GAMES WITH SEQUENTIAL MOVES

• Chess, card games, bidding in some auctions, voting in some committees, entry-exit models in industrial organization,…

Players take actions sequentially

When I am choosing my action, I know what people who acted before me has done
The Senate Race

• Two candidates running to represent Tuzla in the parliament.

  • Gray: the incumbent senator
         the stronger one

  • Green: the newcomer (from the Greens party)
         the weaker one
They play the following game:

**First:** Gray chooses whether to give **ads** or **not**

**Second:** Green chooses whether to stay **in** or **out** of the race

**NOTE:**

Green chooses between the actions in and out after she observes Gray’s choice

Possible **outcomes** of the game are:

(No Ads, Out)  (Ads, Out)  (No Ads, In)  (Ads, In)
To describe a game, we need to specify:

1. Players (agents)

   Gray and Green

2. Strategies for each player

   ?????????????

3. Payoffs

   Gray’s ranking of the outcomes:
   
   (No Ads, Out)   (Ads, Out)   (No Ads, In)   (Ads, In)

   Gray’s payoffs (choose numbers that represent her ranking)
   
   4  3  2  1
To describe a game, we need to specify:

1. Players (agents)
   - Gray and Green

2. Strategies for each player
   - ??????????????

3. Payoffs
   - Green’s ranking of the outcomes:
     (No Ads, In)  (Ads, Out)  (No Ads, Out)  (Ads, In)
   - Green’s payoffs (choose numbers that represent her ranking)
     4  3  2  1
Is there a nice way to summarize this story in a figure? 

YES!!!!!

Use a game tree (a.k.a, the extensive form)
Game Tree:

Made up of two things

1. **Nodes** there are two types of nodes

   **Decision nodes**
   nodes at which a player has to take an action

   **Terminal nodes**
   nodes at which there is no action to take
   they represent possible outcomes of the game

2. **Branches**

   they represent actions

   they come out of decision nodes and connect nodes
The Game Tree in the Senate Race game:

Made up of two things

1. Nodes
   - Decision nodes
   
   Gray has a node in which he has to act between Ads and No

   What about Green?

   - Terminal nodes
     
     Each of (No Ads, Out) (Ads, Out) (No Ads, In) (Ads, In)
     is represented by a terminal node

2. Branches

   “Ads” is a branch
NODES:

Gray, Green

Decision nodes

Terminal nodes

Nodes:
- Gray, a
- Green, b
- Green, c
NODES AND BRANCHES:

GRAY, GREEN

FIGURE 3.1 Tree for Senate Race Game
(a) Pruning at terminal nodes

Backward induction = Rollback technique

FIGURE 3.2 A Using Rollback
(b) Fully pruned tree

FIGURE 3.2 B Using Rollback
The senate game: formal representation

$$G = (N, S_{\text{gray}}, S_{\text{green}}, u_{\text{gray}}, u_{\text{green}})$$

where

\(N = \{ \text{Gray}, \text{Green} \}\)

\(S_{\text{gray}} = \{ \text{Ads}, \text{NoAds} \}\)

\(S_{\text{green}} = \{ \text{If Ads then In and if NoAds then In},
\text{If Ads then In and if NoAds then Out,}
\text{If Ads then Out and if NoAds then In,}
\text{If Ads then Out and if NoAds then Out} \} \)

and
\[ u_{\text{gray}}(\text{Ads}, \text{If Ads then In and if NoAds then In}) = 1 \]

\[ u_{\text{gray}}(\text{Ads}, \text{If Ads then In and if NoAds then Out}) = 1 \]

\[ u_{\text{gray}}(\text{Ads}, \text{If Ads then Out and if NoAds then In}) = 3 \]

\[ u_{\text{gray}}(\text{Ads}, \text{If Ads then Out and if NoAds then Out}) = 3 \]

\[ u_{\text{gray}}(\text{NoAds}, \text{If Ads then In and if NoAds then In}) = 2 \]

\[ u_{\text{gray}}(\text{NoAds}, \text{If Ads then In and if NoAds then Out}) = 4 \]

\[ u_{\text{gray}}(\text{NoAds}, \text{If Ads then Out and if NoAds then In}) = 2 \]

\[ u_{\text{gray}}(\text{NoAds}, \text{If Ads then Out and if NoAds then Out}) = 4 \]
\( u_{\text{green}} (\text{Ads} , \text{If Ads then In and if NoAds then In} ) \) = 1

