

# A Trilevel $r$ -Interdiction Selective Multi-Depot Vehicle Routing Problem

## Supplementary Material (C)

### Routing Neighborhood Structures

#### 1-0 Move

In this move, a node is removed from its position and inserted in another position either on the same route or different routes. This move is represented visually in Fig. C1. and Fig. C2.

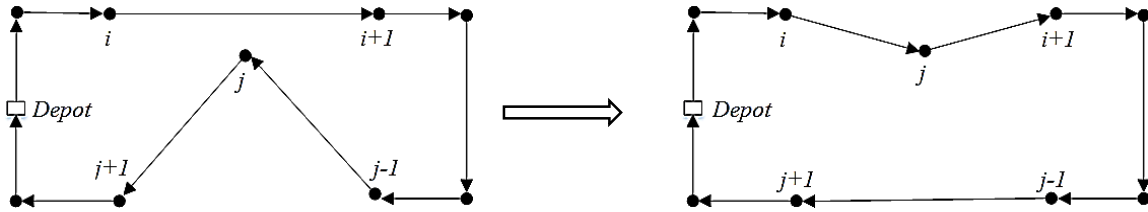


Fig. C1. 1-0 Move on the same route: Remove node  $j$  from its position and insert it between nodes  $i$  and  $i+1$ .

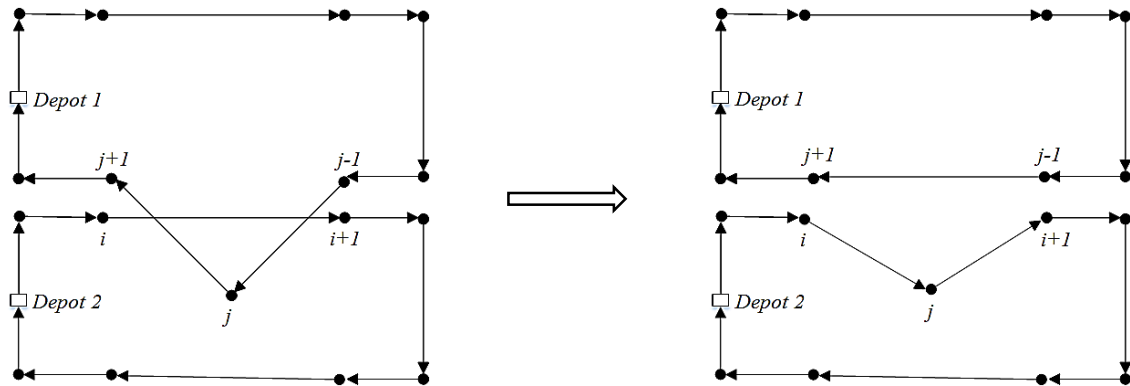


Fig. C2. 1-0 Move on different routes: Remove node  $j$  from its position on route 1 and insert it between nodes  $i$  and  $i+1$  on route 2.

### 2-0 Move

In this move, a node and its successor are removed from their positions and inserted in another position either on the same route or different routes. This move is represented visually in Fig. C3 and Fig. C4.

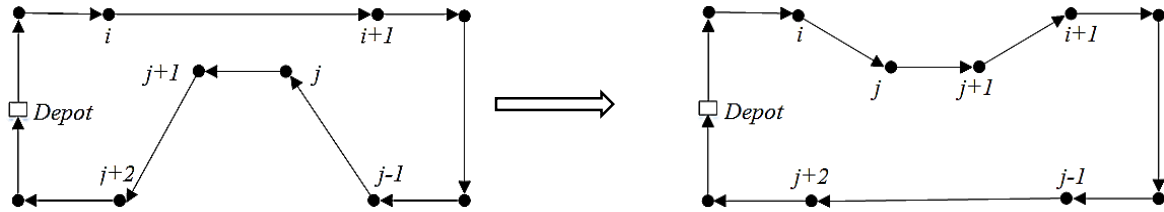


Fig. C3. 2-0 Move on the same route: Remove node  $j$  and its successor node  $j+1$  from their positions and insert them between nodes  $i$  and  $i+1$  (directions of  $j$  and  $j+1$  are reversed).

