What is Symmergent Gravity?

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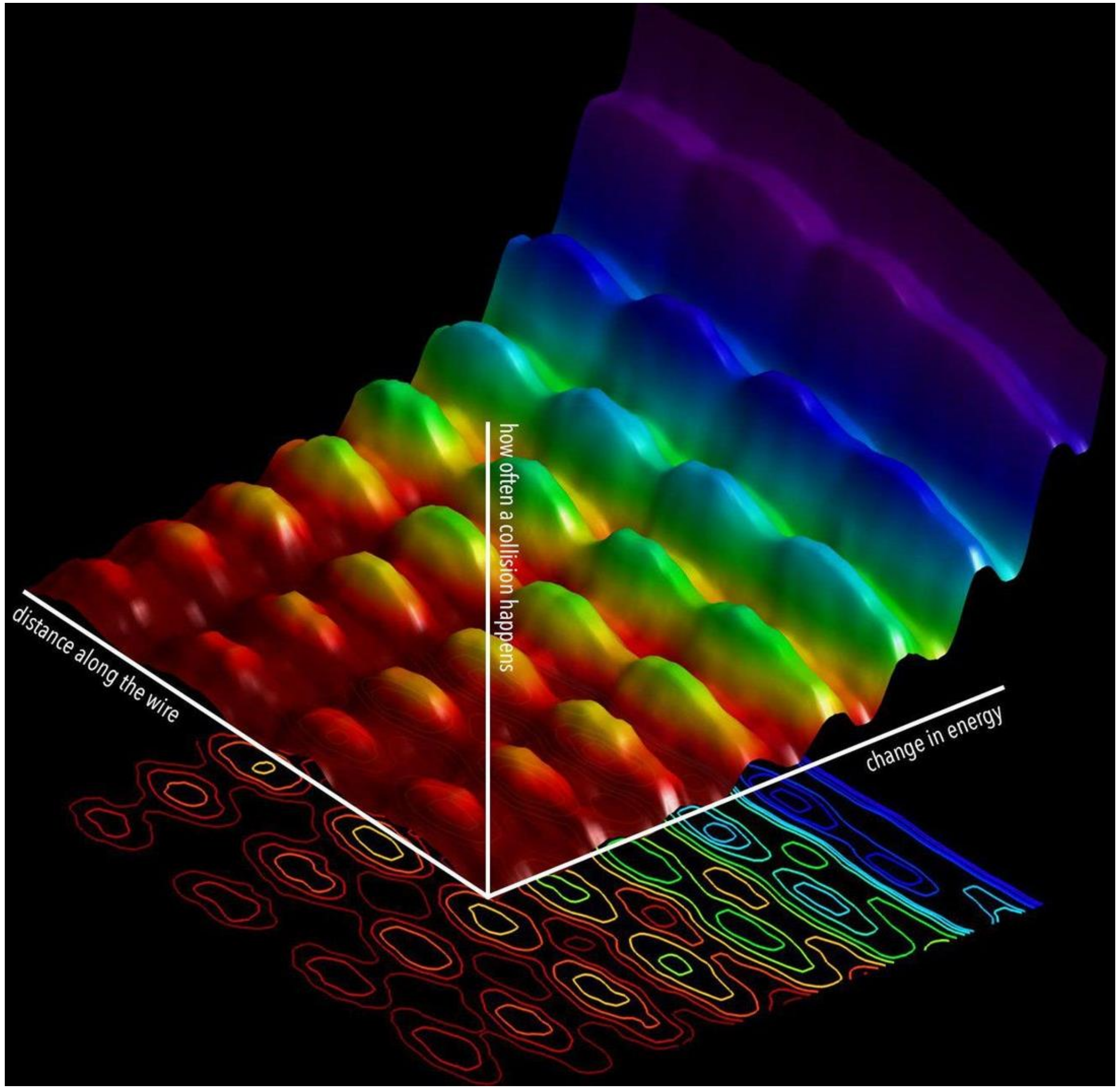
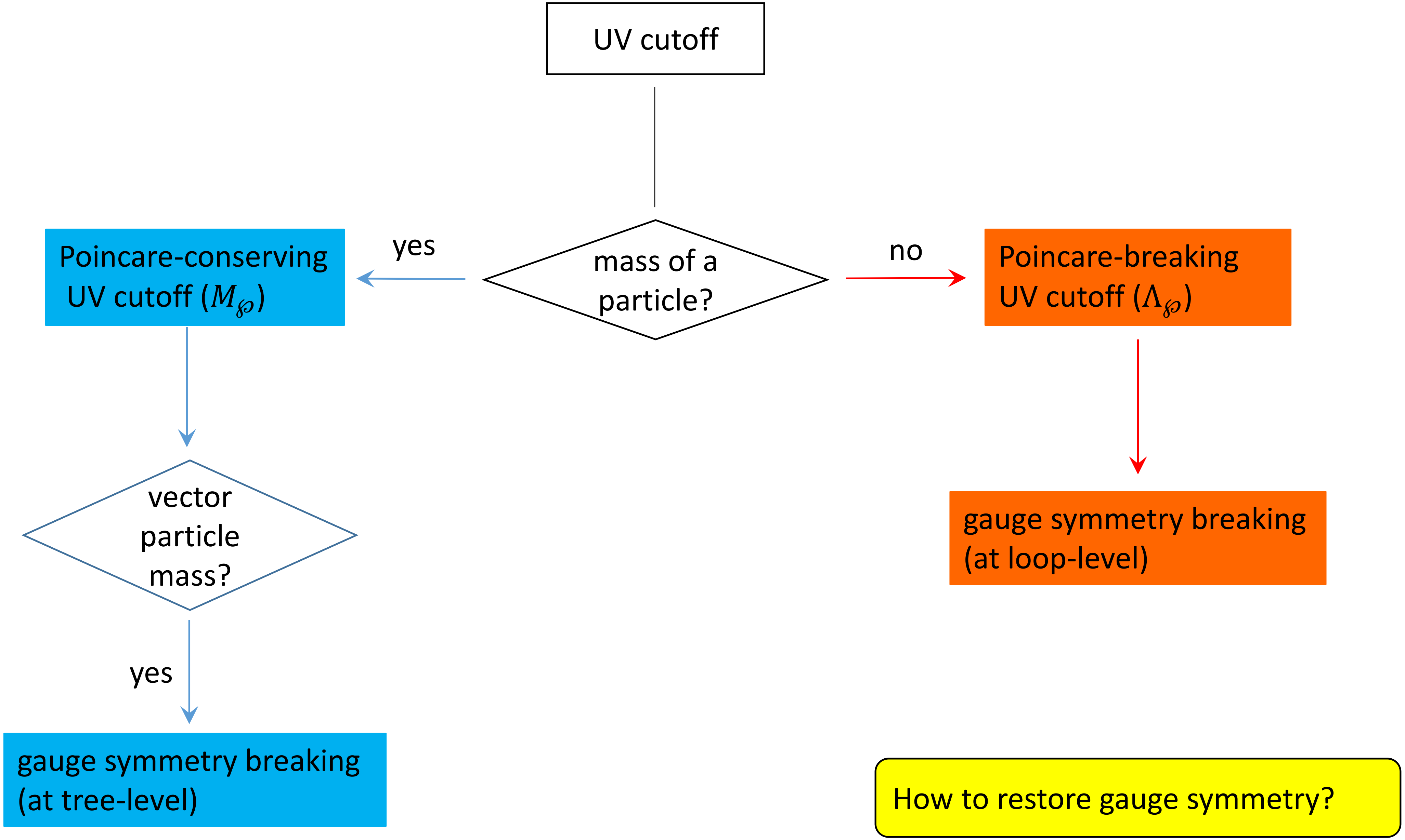
- Quantum behavior rests on wave-particle duality, and this duality holds in flat spacetime.
- QFTs are inherent to flat spacetime.
- QFTs are defined with an invariant action.
- QFTs make sense with a UV cutoff.