\( u_{\text{green}} (\text{Ads} , \text{If Ads then In and if NoAds then Out} ) \) = 1

\( u_{\text{green}} (\text{Ads} , \text{If Ads then Out and if NoAds then In} ) \) = 3

\( u_{\text{green}} (\text{Ads} , \text{If Ads then Out and if NoAds then Out} ) \) = 3

\( u_{\text{green}} (\text{NoAds} , \text{If Ads then In and if NoAds then In} ) \) = 4

\( u_{\text{green}} (\text{NoAds} , \text{If Ads then In and if NoAds then Out} ) \) = 2

\( u_{\text{green}} (\text{NoAds} , \text{If Ads then Out and if NoAds then In} ) \) = 4

\( u_{\text{green}} (\text{NoAds} , \text{If Ads then Out and if NoAds then Out} ) \) = 2
A **strategy profile** is a list of strategies, **one for each agent**

That is,

one from \( S_{\text{gray}} = \{ \text{Ads, NoAds} \} \) and

one from \( S_{\text{green}} = \{ \text{If Ads then In and if NoAds then In,} \)  
\text{If Ads then In and if NoAds then Out,} 
\text{If Ads then Out and if NoAds then In,} 
\text{If Ads then Out and if NoAds then Out } \}

**Example:**

\((\text{NoAds} ; \text{If Ads then Out and if NoAds then In})\)
A Nash equilibrium is a strategy profile with the property that no agent can increase her payoff by changing her strategy in the profile.

More on this to come later.

The rollback (a.k.a. backward induction) technique gives you an equilibrium:

\[( \text{Ads} ; \text{If Ads then Out and if NoAds then In} )\]

To verify, we ask two questions:

1. Can Gray increase her payoff by changing from Ads to NoAds?
2. Can Green increase her payoff by changing from

   \[\text{If Ads then Out and if NoAds then In}\]

   to any one of her other strategies?
FIGURE 3.3 “One”-Player Game

CURRENT SELF
- law school
  - public prosecutor
    - $50,000/yr
  - politics
    - $35,000–$200,000/yr
- medical school
  - community medicine
    - $50,000/yr
  - family practice
    - $150,000/yr
  - neurosurgery
    - $500,000/yr
- business school
  - corporate law
    - $600,000/yr
  - marketing/nonprofit
    - $100,000/yr
  - management
    - $250,000/yr on average
  - finance
    - $0–$5m/yr

FUTURE SELF
- management
  - $250,000/yr on average
First-mover advantage
Second-mover advantage

FIGURE 3.4 Change of Move Order in the Senate Race Game
FIGURE 3.5 Three-Player Game Tree
FIGURE 3.6 A More Complex Tree
Take Dimes
Take Dimes
Take Dimes
Take Dimes
Take Dime
Take Dimes
Take Dimes
Take Dimes
Take Dimes
Pass
Pass
Pass
Pass
Pass

Payoffs all shown as A, B

FIGURE 3.7 The Centipede Game
(c)
Sequential-move games

extensive form (i.e. the game tree).

rollback technique

Simultaneous-move games

strategic form (i.e. the game table).

dominant or dominated strategies, cell-by-cell,
and minimax techniques
Next: analyse relationship between the two

WHY ???

To be able to

1. relate the solution techniques used for the two types of games.

2. solve games that are mixtures of sequential and simultaneous moves.
(a) **Extensive form** of the Senate Race Game

**FIGURE 6.1 A** Senate Race Game

Copyright © 2000 by W.W. Norton & Company
This table represents an interaction different than the Senate Race game!

<table>
<thead>
<tr>
<th></th>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ads</td>
<td>1, 1</td>
<td>3, 3</td>
</tr>
<tr>
<td>No Ads</td>
<td>2, 4</td>
<td>4, 2</td>
</tr>
</tbody>
</table>
How to translate an extensive-form game into strategic-form?

1. Determine the agents’ strategies
   a. What are the pure strategies of GRAY?
   b. What are the pure strategies of GREEN?