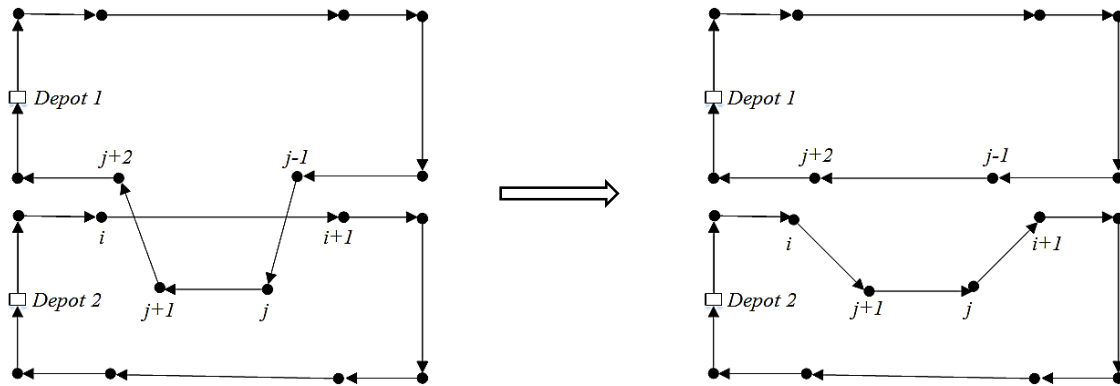


Fig. C4. 2-0 Move on different routes: Remove node  $j$  and its successor  $j+1$  from their positions on route 1 and insert them between nodes  $i$  and  $i+1$  on route 2 (directions of  $j$  and  $j+1$  are reversed).

### 1-1 Exchange

Given two nodes either on the same or different routes, their positions are swapped. 1-1 Exchange move is represented visually in Fig. C5 and Fig. C6.

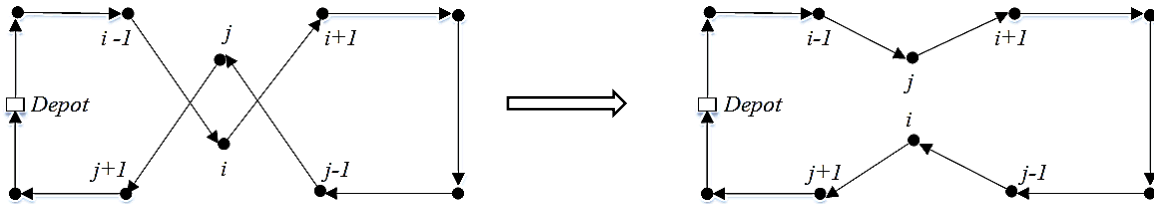


Fig. C5. 1-1 Exchange on the same route: Swap node  $j$  from its position with node  $i$ .

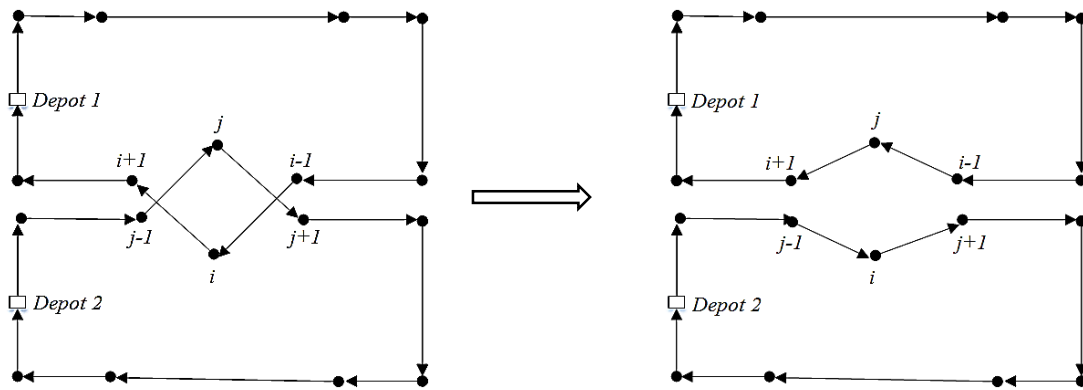


Fig. C6. 1-1 Exchange on different routes: Swap node  $j$ 's position on route 2 with node  $i$  on route 1.

### Or-Opt on the same route (Or, 1976):

Or-Opt relocates a chain of  $k$  consecutive nodes. This move is obtained by replacing three arcs by three new ones without reversing the subroutes. Or-Opt is represented visually in Fig. C7.

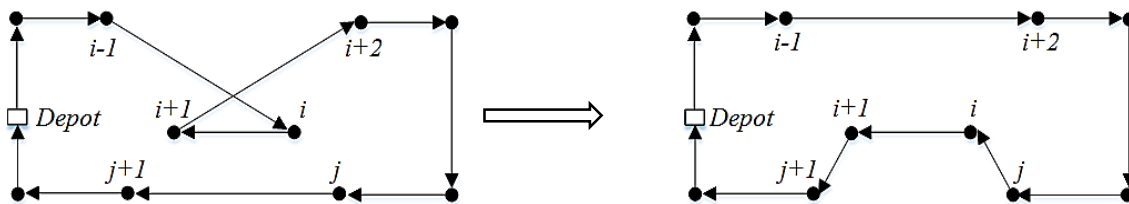


Fig. C7. Or-Opt: Replacing three arcs  $(i-1, i), (i+1, i+2), (j, j+1)$  by three new arcs  $(i-1, i+2), (j, i), (i+1, j+1)$ .

