_N V_{tot})\)
  - dS solution \((V_{tot} > 0\) or \(n_B > n_F\))
  - AdS solution \((V_{tot} < 0\) or \(n_B < n_F\))
  - \(c_O\) disappears from asymptotically-flat zero-\(R\) solution

- Another class corresponds to variable scalar curvature \((R \neq \text{constant})\)
  - There exist asymptotically-flat solutions explicitly involving \(c_O\)
**ASYMTOTICALLY-FLAT SYMMERGENT BLACK HOLES**

<table>
<thead>
<tr>
<th>gravity theory ($V_{tot} = 0$)</th>
<th>static spherically-symmetric solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmergent gravity action:</td>
<td>Buchdahl-Nguyen solution:</td>
</tr>
<tr>
<td>$\int d^4x \sqrt{-g} \left( -\frac{R}{16\pi G_N} - \frac{c_0}{16} R^2 \right)$</td>
<td>$(ds)^2 = A(r) ,(dt)^2 - \frac{(dr)^2}{B(r)} - C(r) \left( (d,\theta)^2 + \sin^2(\theta) ,(d\phi)^2 \right)$</td>
</tr>
</tbody>
</table>

\[
A(r) = e^{-\varphi(r)} \left( 1 - \frac{r_s}{r} \right) \quad B(r) = e^{\varphi(r)} \left( 1 - \frac{r_s}{r} \right) \quad C(r) = e^{-\varphi(r)} r^2
\]

\[
\frac{d}{dr} \left( r^2 - rr_s \frac{d\varphi(r)}{dr} \right) = -\gamma r^2 \varphi(r)
\]

\[
\gamma = -\frac{1}{6\pi c_0} = -\frac{64\pi}{3(n_b - n_f)}
\]

H. Buchdahl, Nuovo Cim. 23, 141 (1962)
H. Nguyen, Phys. Rev. D 107, 104009 (2023)
B. Puliçe, R. Pantig, A. Övgün, DD, work in progress (2023)
Conformal factor $\varphi(r)$ diverges at the Schwarzschild horizon $r = r_s \equiv 2M$ and gets suppressed exponentially (sinusoidally) at large $r$ for $n_B - n_F < 0$ ($n_B - n_F < 0$).
Metric potentials $A(r)$ and $B(r)$ approach to the flat spacetime limit of $A(r) = B(r) = 1$ at large $r$. The approach is exponential and different $n_B - n_F < 0$ values are hard to distinguish observationally.
Metric potentials $A(r)$ and $B(r)$ approach to the flat spacetime limit of $A(r) = B(r) = 1$ at large $r$. The approach is sinusoidal and gradual and different $n_B - n_F > 0$ values could be distinguished observationally.

B. Pulice, R. Pantig, A. Övgün, DD, work in progress (2023)
 ➢ Photonsphere radius $r_{PS}$ for $n_B - n_F < 0$ (left) and $n_B - n_F > 0$ (right).
Hawking temperature as a function of the radial coordinate $r$ for $n_B - n_F < 0$ (left) and $n_B - n_F > 0$ (right).
Gravity theory ($V_{tree} = 0$) | Static spherically-symmetric solutions
---|---

Symmergent gravity action:

$$\int d^4x \sqrt{-g} \left(- \frac{R}{16\pi G_N} - \frac{c_O}{16} R^2 - \frac{1-\hat{\alpha}}{(8\pi G_N)^2 c_O} \right)$$

Schwarzschild-dS/AdS solution:

$$(ds)^2 = h(r)(cdt)^2 - \frac{(dr)^2}{h(r)} - r^2 \left((d\theta)^2 + \sin^2 \theta (d\phi)^2\right)$$

$h(r) = 1 - \frac{r_s}{r} - \frac{(1-\hat{\alpha})r^2}{24\pi G_N c_O}$

$\hat{\alpha} = \text{a constant parametrizing symmergent vacuum energy}$

$C_O = \frac{n_b-n_f}{248\pi^2}$

I. Çimdiker, DD, A. Övgün, Phys. Dark Univ. 34, 100900 (2021)
SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

(J. Raimbayev et al 2206.06599 (Annals of Physics))
(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)
the regions in $a - c_o$ plane admitting BH solutions

(J. Raimbayev et al 2020.06599 (Annals of Physics)
(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)
event horizon: $r_h = 2M$ is the Schwarzschild radius

(J. Raimbayev et al. 2206.06599 (Annals of Physics)
(I. Çimdiker, A. Övgün, DD. Phys.Dark Univ. 34 (2021) 100900)
Photon sphere radius for a rotating symmergent black hole of angular momentum number $s$.

Effective potential for $\ell = 4.5M$

Schwarzschild-dS/AdS Symmergent Black Hole

(J. Raimbayev et al 2206.06599 (Annals of Physics)
(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)
innermost (ISCO) and outermost (OSCO) stable circular orbits

SCHWARZSCHILD-dS/AdS SYMMERGENT BLACK HOLE

(J. Raimbayev et al 2206.06599 (Annals of Physics)
(I. Çimdiker, A. Övgün, DD, Phys.Dark Univ. 34 (2021) 100900)
bounds on model parameters form shadow radii of Sgr.A* (left) and M87* (right)
bending angle ($\mu as$) for a nearby source ($r_0/20$) and an observer with impact parameter $b$.

(J. Raimbayev et al 2206.06599 (Annals of Physics)
(I. Çimdiker, A. Övgün, DD. Phys.Dark Univ. 34 (2021) 100900)
Hawking temperature \( r_s = 1 \) for a charged symmergent black hole \( (c_o = 0.9 \text{ and } a = 0.5) \).

(B. Puliçe, R. Pantig, A. Ögün, DD, under review CQG)
Hawking radiation from photon, neutrino etc.:

Hawking radiation from new dark fields:

Black hole temperature and evaporation rate change if symmergent particles are included!

Hawking radiation=\{\gamma, \nu\} + \{\text{light symmergent particles}\}

D. Gogoi, A. Övgün, DD, work in progress (2023)

B. Pulice, R. Pantig, A. Övgün, DD, work in progress (2023)
GAIA: A sun-like star is orbiting about a dark object.

Calculations: The dark object is less likely to be a black hole.

Analyses: The dark object is likely to be a dark star.

dark stars and even dark galaxies are expected in symmergent gravity

\[ M = 11.7 \, M_{\text{sun}} \]

\[ 1.4 \, \text{AU} \]

A. Pombo & I. Saltas (2023)
S. Das, S. Chattopadhyay, A. Övgün, DD (2023)
Thank you.