On the Investment Implications of Bankruptcy Laws

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Introduction:
Bankruptcy Problem

• A firm goes bankrupt
  – Liquidated assets worth $E$

• The bankrupt firm owes money to agents in $N$
  – Each agent has a verifiable claim of $c_i$

• There isn’t enough to honour all claims

  How to allocate $E$ among agents in $N$?
The Axiomatic Approach

• Analyzes \((c,E)\) as a “normative problem”

• Proposes solution rules:

\[
F : (c_1, \ldots, c_n, E) \rightarrow (x_1, \ldots, x_n)
\]

\[
\text{s.t. } x_1 + \ldots + x_n = E
\]

• Looks for rules with desirable properties
  E.g. Pareto optimality
  Claims monotonicity
Three Central Principles

- **Proportionality**
  - Proportional Rule, Weighted Proportional Rules

- **Equal Awards**
  - Constrained Equal Awards Rule, Talmudic rule, Equal Gains Rule, Piniles’ Rule, Random Arrival rule, Minimal Overlap Rule

- **Equal Losses**
  - Constrained Equal Losses Rule, Talmudic rule, Random Arrival Rule, Minimal Overlap Rule
Axiomatic Literature

• In support of CEA:

• In support of CEL:

• In support of TAL:

• In support of PRO:

NOTE:
All three principles
- Proportionality
- Equal Awards
- Equal Losees
more or less equally predominant
Bankruptcy in real life

• Between 1999 – 2009 in US
  – 551,000 + firms filed for Chapter 7 bankruptcy
  – 22 + billion $ allocated

– Chapter 7 bankruptcy:
  • liquidate the remaining assets
    – as a whole or piecewise
  • allocate among claimants
  • similar to the axiomatic literature

– Chapter 11 bankruptcy:
  • reorganize the firm
The Empirical / Finance Literature

• Describe alternative practices
  • Atiyas (1995)
  • Hotchkiss, John, Mooradian, Thorburn (2008)
• Literature mostly on Chapter 11
• Comparisons of Chapter 7 vs Chapter 11
  – Hart (1999)
  – Stiglitz (2001)
  – Bris, Welch, and Zhu (2006)
Chapter 7 bankruptcy

- Everywhere around the world
- the common way to allocate liquidated assets among claimants:
  - Proportional Rule
    (combined with a priority rule)
This Paper:
asks the following question

Why is proportionality preferred
over alternative principles in real-life bankruptcy problems?

The finance literature remains silent on this issue
Possible explanations

• Historical reasons
  – Counter-argument: Talmudic rule (Aumann and Maschler, 1985)
  – although Rabbi Abraham Ibn Ezra (1140) also mentions PRO

• Axiomatic reasons: maybe governments prefer the axioms that characterize PRO

• Incentive reasons: maybe the investment incentives created by the PRO are superior to that of others

We check this third explanation.
We study noncooperative investment games with possible bankruptcy

- Araujo and Pascua (2002)
  - 2 period general equilibrium model with bankruptcy
  - Conditions under which equilibrium exists and is efficient
  - No comparison of bankruptcy rules

- Karagözoğlu (2010)
  - Noncooperative investment game
  - Two types of agents: high/low income
  - Invest zero or everything
  - Linear utilities (risk neutrality)
The Investment Game under $F$

- **(t=1) n investors**
  - Simultaneously choose their investments on a firm: $s_1, \ldots, s_n \geq 0$
  - Value of the firm: $s_1 + \ldots + s_n$

- **(t=2) Firm**
  - Succeeds with probability $p$ : return of $r$
  - Fails with probability $1-p$ : bankruptcy

- **Bankruptcy**
  - The value of the firm becomes $\beta (s_1 + \ldots + s_n)$
  - Allocated among the investors according to a prespecified bankruptcy rule $F$

\[ \beta \in (0, 1) \]

Supported by Bris et al (2006)
The Investment Game Under $F$:

- Success with probability $p$:
  - Net Return: $rs_i$

- Bankruptcy with probability $1-p$:
  - Net Return: $F_i(s, \beta(s_1 + ... + s_n)) - s_i$

Selected investments $s_1, \ldots, s_n$ chosen simultaneously.

Total value $V = s_1 + ... + s_2$.
Parameters of the Game

- The bankruptcy rule used \( F \)
- Probability of success: \( \rho \)
- Return in case of success: \( r \)
- Fraction that survives bankruptcy: \( \beta \)
- Agents’ risk aversion levels: \( a_i \)
Agents

- CARA utilities
  - Risk aversion level independent of income
  - Agents possibly heterogenous in risk aversion

- No income constraints
  - Initially all agents have zero income
  - Agents borrow at the market rate (norm. to 0)
  - Simplifies the agents’ optimization problems by eliminating the boundary conditions
The agents’ CARA utilities

\[ U_i^F(s) = -pe^{-a_i rs_i} - (1 - p)e^{-a_i F_i(s_i, s_{-i}) + a_i s_i} \]
We do