L. Ford, arXiv: 9707.062 [gr-qc]
R. Wald, arXiv: 0907.0416 [gr-qc]



L. Piazza *et al.* Nat Commun 6, 6407 (2015)



UV cutoff

mass of a particle?

yes

Poincare-conserving UV cutoff ($M_{\wp}$)

vector particle mass?

yes

gauge symmetry breaking (at tree-level)

no

Poincare-breaking UV cutoff ($\Lambda_{\wp}$)

gauge symmetry breaking (at loop-level)

How to restore gauge symmetry?

UV COMPLETION BY GAUGE INVARIANCE

Poincare-conserving UV cutoff $M_{\wp}$:

tree-level mass term = $M_{\wp}^{2}\, V_{\mu}V^{\mu}$

$M_{\wp}^{2} \Longrightarrow$ "Higgs field ϕ"
(Poincare-conserving)

$$(D_{\mu}\phi)^{\dagger}D^{\mu}\phi$$

P. Anderson, Phys. Rev. Phys. **130**, 439 (1962)
F.Englert & R. Brout, Phys. Rev. Lett. **13,** 321 (1964)
P. Higgs, Phys. Rev. Lett. **13,** 508 (1964)

Poincare-breaking UV cutoff $\Lambda_{\wp}$:

loop-induced mass term= $\Lambda_{\wp}^{2}\, V_{\mu}V^{\mu}$

$\Lambda_{\wp}^{2} \Longrightarrow$ "Higgs-like field $\mathbb{R}$"
(Poincare-breaking)

$$V^{\mu}\mathbb{R} \quad V^{\nu}$$

What is this Higgs-like field $\mathbb{R}_{\mu\nu}$?

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)
DD, Adv. High En. Phys. 4652048 (2019)
DD, Adv. High En. Phys. 6727805 (2016)

EFFECTIVE QFT: Dimensional Regularization

QFTs without UV cutoff:

- Dimensional Regularization $(D \to 4)$: $I_n = \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2 + i0)^n} \Longrightarrow \mu^{4-D} \int \frac{d^Dp}{(2\pi)^D} \frac{1}{(p^2 - m^2 + i0)^n}$

C. Bollini & J. Giambiagi, Nuovo Cim. B12, 20 (1972)
G. 't Hooft & M. Veltman, Nucl. Phys. B44, 189 (1972)

QFTs with UV cutoff:

- Question: How to extend the Dimensional Regularization to QFTs with UV cutoff such that logarithmic (global) and power-law (local) UV sensitivities come independently?

- Useful hint: Dimensional Regularization with $D \to 0$ and $D \to 2$ gather, respectively, the quartic and quadratic UV sensitivities.

I. Jack & D. Jones, Nucl. Phys. B342, 127 (1990)
M. Al-Sarhi, D. Jones & I. Jack, Nucl. Phys. B345, 431 (1990)

EFFECTIVE QFT: Detached Regularization

- The cutoff $\Lambda_\wp$ acts as "mass" if it does not show up in "log" terms. And this is accomplished Detached Regularization:

$$\int^{(\Lambda_\wp)} \frac{d^4p}{(2\pi)^4}\,\frac{1}{(p^2-m^2+i0)^n} \Longrightarrow \left[\frac{1}{(8\pi)^{2-n}}\left(\delta_{[D]0}+\delta_{[D]2}\right)\Lambda_\wp^{4-2n}\,\mu^{2n-D} + \delta_{[D]4}\,\mu^{4-D}\right]\int \frac{d^Dp}{(2\pi)^D}\,\frac{1}{(p^2-m^2+i0)^n}$$

$D \to 0$ and $D \to 2$

$D \to 4$

- Typical loop integral in Detached Regularization with $\overline{MS}$ Renormalization:

$$\int^{(\Lambda_\wp)} \frac{d^4p}{(2\pi)^4}\,\frac{1}{(p^2-m^2+i0)^n} = \begin{cases} \dfrac{i\,\Lambda_\wp^4}{32\pi^2} & D=0, n=0 \\ \dfrac{-i\Lambda_\wp^2}{16\pi^2}\left(1-\log\dfrac{\mu}{m}\right) & D=2, n=1 \\ \dfrac{im^2}{16\pi^2}\left(1+2\log\dfrac{\mu}{m}\right) & D=4, n=1 \\ \dfrac{i}{8\pi^2}\log\dfrac{\mu}{m} & D=4, n=2 \end{cases}$$