3. Use these strategies to form the game table.

4. Determine the payoffs of each outcome from the original tree.
(b) **Strategic form** of the Senate Race Game

<table>
<thead>
<tr>
<th>GRAY</th>
<th>Ads</th>
<th>In, In</th>
<th>In, Out</th>
<th>Out, In</th>
<th>Out, Out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1, 1</td>
<td>1, 1</td>
<td>3, 3</td>
<td>3, 3</td>
</tr>
<tr>
<td>No Ads</td>
<td>2, 4</td>
<td>4, 2</td>
<td>2, 4</td>
<td>4, 2</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 6.1 B** Senate Race Game
Applying rollback to the game tree gives us the strategy profile

( Ads ; (Ads=>Out, NoAds=>In ) )

This profile is a Nash equilibrium of the Senate Race game

(verify this claim by using the strategic form representation)

this is not a coincidence

**Theorem:** For every game, every strategy profile that is obtained by applying the rollback (a.k.a. backward induction) technique to the game tree is a Nash equilibrium of the game.
Checking the strategic form of the Senate Race game however gives us another Nash equilibrium

\[(\text{NoAds} ; (\text{Ads}=>\text{In}, \text{NoAds}=>\text{In}) )\]

Thus

**Theorem:** The rollback technique does not give you all Nash equilibria of a game.

Note that this new equilibrium is based on a noncredible threat

**Claim:** every Nash equilibrium which is not obtained by the rollback technique is based on a noncredible threat.

Let us formalize this idea:
**Definition:** Given an extensive form game and a decision node that is not part of an information set, the part of the game tree that follows from that decision node is called a **subgame**.

Note that every subgame of a game are themselves games and they have their own Nash equilibria.

**Definition:** A Nash equilibrium \((s_1, \ldots, s_n)\) for the game \(G\) is called a **subgame perfect Nash equilibrium of \(G\)** if for every subgame \(G'\) of the game \(G\), the restriction of \((s_1, \ldots, s_n)\) to \(G'\) is a Nash equilibrium of \(G'\).
The relation between *subgame perfect Nash equilibria* and the *rollback technique*:

**Theorem:** Given an extensive form game $G$, the strategy profile $(s_1, ..., s_n)$ is a subgame perfect Nash equilibrium of $G$ if and only if $(s_1, ..., s_n)$ is obtained by the rollback technique.

Subgame perfect Nash equilibria are those Nash equilibria that are not based on noncredible threats.

The Theorem follows since the rollback technique does not allow noncredible threats.
(a) Strategic form

<table>
<thead>
<tr>
<th>USAF</th>
<th>JAPANESE NAVY</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North</td>
<td>South</td>
</tr>
<tr>
<td>North</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>South</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
How to translate a strategic-form game into extensive form?

1. Choose a player to be the first-mover, say USAF.  
   *(He won’t actually move first, doesn’t matter who you choose)*

2. Draw USAF’s decision node and branches denoting its actions.

3. At the end of each branch, there will be a decision node for Japan.

4. Draw branches denoting Japan’s actions.

5. Write down the payoffs corresponding to each outcome.
In the story, Japan did not know which action USAF took.

How to denote this in the game tree?

Use an information set:

*it is a set of decision nodes (of a single player), the player cannot observe* which of these decision nodes the game has reached.

Information sets indicate incomplete information.

The player **has to choose the same action** in every decision node in the same information set.

**NOTE:** You can not apply the rollback technique when there are (non-singleton) information sets.
(b) Extensive form

FIGURE 6.2 B The Battle of the Bismarck Sea
A game with both sequential and simultaneous moves

Two electronics firms: KUMQUAT and KIWIFRUIT

1. period: firms simultaneously choose their R&D budgets
   this determines the quality of their products

2. period: firms simultaneously choose their prices
   this determines their sales and profits

How to write down the strategies available to these firms?
How to find the equilibria of this game?
Kumquat shows full features; Kiwifruit shows limited features.
Both announce full-featured product.

Kumquat shows full features; Kiwifruit shows limited features.
Both announce limited features product.
FIGURE 6.3 A Combination Game in Extensive Form
<table>
<thead>
<tr>
<th>OFFENSE</th>
<th>DEFENSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>4/5</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
</tr>
</tbody>
</table>

**FIGURE 6.4** Success Probability Table in Two-Play Football Example
<table>
<thead>
<tr>
<th>OFFENSE</th>
<th>DEFENSE</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1</td>
<td>1/2</td>
</tr>
</tbody>
</table>

**FIGURE 6.5** Payoff Table for Fourth Down, 20 Yards to Go
<table>
<thead>
<tr>
<th>OFFENSE</th>
<th>DEFENSE</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>4/5</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td></td>
<td>1/2</td>
</tr>
</tbody>
</table>