*2-Opt on the same route*

Given a set of two arcs on the same route that form a crisscross, the 2-Opt replaces these two arcs with two new arcs. This move is represented visually in Fig. C8.

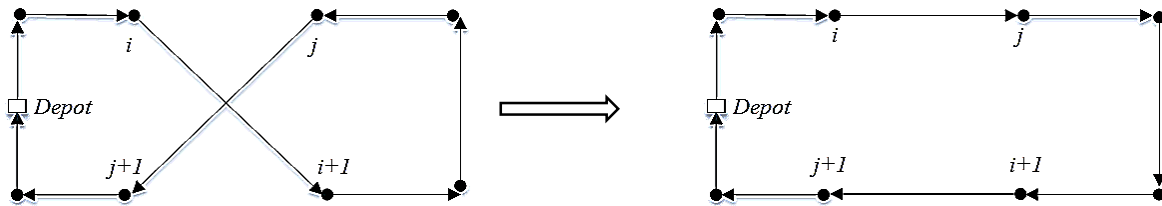


Fig. C8. 2-Opt on the same route: arcs  $(i, i+1)$  and  $(j, j+1)$  are removed, and nodes  $i$  and its successor  $i+1$  are connected to node  $j$  and its successor  $j+1$ , respectively. (Subroute  $(i+1, i+2, \dots, j)$  is reversed).

*2-Opt on different routes*

Arcs  $(i, i+1)$  and  $(j, j+1)$  form a crisscross, but they belong to different routes. The 2-Opt move, which is implemented in exactly the same way as in the same route case, is represented visually in Fig. C9.

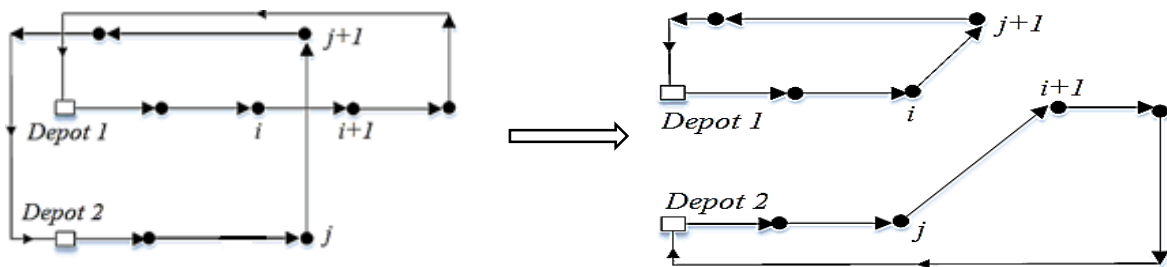


Fig. C9. 2-Opt on different routes: The nodes after node  $i$  on the first route are inserted after node  $j$  on the second route and the nodes after node  $j$  on the second route are moved after node  $i$  on the first route.

### 2-2 Exchange

Given two nodes either on the same or different routes, the positions of the first node and its successor are swapped by the second node and its successor. 2-2 Exchange move is represented visually in Fig. C10 and Fig. C11.

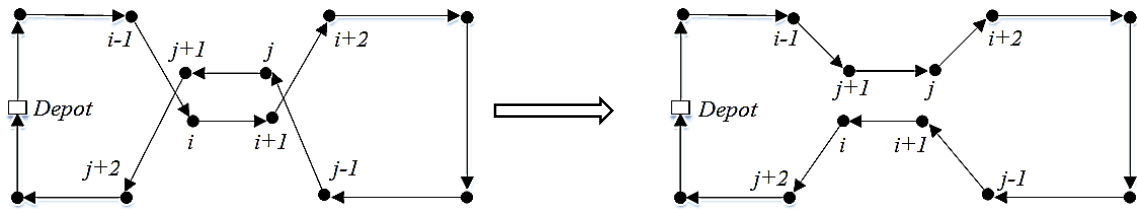


Fig. C10. 2-2 Exchange on the same route: Swap node  $j$  and its successor  $j+1$  from their positions with nodes  $i$  and its successor  $i+1$  (directions of arcs  $(j, j+1)$  and  $(i, i+1)$  are reversed).