- $\Lambda_\wp$ for power-law divergences
- μ for logarithmic divergences

DD, C. Karahan & O. Sargın, Phys. Rev. D 107, 045003 (2023)

EFFECTIVE QFT: Power-Law Corrections

$$\delta S_{pow} = \int d^4x \sqrt{-\eta} \left\{ -c_O(\log\mu)\Lambda_{\wp}^4 - \sum_m c_m(\log\mu)\, \Lambda_{\wp}^2 m^2 - c_\phi(\log\mu)\, \Lambda_{\wp}^2 \phi^\dagger \phi + c_V(\log\mu)\Lambda_{\wp}^2 V_\mu V^\mu \right\}$$

flat metric $\eta_{\mu\nu}$

$$c^{(SM)}{}_{\phi=H} \approx \frac{3h_t^2}{4\pi^2}$$

$c^{(SM)}_{V=g^a}$	$\frac{21g_3^2}{16\pi^2}$
$c^{(SM)}_{V=W^i}$	$\frac{21g_2^2}{16\pi^2}$
$c^{(SM)}_{V=B}$	$\frac{39g_1^2}{32\pi^2}$

$$\sum_m c_m(\log\mu)\, m^2 = \frac{1}{32\pi^2}\mathrm{str}[M^2]$$

$$S_{tot} = S_{tree}(\eta,\psi) + \delta S_{logarithmic}(\eta,\psi,\log\mu) + \delta S_{pow}(\eta,\psi,\log\mu\,,\Lambda_{\wp})$$

$$c_O = \frac{n_b - n_f}{128\pi^2}$$

(mass)^2 matrix of fields

n_b bosonic and n_f fermionic degrees of freedom

DD, Phys. Rev. D 107, 105014 (2023)
DD, C. Karahan & O. Sargın, Phys. Rev. D 107, 045003 (2023)

EFFECTIVE QFT: Flat Spacetime to Curved Spacetime

- General covariance
  - $\eta_{\mu\nu} \longrightarrow g_{\mu\nu}$ (switching to curved metric $g_{\mu\nu}$ with $\partial_\mu g_{\alpha\beta} \neq 0$)
  - $\partial_\mu \longrightarrow \nabla_\mu$ (switching to covariant derivative ∇_μ with $\nabla_\mu g_{\alpha\beta} = 0$)
    - Levi-Civita connection ${}^{g}\Gamma^{\lambda}_{\mu\nu}$

- Problem with effective QFTs:
  - adding curvature terms to make $g_{\mu\nu}$ dynamical is inconsistent (adding bare constants to a loop-level effective QFT)
  - curvature can arise only with the gauge fields V^μ through the defining relation $\left[\nabla_\mu, \nabla_\nu\right] V^\lambda = R^{\lambda}_{\rho\mu\nu}({}^{g}\Gamma) V^\rho$

DD, Phys. Rev. D 107, 105014 (2023)
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DD, Adv. High En. Phys. 4652048 (2019)

EFFECTIVE QFT: Flat Spacetime to Curved Spacetime

Sabancı Üniversitesi

Kinetic construct (bulk)	Kinetic construct (bulk+boundary)
$I_V(\eta) = \int d^4x\sqrt{-\eta}\; c_V V_{\mu\nu}V^{\mu\nu}$	$\tilde{I}_V(\eta) = \int d^4x\sqrt{-\eta}\; c_V\, V^{\mu}(-D^2\eta_{\mu\nu} + D_\mu D_\nu + iV_{\mu\nu})V^{\nu} + \partial_\mu(V_\nu V^{\mu\nu})$

$$\begin{tikzcd} \cdot \arrow[r, leftrightarrow, bend right, "I_V(\eta) = \tilde{I}_V(\eta)"'] & \cdot \end{tikzcd}$$