**FIGURE 6.6** Payoff Table for Fourth Down, 10 Yards to Go
FIGURE 6.7  Payoff Table for Third Down

<table>
<thead>
<tr>
<th>OFFENSE</th>
<th>DEFENSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11/14</td>
<td>6/7</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>3/4</td>
</tr>
</tbody>
</table>
## Prisoners’ Dilemma

<table>
<thead>
<tr>
<th>Student 1</th>
<th>Goof off (Defect)</th>
<th>Work hard (Cooperate)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goof off (Defect)</td>
<td>1, 1</td>
<td>3, 0</td>
</tr>
<tr>
<td>Work hard (Cooperate)</td>
<td>0, 3</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
A twice-repeated prisoners’ dilemma game

1. Draw the game tree.

2. How many subgames does this game have?

3. What are the players’ strategies?

4. Apply rollback to find all subgame perfect Nash equilibria.
### Battle of the Sexes

<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Football</td>
<td>Soap opera</td>
</tr>
<tr>
<td><strong>Husband</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Soap opera</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

Other examples: two politicians determining position on an issue

two merging firms choosing between PC and MAC
A twice-repeated battle-of-the-sexes game

1. Draw the game tree.

2. How many subgames does this game have?

3. What are the players’ strategies?

4. Apply rollback to find all subgame perfect Nash equilibria.
Sequential moves and infinitely many actions

• Analyze the sequential versions of the following games.

• Relate your reasoning to the best-response analysis
The Sequential Price-setting Game Between Donna and Pierce

Donna’s Deep Dish: moves first and chooses \( P_{Donna} \)

Pierce’s Pizza Pies: moves second and chooses \( P_{Pierce} \)

Market surveys show that given the prices each sells (in 1000 pizzas per week):

\[
Q_{Donna} = 12 - P_{Donna} + 0.5P_{Pierce}
\]

\[
Q_{Pierce} = 12 - P_{Pierce} + 0.5P_{Donna}
\]

Note: If Pierce increases his price, his sales go down and Donna’s sales go up

• Cost of each pizza: 3 USD
• Pierce’s profit (i.e. his payoff) (in 1000 USD) is then

\[ Y_{Pierce} = P_{Pierce}Q_{Pierce} - 3Q_{Pierce} \]
\[ = (P_{Pierce} - 3)Q_{Pierce} \]
\[ = (P_{Pierce} - 3)(12 - P_{Pierce} + 0.5P_{Donna}) \]
\[ = (15 + 0.5P_{Donna})P_{Pierce} - P_{Pierce}^2 - 36 - 1.5P_{Donna} \]

• Given \( P_{Donna} \), Pierce will choose his price to maximize his payoff

Taking the derivative of \( Y_{Pierce} \) with respect to \( P_{Pierce} \)

\[ \frac{dY_{Pierce}}{dP_{Pierce}} = 15 + 0.5P_{Donna} - 2P_{Pierce} \]

When \( Y_{Pierce} \) is maximized, this derivative is equal to 0

\[ \frac{dY_{Pierce}}{dP_{Pierce}} = 15 + 0.5P_{Donna} - 2P_{Pierce} = 0 \]
Solving for $P_{Pierce}$ we have

$$p^*_{Pierce} = \frac{15 + 0.5P_{Donna}}{2} = 7.5 + 0.25P_{Donna}$$

This is the best-response function of Pierce.

**NOTE:** We have to verify that what we found by equating the derivative to 0 is a maximum (it can also be a minimum or a saddle-point). For this, we must check if the second derivative at $P^*_{Pierce}$ is negative:

$$\frac{d^2Y_{Pierce}}{dP^2_{Pierce}} = -2$$

So it’s O.K.. We have maximized $Y_{Pierce}$ at $P^*_{Pierce}$ and therefore, we have a best-response function.
What about Donna?

• Donna knows that whatever price she chooses, Pierce will observe it and play a best response.

• So Donna’s problem is to maximize her payoff when $P_{Pierce}$ is given by the previous formula.
Donna’s payoff function is

\[ Y_D = \left( 15 + \frac{1}{2} P_P \right) P_D - P_D^2 - 36 - \frac{3}{2} P_P \]

\[ = \left( 15 + \frac{1}{2} \left( \frac{15}{2} + \frac{1}{4} P_D \right) \right) P_D - P_D^2 - 36 - \frac{3}{2} \left( \frac{15}{2} + \frac{1}{4} P_D \right) \]

\[ = -\frac{7}{8} P_D^2 + \frac{147}{8} P_D - \frac{189}{4} \]