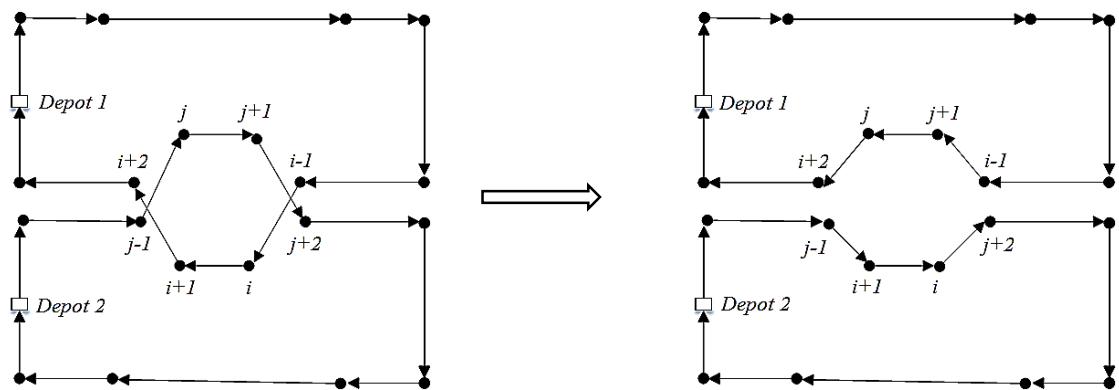


Fig. C11. 2-2 Exchange on different routes: Swap node  $j$  and its successor  $j+1$  from their positions on route 2 with nodes  $i$  and its successor  $i+1$  on route 1 (directions of arcs  $(j, j+1)$  and  $(i, i+1)$  are reversed).

3-Opt on the same route (Lin, 1965)

This move improves the routes by removing three of its arcs, and then reconnect the network in all possible ways. This move is applied on the same route and there are two possible new routes which Fig. C12 and Fig. C13 represent visually.

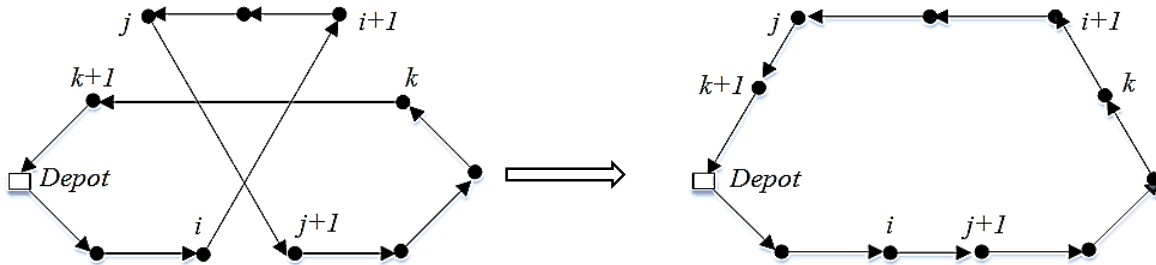


Fig. C12. 3-Opt: Remove three arcs  $(i, i+1)$ ,  $(j, j+1)$ ,  $(k, k+1)$  and reconnect the network by adding the arcs of  $(i, j+1)$ ,  $(j, k+1)$ ,  $(k, i+1)$ .

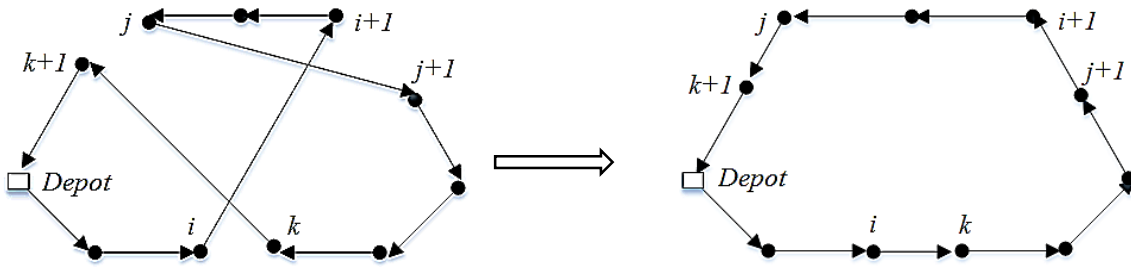


Fig. C13. 3-Opt: Removing three arcs  $(i, i+1)$ ,  $(j, j+1)$ ,  $(k, k+1)$  and reconnect the network by adding the arcs of  $(i, k)$ ,  $(j+1, i+1)$ ,  $(j, k+1)$ . (Subroute  $(j+1, j+1, \dots, k)$  is reversed).