Compare the Nash equilibria of the investment games under

1. proportionality
2. equal awards
   mixtures of prop. and equal awards
   constrained equal awards
3. equal losses
   mixtures of prop. and equal losses
   constrained equal losses
We do

Compare them in terms of

1. total equilibrium investment
2. equilibrium social welfare
   - egalitarian
   - utilitarian
3. the effect of possibly heterogenous risk attitudes
RESULTS I

CALCULATING EQUILIBRIUM INVESTMENT LEVELS UNDER ALTERNATIVE BANKRUPTCY RULES
Proportionality (PRO)

\[ x_i = q \, s_i \]
EQUILIBRIUM UNDER PRO

\[ F_i^P(s) = \beta s_i \]

Proportional shares in case of bankruptcy

\[ U_i^P(s) = -pe^{-ai rs_i} - (1 - p)e^{ai s_i(1-\beta)} \]

• Independent of agent j’s strategy
• Well-behaved => unique best response

\[ s_i^* = \frac{1}{a_i (r + 1 - \beta)} \ln \left( \frac{pr}{(1 - p)(1 - \beta)} \right) \]

Common term for nonnegative investment
EQUILIBRIUM UNDER PRO

The investment game under PRO
unique dominant strategy equilibrium

Equilibrium investment level is
increasing in $p$ and $\beta$
decreasing in own risk aversion
independent of other’s risk aversion
Numerical example:
Equilibrium investment levels under PRO

\[ p = 0.8 \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
Equal Awards (EA)

\[ x_i = E / 2 \]
EQUILIBRIUM UNDER EA

• Agents are awarded equal shares in case of bankruptcy

\[ E A_i(s) = \frac{\beta}{n} \sum_{N} s_i \]

• Well-behaved payoff functions
• Unique best response
• Unique NE always exists
MIXTURES OF PRO and EA

Agents receive a convex combination of PRO and EA in case of bankruptcy

\[ AP[\alpha]_i(s) = \alpha PRO_i(s) + (1 - \alpha) EA_i(s) \]

\( \alpha = 1 \) is PRO \hspace{1cm} \( \alpha = 0 \) is EA

• **Unique NE:**

\[ s_i^* = \frac{n (1 + r - \beta) + \beta (1 - \alpha) + \beta (1 - \alpha) a_i \sum_{N-i} \frac{1}{a_j} \ln \left( \frac{npr}{(1 - p) (n - \beta - (n - 1) \alpha \beta)} \right)}{a_i n (1 + r - \beta) (1 + r - \alpha \beta)} \]

Common term for nonnegative investment
Numerical example:
Equilibrium investment levels under EA

\[ a_1 = 3 \]

\[ a_2 \]

\[ p = 0.8 \]

\[ r = 0.6 \]

\[ \beta = 0.7 \]
Problematic Parameter Values

• Want to rule out cases where equilibrium investment < share in case of bankruptcy

• This implies:

\[
\frac{1}{a_n} \sum_{N} \frac{1}{a_j} \geq \frac{r \beta (1 - \alpha)}{n \left(1 - \alpha \beta \right) \left(1 + r - \beta \right)}
\]

• Alternatively: use CEA instead of EA
Equal Losses (EL)

\[ x_i = s_i - q \]
EQUILIBRIUM UNDER EL

- Agents forego equal shares in case of bankruptcy

\[ EL_i(s) = s_i - \frac{1-\beta}{n} \sum_{j=1}^{N} s_j \]

- Well-behaved payoff functions
- Unique best response
- Unique NE always exists
MIXTURES OF PRO and EL

- Agents receive a convex combination of PRO and EL in case of bankruptcy

\[ LP[\alpha]_i(s) = \alpha PRO_i(s) + (1 - \alpha) EL_i(s) \]

\( \alpha = 1 \) is PRO and \( \alpha = 0 \) is EL

- Unique NE:

\[ s^*_i = \left( \frac{1}{a_i} - \frac{(1 - \alpha)(1 - \beta)}{n(1 + r - \beta)} \sum_{j \neq i} \frac{1}{a_j} \right) \ln \left( \frac{npr^r}{(1 - \beta)(1 - p)(1 + (n - 1)\alpha)} \right) \frac{1}{r + \alpha(1 - \beta)} \]

Common term for nonnegative investment
Numerical example:

Equilibrium investment levels under EL

\[ p = 0.8, \quad r = 0.6, \quad \beta = 0.7, \quad a_1 = 3 \]
Problematic Parameter Values

• Want to rule out cases where
  
equilibrium share in case of bankruptcy < 0
• This implies:

\[
\frac{1}{a_n} \sum_{N} \frac{1}{a_j} \geq \frac{(r + 1)(1 - \alpha)(1 - \beta)}{n (1 - \beta + r)(1 - \alpha + \alpha\beta)}
\]

• Alternatively: use CEL instead of EL
Summary of Part I
Agent 1’s investment levels

$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$
Agent 2’s investment levels

HOW ABOUT TOTAL INVESTMENT?

\[ \text{How about total investment?} \]

\[ a_2 \]

\[ \text{p = 0.8} \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
RESULTS II