Flat spacetime (metric=$\eta_{\mu\nu}$)	Curved spacetime (metric=$g_{\mu\nu}$)
$-I_V(\eta) + \tilde{I}_V(\eta) = 0$	$-I_V(g) + \tilde{I}_V(g) = -\int d^4x\sqrt{-g}\; c_V\, V^{\mu}R_{\mu\nu}({}^{g}\Gamma)V^{\nu}$

$$\begin{tikzcd} \cdot \arrow[r, bend right, "\begin{array}{c}\eta_{\alpha\beta} \to g_{\alpha\beta} \\ \partial_\alpha \to \nabla_\alpha\end{array}"'] & \cdot \end{tikzcd}$$

Flat spacetime effective action	Curved spacetime effective action
$S_{eff}(\eta, \psi, \log\mu, \Lambda_{\wp}) - I_V(\eta) + \tilde{I}_V(\eta)$ $= S_{eff}(\eta, \psi, \log\mu, \Lambda_{\wp})$	$S_{eff}(g, \psi, \log\mu, \Lambda_{\wp}) - I_V(g) + \tilde{I}_V(g)$ $= S_{eff}(g, \psi, \log\mu, \Lambda_{\wp}) \boxed{- \int d^4x\sqrt{-g}\, c_V V^{\mu}R_{\mu\nu}({}^{g}\Gamma)V^{\nu}}$

no change !

change with curvature !

EFFECTIVE QFT: Poincare-Breaking Higgs-like field = Affine Curvature

- In an arbitrary second-quantized theory with no presumed properties, "… lack of translational invariance would just be interpreted as the effect of gravitational fields being present, which are not translational invariant."

C. Froggatt & H. Nielsen, Ann. Phys. 517, 115 (2007)

- The UV cutoff $\Lambda_{\wp}$ is the only translation (Poincare) breaking source in flat spacetime. There must be an affinity between the UV cutoff $\Lambda_{\wp}$ and curvature.

$$\boxed{\Lambda_{\wp}^2 \; \Leftrightarrow \; \text{curvature}}$$

- This affinity makes sense only if the said curvature can exist in flat spacetime. This can happen only with an affine connection field $\Gamma_{\mu\nu}^{\lambda}$ (which is independent of ${}^{g}\Gamma_{\mu\nu}^{\lambda}$).

$$\boxed{\Lambda_{\wp}^2 \; \Leftrightarrow \; \text{affine curvature}}$$

- Thus, Higgs-like field $\mathbb{R}_{\mu\nu}$ can be identified with the Ricci curvature of an affine connection $\Gamma_{\mu\nu}^{\lambda}$:

$$\mathbb{R}_{\mu\nu} \equiv \mathbb{R}_{\mu\nu}(\Gamma) = \partial_{\rho}\Gamma_{\nu\mu}^{\rho} - \partial_{\nu}\Gamma_{\rho\mu}^{\rho} + \Gamma_{\rho\lambda}^{\rho}\Gamma_{\nu\mu}^{\lambda} - \Gamma_{\nu\lambda}^{\rho}\Gamma_{\rho\mu}^{\lambda}$$

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)

EFFECTIVE QFT: Power-Law Action to Metric-Palatini

$$\delta S_{pow} = \int d^4x \sqrt{-\eta} \left\{ -c_O \Lambda_{\wp}^4 - \sum_m c_m m^2 \Lambda_{\wp}^2 - c_\phi \Lambda_{\wp}^2 \phi^\dagger \phi + c_V \Lambda_{\wp}^2 \mathrm{tr}\left[V_\mu V^\mu\right] \right\}$$

Higgs-like field: $\Lambda_{\wp}^2 \eta_{\mu\nu} \longrightarrow \mathbb{R}_{\mu\nu}(\Gamma)$

General covariance:
$\eta_{\mu\nu} \longrightarrow g_{\mu\nu}$
$\partial_\mu \rightarrow \nabla_\mu$

$$\delta S_{pow} = \int d^4x \sqrt{-g} \left\{ -\frac{c_O}{16}\left(g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma)\right)^2 - \frac{1}{4}\sum_m c_m m^2 g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma) - \frac{c_\phi}{4} g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma)\phi^\dagger\phi + c_V\left(\mathbb{R}_{\mu\nu}(\Gamma) - R_{\mu\nu}({}^g\Gamma)\right) V_\mu V_\nu \right\}$$

metric-Palatini gravity

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EFFECTIVE QFT: Metric-Palatini Gravity