The first and the second derivatives are

\[ \frac{\partial Y_D}{\partial P_D} = \frac{147}{8} - \frac{7}{4} P_D \]

\[ \frac{\partial^2 Y_D}{\partial P_D^2} = -\frac{7}{4} \]
Thus, Donna’s optimal price choice is

$$P_D = \frac{147.4}{8.7} = 10.5$$

Seeing this price, Pierce responds with

$$P_P = \frac{15}{2} + \frac{1}{4}P_D = 10.125$$

In the simultaneous-move version of this game, we had calculated the Nash equilibrium

$$(10, 10).$$

That is, being the first mover gives Donna the upper hand. She can now charge a higher price.
Comparing the outcomes of sequential and simultaneous move interactions
### FIGURE 6.9  Simultaneous-Move Mall Location Game

<table>
<thead>
<tr>
<th></th>
<th>Urban</th>
<th>Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BIG GIANT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>5, 5, 1</td>
<td>5, 2, 5</td>
</tr>
<tr>
<td>Rural</td>
<td>2, 5, 5</td>
<td>4, 4, 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Urban</th>
<th>Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BIG GIANT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>5, 5, 2</td>
<td>3, 4, 4</td>
</tr>
<tr>
<td>Rural</td>
<td>4, 3, 4</td>
<td>4, 4, 4</td>
</tr>
<tr>
<td>TITAN (Row player)</td>
<td>GIANT (Column player)</td>
<td>GIANT (Column player)</td>
</tr>
<tr>
<td>--------------------</td>
<td>-----------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>UU</td>
<td>UR</td>
</tr>
<tr>
<td>1: UUUU</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>2: UUUR</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>3: UURU</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>4: URUU</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>5: RUUU</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>6: UURR</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>7: URRU</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>8: RRUU</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>9: URUR</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>10: RURU</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>11: RUUR</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>12: URRR</td>
<td>5,5,1</td>
<td>5,5,1</td>
</tr>
<tr>
<td>13: RRRR</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>14: RRUR</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>15: RRRU</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
<tr>
<td>16: RRRR</td>
<td>2,5,5</td>
<td>2,5,5</td>
</tr>
</tbody>
</table>

**FIGURE 6.11** Mall Location Game in Strategic Form
(a) Simultaneous play

<table>
<thead>
<tr>
<th>HUSBAND</th>
<th>WIFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>10 yr, 10 yr</td>
</tr>
<tr>
<td>Deny</td>
<td>25 yr, 1 yr</td>
</tr>
</tbody>
</table>
(b) Sequential play—Husband moves first

FIGURE 6.12 B  Three Versions of the Prisoners' Dilemma Game
(c) Sequential play—Wife moves first

**FIGURE 6.12 C** Three Versions of the Prisoners' Dilemma Game
(a) Simultaneous play

<table>
<thead>
<tr>
<th></th>
<th>DEAN</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Swerve</td>
<td>Straight</td>
</tr>
<tr>
<td>JAMES</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Swerve</td>
<td>0, 0</td>
</tr>
<tr>
<td></td>
<td>Straight</td>
<td>1, –1</td>
</tr>
</tbody>
</table>
(b) Sequential play—James moves first

JAMES, DEAN

\[
\begin{array}{c}
\text{JAMES} \\
\text{DEAN}
\end{array}
\]

\[
\begin{array}{c}
\text{Swerve} \\
\text{Straight}
\end{array}
\]

\[
\begin{array}{c}
0, 0 \\
-1, 1 \\
1, -1 \\
-2, -2
\end{array}
\]

FIGURE 6.14 B  Chicken in Simultaneous- and Sequential-Play Versions

Copyright © 2000 by W.W. Norton & Company
(c) Sequential play—Dean moves first

DEAN, JAMES

FIGURE 6.14 C  Chicken in Simultaneous- and Sequential-Play Versions
(a) Simultaneous play

<table>
<thead>
<tr>
<th></th>
<th>HINGIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DL</td>
</tr>
<tr>
<td>SELES</td>
<td></td>
</tr>
<tr>
<td>DL</td>
<td>50</td>
</tr>
<tr>
<td>CC</td>
<td>90</td>
</tr>
</tbody>
</table>

FIGURE 6.15 A  Tennis Game in Simultaneous and Sequential Versions
(b) Sequential play—Seles moves first

Figure 6.15B: Tennis Game in Simultaneous and Sequential Versions
(c) Sequential play—Hingis moves first

FIGURE 6.15 C Tennis Game in Simultaneous and Sequential Versions