COMPARING TOTAL INVESTMENT LEVELS UNDER ALTERNATIVE BANKRUPTCY RULES
Total investment levels for our numerical example

\[ p = 0.8 \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
PRO vs. EA

• An agent’s equilibrium investment level
  – Decreasing in risk aversion
  – Cutoff risk aversion level
    • Below cutoff: invests more under PRO
    • Above cutoff: invests more under EA
Agent i’s NE investment as a function of his risk aversion is the solid curve under EA and the dotted curve under PRO \((a_j = 1)\).
Investment Under 
PRO vs. EA

• Small investors: invest more under EA
• Big investors: invest more under PRO
• How about total investment?
  – Independent of the parameters, the following is always true:
PRO vs. EA

THM: In terms of total investment,

PRO > EA

GENERALIZE IT FURTHER?
Mixtures of PRO and EA

Total investment is an increasing function of $\alpha$

**THM:**

$\alpha > \alpha'$

implies

Total Investment under $AP[\alpha]$ > Total Investment under $AP[\alpha']$

PRO and EA are the two extremes
PRO vs. EL

• An agent’s equilibrium investment level
  – Decreasing in risk aversion
  – Cutoff risk aversion level
    • Below cutoff: invests more under EL
    • Above cutoff: invests more under PRO
Agent i’s NE investment as a function of his risk aversion is the solid curve under EL and the dotted curve under PRO ($a_j = 1$)
Investment Under
PRO vs. EL

• Small investors: invest more under PRO

• Big investors: invest more under EL

• How about total investment?
  – Independent of the parameters, the following is always true:
PRO vs. EL

**THM:** In terms of total investment, EL > PRO

GENERALIZE IT FURTHER?
Mixtures of PRO and EL

Total investment is a decreasing function of $\alpha$

**THM:**

$$\alpha > \alpha'$$

implies

Total Investment under $LP[\alpha] <$ Total Investment under $LP[\alpha']$

PRO and EL are the two extremes
OVERALL

In terms of total investment

EL > PRO > EA

Mixtures of EL and PRO

Mixtures of EA and PRO
RESULTS: III

COMPARING SOCIAL WELFARE UNDER THE THREE MAIN RULES
Welfare Calculation

- Messy expressions
- Restrict analysis to
  - Three main rules:
    - PRO
    - EA
    - EL
  - Two agents
Example: Agent 1’s welfare levels

\[ p = 0.8 \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
Example: Agent 2's welfare levels

\[ a_2 \]

\[ p = 0.8 \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
Egalitarian Social Welfare
induced by $F$

$$EG^F(p, r, \beta, a_1, a_2) = \min \{ U_1^F(\epsilon (G^F)), U_2^F(\epsilon (G^F)) \}$$

- a function of the parameters
- Agent 1’s equilibrium utility
- Agent 2’s equilibrium utility
Egalitarian social welfare for our example

\[ p = 0.8^{1.4} \]
\[ r = 0.6^{0.5} \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
THEOREM

Assume parameter values are such that there is an interior equilibrium under all three rules.

Then in terms of egalitarian social welfare

\[ \text{PRO} > \text{EL} \quad \text{and} \quad \text{PRO} > \text{EA} \]
Egalitarian Social Welfare

EA vs EL

Numerical comparison of interior equilibria

1.3 million parameter combinations

EA > EL on 73% of the parameter space

EL > EA on 27% of the parameter space

Never equal
Utilitarian Social Welfare induced by $F$

$$UT^F (p, r, \beta, a_1, a_2) = U_1^F (\epsilon (G^F)) + U_2^F (\epsilon (G^F))$$

- a function of the parameters
- Agent 1’s equilibrium utility
- Agent 2’s equilibrium utility
Utilitarian social welfare for our example

\[ p = 0.8 \]
\[ r = 0.6 \]
\[ \beta = 0.7 \]
\[ a_1 = 3 \]
THEOREM

Assume parameter values are such that there is an interior equilibrium under all three rules.

Then in terms of utilitarian social welfare

PRO > EL
Utilitarian Social Welfare
PRO vs EA

Proposition:
Assume agents equally risk averse
Then
in terms of utilitarian social welfare

PRO > EA
Utilitarian Social Welfare

PRO vs EA

Numerical comparison of interior equilibria

2.7 million parameter combinations

PRO > EA on 61% of the parameter space

EA > PRO on 39% of the parameter space

Never equal
Utilitarian Social Welfare
EA vs EL

Numerical comparison of interior equilibria

1.3 million parameter combinations

EA > EL on 66% of the parameter space

EL > EA on 34% of the parameter space

Never equal
SUMMARY

In terms of total investment
  EL > PRO > EA

In terms of egalitarian social welfare
  PRO > EL and EA

In terms of utilitarian social welfare
  PRO > EL
SUMMARY

1. Switching from PRO to EL
   increases total investment
   but
   decreases social welfare

2. Switching from PRO to EA
   decreases total investment
   decreases egalitarian social welfare
   might increase utilitarian social welfare
THANK YOU!