- The affine connection $\Gamma^{\lambda}_{\mu\nu}$ can be integrated out of the spectrum by solving its equation of motion:

$$ {}^{\Gamma}\nabla_{\lambda}\ \mathbb{D}_{\mu\nu} = 0 $$

$$ \mathbb{D}_{\mu\nu} = \left(\frac{g}{\mathrm{Det}[\mathbb{Q}]}\right)^{\frac{1}{6}} \mathbb{Q}_{\mu\nu} $$

$$ \mathbb{Q}_{\mu\nu} = \left(\frac{1}{16\pi G_N} + \frac{c_{\phi}}{4}\phi^{\dagger}\phi + \frac{c_{O}}{8} g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma)\right) g_{\mu\nu} - c_{V} V_{\mu} V_{\nu} $$

$$ G_N^{-1} = 4\pi \sum_{m} c_m m^2 \xrightarrow[\text{1−loop}]{} \frac{\mathrm{str}[M^2]}{8\pi} $$

For $\mathrm{str}[M^2]$ to generate $G_N^{-1} = (10^{19}\ \mathrm{GeV})^2$ it is necessary to have new massive particles in excess of the Standard Model spectrum.

DD, Phys. Rev. D 107, 105014 (2023)

G. Bostan, C. Karahan, O. Sargın, work in progress (2023)

EFFECTIVE QFT: Emergent General Relativity

- Exact solution of the $\Gamma^{\lambda}_{\mu\nu}$ equation of motion ${}^{\Gamma}\nabla_{\lambda}\ \mathbb{D}_{\mu\nu} = 0$:

$$\Gamma^{\lambda}_{\mu\nu} = {}^{g}\Gamma^{\lambda}_{\mu\nu} + \frac{1}{2}(\mathbb{D}^{-1})^{\lambda\rho}\left(\nabla_{\mu}\mathbb{D}_{\nu\rho} + \nabla_{\nu}\mathbb{D}_{\rho\mu} - \nabla_{\rho}\mathbb{D}_{\mu\nu}\right)$$

- Enormity of the Planck scale G_N^{-1} enables one to make the expansions:
  - $\Gamma^{\lambda}_{\mu\nu} = {}^{g}\Gamma^{\lambda}_{\mu\nu} + 8\pi G_N\left(\nabla_{\mu}\overline{\mathbb{D}}^{\lambda}_{\nu} + \nabla_{\nu}\overline{\mathbb{D}}^{\lambda}_{\mu} - \nabla^{\lambda}\overline{\mathbb{D}}_{\mu\nu}\right) + \mathcal{O}(G_N^2)$
  - $\mathbb{R}_{\mu\nu}(\Gamma) = R_{\mu\nu}({}^{g}\Gamma) + 8\pi G_N\left(\nabla^{\alpha}\nabla_{\mu}\delta^{\beta}_{\nu} + \nabla^{\beta}\nabla_{\mu}\delta^{\alpha}_{\nu} - \Box\delta^{\alpha}_{\mu}\,\delta^{\beta}_{\nu} - \nabla_{\mu}\nabla_{\nu}g^{\alpha\beta} + (\alpha \leftrightarrow \beta)\right)\overline{\mathbb{D}}_{\alpha\beta} + \mathcal{O}(G_N^2)$

$$\overline{\mathbb{D}}_{\mu\nu} = \left(\frac{c_{\phi}}{12}\phi^{\dagger}\phi + \frac{c_{O}}{24}g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma) + \frac{c_V}{6}V_{\alpha}V^{\alpha}\right)g_{\mu\nu} - c_V V_{\mu}V_{\nu}$$

DD, Phys. Rev. D 107, 105014 (2023)

G. Bostan, C. Karahan, O. Sargın, work in progress (2023)

EFFECTIVE QFT: Emergent General Relativity

gauge symmetries got restored!

- $\int d^4x\,\sqrt{-g}\{c_V(\mathbb{R}_{\alpha\beta}(\Gamma) - R_{\alpha\beta}({}^{g}\Gamma))\,\mathrm{tr}[V^\alpha V^\beta]\} = \int d^4x\,\sqrt{-g}\,\{0 + \mathcal{O}(G_N)\}$

GR emerged!

- $\int d^4x\,\sqrt{-g}\left\{-\frac{1}{16\pi G_N}\,g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma)\right\} = \int d^4x\,\sqrt{-g}\,\left\{-\frac{R(g)}{16\pi G_N} + \mathcal{O}(G_N)\right\}$

quadratic corrections to scalar masses give cause to non-minimally coupled scalar fields

- $\int d^4x\,\sqrt{-g}\{-c_\phi g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma)\phi^\dagger\phi\} = \int d^4x\,\sqrt{-g}\,\left\{-\frac{c_\phi}{4}R(g)\phi^\dagger\phi + \mathcal{O}(G_N)\right\}$

quartic corrections to vacuum energy give rise to quadratic curvature terms

- $\int d^4x\,\sqrt{-g}\left\{-\frac{c_O}{16}\left(g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma)\right)^2\right\} = \int d^4x\,\sqrt{-g}\,\left\{-\frac{c_O}{16}(R(g))^2 + \mathcal{O}(G_N)\right\}$

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)

EFFECTIVE QFT: Renormalized QFT + Emergent GR

$$S_{QFT+GR} = \underbrace{S\,(g,\psi\,) + \delta S(g,\psi)}_{\text{QFT sector}} + \int d^4x\,\sqrt{-g}\,\left\{\underbrace{-\frac{R(g)}{16\pi G_N} - \frac{c_O}{16}R(g)^2 - \frac{c_\phi}{4}R(g)\phi^\dagger\phi + \mathcal{O}(G_N)}_{R\,+\,R^2\ \text{gravity sector}}\right\}$$

- symmetry-restoring emergent gravity = “symmergent gravity”
- gravity has emerged from within the flat spacetime effective QFT: Gravity is classical, but all its couplings are generated or improved by matter loops.
- gravitational interactions are as quantum as the matter interactions: there is a chime/concord between the gravitation and QFT.

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)

Symmergent Gravity: Main Properties and Salient Predictions

new massive particles are a must for Newton's constant to take the right value

new particles do not have to couple to the SM particles

Higgs mass stability requires new particles (*e.g.* dark matter) to couple to SM particles weakly/feebly.

Higgs mass stability requires neutrinos to be Dirac

Higgs-curvature coupling (10 % in the SM) can reveal Higgs couplings to new particles

pure Einstein gravity is attained if nature has equal numbers of bosonic and fermionic degree of freedom

cosmic inflation is naturally of the Starobinsky type but scalar field inflation can also be realized

black hole shadow, photon radius, deflection angle and quasiperiodic oscillations can provide viable testbeds

detection of new particles can wait for high-luminosity LHC

the Universe may contain dark stars, dark planets, even dark galaxies.

DD, Phys. Rev. D 107, 105014 (2023)
DD, Gen Relativ Gravit 53, 22 (2021)
DD, Galaxies 9, 2 (2021)
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K. Cankoçak *et al.*, Eur. Phys. J. C80, 1188 (2020)
DD, C. S. Ün, arXiv: 2005.03589 [hep-ph] (2020)

I. Çimdiker, DD, A. Övgün, Phys. Dark Univ. 34, 100900 (2021)
J. Rayimbaev *et al.*, Annals of Physics 454, 169335 (2023)
R. Pantig, A. Övgün, DD, Eur. Phys. J. C83, 250 (2023)
S. Jalalzadeh et al., Phys. Dark. Univ. 40, 101227 (2023)

Symmergent Black Holes: The Action

$$S_{sgr} = \int d^4x \sqrt{-g} \left\{ -\frac{R(g)}{16\pi G_N} - \frac{c_O}{16} R(g)^2 - V_{tot} \right\}$$

$$V_{tot} = V_{tree} + \delta V$$

$$\delta V = \frac{1}{64\pi^2} \mathrm{str}[M^4]$$

$$\delta V = \frac{m_0^4}{64\pi^2}\left(n_b - n_f\right) = \frac{1}{24\pi G_N^2 c_O}$$

(if bosons and fermions
had equal masses m_0)

$$\delta V = \frac{(1-\hat{\alpha})}{24\pi G_N^2 c_O}$$

(one possible
parametrization)

J. Rayimbaev *et al.*, Annals of Physics 454, 169335 (2023)
R. Pantig, A. Övgün, DD, Eur. Phys. J. C83, 250 (2023)

Symmergent Black Holes: Field Equations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} - 2\pi G_N c_O \left(R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} + g_{\mu\nu} - \nabla_\mu \nabla_\nu \right) R - 8\pi G_N V_{tot} = 0$$

- One class of solutions corresponds to constant scalar curvature ($R = R_0 = -8\pi G_N V_{tot}$)
    - dS solution ($V_{tot} > 0$ or $n_B > n_F$)
    - AdS solution ($V_{tot} < 0$ or $n_B < n_F$)
    - c_O disappears from asymptotically-flat zero-R solution

W. Nelson, Phys. Rev. D 82, 104026 (2010)
H. Lü *et al*. Phys. Rev. Lett. 114, 171601 (2015)

- Another class corresponds to corresponds to variable scalar curvature ($R \neq \text{constant}$)
    - There exist asymptotically-flat solutions explicitly involving c_O

H. Buchdahl, Nuovo Cim. 23, 141 (1962)
H. Nguyen, Phys. Rev. D 107, 104009 (2023)
B. Puliçe, R. Pantig, A. Övgün, DD, work in progress (2023)

Symmergent Black Holes: Schwarschild-dS/AdS Solutions

gravity theory ($V_{tree} = 0$)	static spherically-symmetric solutions
Symmergent gravity action: $\int d^4x \sqrt{-g}\left(-\frac{R}{16\pi G_N} - \frac{c_O}{16}R^2 - \frac{1-\hat{\alpha}}{(8\pi G_N)^2 c_O}\right)$	Schwarzschild-dS/AdS solution: $(ds)^2 = h(r)(cdt)^2 - \frac{(dr)^2}{h(r)} - r^2\left((d\theta)^2 + \sin^2\theta\,(d\phi)^2\right)$

$$c_O = \frac{n_b - n_f}{248\pi^2}$$

$\hat{\alpha}$ = a constant parametrizing symmergent vacuum energy

$$h(r) = 1 - \frac{r_s}{r} - \frac{(1-\hat{\alpha})r^2}{24\pi G_N c_O}$$

I. Çimdiker, DD, A. Övgün, Phys. Dark Univ. 34, 100900 (2021)

Symmergent Black Holes: Asymptotically-Flat Variable-R Solutions

gravity theory ($V_{tot} = 0$)	static spherically-symmetric solutions
Symmergent gravity action: $\int d^4x \, \sqrt{-g} \left(-\frac{R}{16\pi G_N} - \frac{c_O}{16} R^2\right)$	Buchdahl-Nguyen solution: $(ds)^2 = A(r)\,(dt)^2 - \frac{(dr)^2}{B(r)} - C(r)\left((d\,\theta)^2 + \sin^2\theta\,(d\phi)^2\right)$

$$A(r) = e^{-\varphi(r)}\left(1 - \frac{r_s}{r}\right)$$

$$B(r) = e^{\varphi(r)}\left(1 - \frac{r_s}{r}\right)$$

$$C(r) = e^{-\varphi(r)} r^2$$

$$\frac{d}{dr}\left((r^2 - r r_s)\frac{d\varphi(r)}{dr}\right) = -\gamma r^2 \varphi(r)$$

$$\gamma = -\frac{1}{6\pi c_O} = -\frac{64\pi}{3(n_b - n_f)}$$

H. Buchdahl, Nuovo Cim. 23, 141 (1962)
H. Nguyen, Phys. Rev. D 107, 104009 (2023)
B. Puliçe, R. Pantig, A. Övgün, DD, work in progress (2023)

Thank you for your attention.