On the Investment Implications of Bankruptcy Laws

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Introduction:

Bankruptcy Problem

- A firm goes banktrupt
  - Liquidated assets worth *E* $
- The bankrupt firm owes money to agents in *N*
  - Each agent has a verifiable claim of c_i *$*
- There isn't enough to honour all claims

How to allocate E among agents in N?

The Axiomatic Approach

- Analyzes (c,E) as a “ normative problem”

- Proposes solution rules:

$$F : (c_1 , \dots , c_n , E) \rightarrow (x_1 , \dots , x_n)$$

$$\text{s.t.} \quad x_1 + \dots + x_n = E$$

- Looks for rules with desirable properties

  E.g. Pareto optimality

  Claims monotonicity

Three Central Principles

- Proportionality
  - Proportional Rule, Weighted Proportional Rules
- Equal Awards
  - Constrained Equal Awards Rule, Talmudic rule, Equal Gains Rule, Piniles' Rule, Random Arrival rule, Minimal Overlap Rule
- Equal Losses
  - Constrained Equal Losses Rule , Talmudic rule, Random Arrival Rule, Minimal Overlap Rule

Axiomatic Literature

- In support of CEA:
  - Dagan (1996), Schummer anc
    and Villar (2002), Yeh (2001)
- In support of CEL:
  - Yeh (2001), Herrero and Villar
- In support of TAL:
  - O'Neill (1982), Aumann and M
- In support of PRO:
  - de Frutos (1999), Ching and Kakkar (2000), Chambers and Thomson (2002), Ju, Miyagawa, and Sakai (2007)

NOTE:

All three principles

- Proportionality
- Equal Awards
- Equal Losees

more or less equally predominant

Bankruptcy in real life

- Between 1999 – 2009 in US
  - 551000 + firms filed for Chapter 7 bankruptcy
  - 22 + billion $ allocated
  - Chapter 7 bankruptcy:
    - liquidate the remaining assets
      - as a whole or piecewise
    - allocate among claimants
    - similar to the axiomatic literature
  - Chapter 11 bankruptcy:
    - reorganize the firm

The Empirical / Finance Literature

- Describe alternative practices
    - Atiyas (1995)
    - Hotchkiss, John, Mooradian, Thorburn (2008)
- Literature mostly on Chapter 11
- Comparisons of Chapter 7 vs Chapter 11
    - Hart (1999)
    - Stiglitz (2001)
    - Bris, Welch, and Zhu (2006)

Chapter 7 bankruptcy

- Everywhere around the world
- the common way to allocate liquidated assets among claimants:
- Proportional Rule

  (combined with a priority rule)

This Paper:

asks the following question

Why is proportionality preferred over alternative principles in real-life bankruptcy problems?

The finance literature remains silent on this issue

Possible explanations

- Historical reasons
  - Counter-argument: Talmudic rule (Aumann and Maschler, 1985)
  - although Rabbi Abraham Ibn Ezra (1140) also mentions PRO
- Axiomatic reasons: maybe governments prefer the axioms that characterize PRO
- Incentive reasons: maybe the investment incentives created by the PRO are superior to that of others

We check this third explanation.

We study noncooperative investment games with possible bankruptcy

- Araujo and Pascua (2002)
  - 2 period general equilibrium model with bankruptcy
  - Conditions under which equilibrium exists and is efficient
  - No comparison of bankruptcy rules

- Karagözoğlu (2010)
  - Noncooperative investment game
  - Two types of agents: high/low income
  - Invest zero or everything
  - Linear utilities (risk neutrality)

The Investment Game under *F*

- (t=1) n investors
  - Simultaneously choose their investments on a firm: $s_1, \dots, s_n \geq 0$
  - Value of the firm: $s_1 + \dots + s_n$
- (t=2) Firm
  - Succeeds with probability p : return of r
  - Fails with probability $1-p$: bankruptcy
- Bankruptcy
  - The value of the firm becomes $\beta(s_1 + \dots + s_n)$

$$\beta \in (0,1)$$

Supported by Bris et al (2006)

  - Allocated among the investors according to

a **prespecified bankruptcy rule *F***

The Investment Game
Under F :

s_1 , … , s_n

Chosen

Simultaneously

$V = s_1 + \ldots + s_2$

Success

[p]

Net Return:

$r\, s_i$

Bankruptcy

[1-p]

Net Return:

$F_i\,(s,\beta(s_1+\ldots+s_n)) \;-\; s_i$

Parameters of the Game

- The bankruptcy rule used F
- Probability of success: p
- Return in case of success: r
- Fraction that survives bankruptcy: β
- Agents' risk aversion levels: a_i

Agents

- CARA utilities
  - Risk aversion level independent of income
  - Agents possibly heterogenous in risk aversion

Represents heterogeneity in income

How does a rule treat big vs small investors?

- No income constraints
  - Initially all agents have zero income
  - Agents borrow at the market rate (norm. to 0)
  - Simplifies the agents' optimization problems by eliminating the boundary conditions

The agents' CARA utilities

$$U_i^F(s) = -pe^{-a_i r s_i} - (1-p)e^{-a_i F_i(s_i, s_{-i}) + a_i s_i}$$

Probability of success

Net return in case of success

Probability of bankruptcy

Net return in case of bankruptcy

We do

Compare the Nash equilibria of the investment games under

1. proportionality
2. equal awards

   mixtures of prop. and equal awards

   constrained equal awards

3. equal losses

   mixtures of prop. and equal losses

   constrained equal losses

We do

Compare them interms of

1. total equilibrium investment
2. equilibrium social welfare

   egalitarian

   utilitarian

3. the effect of possibly heterogenous risk attitudes

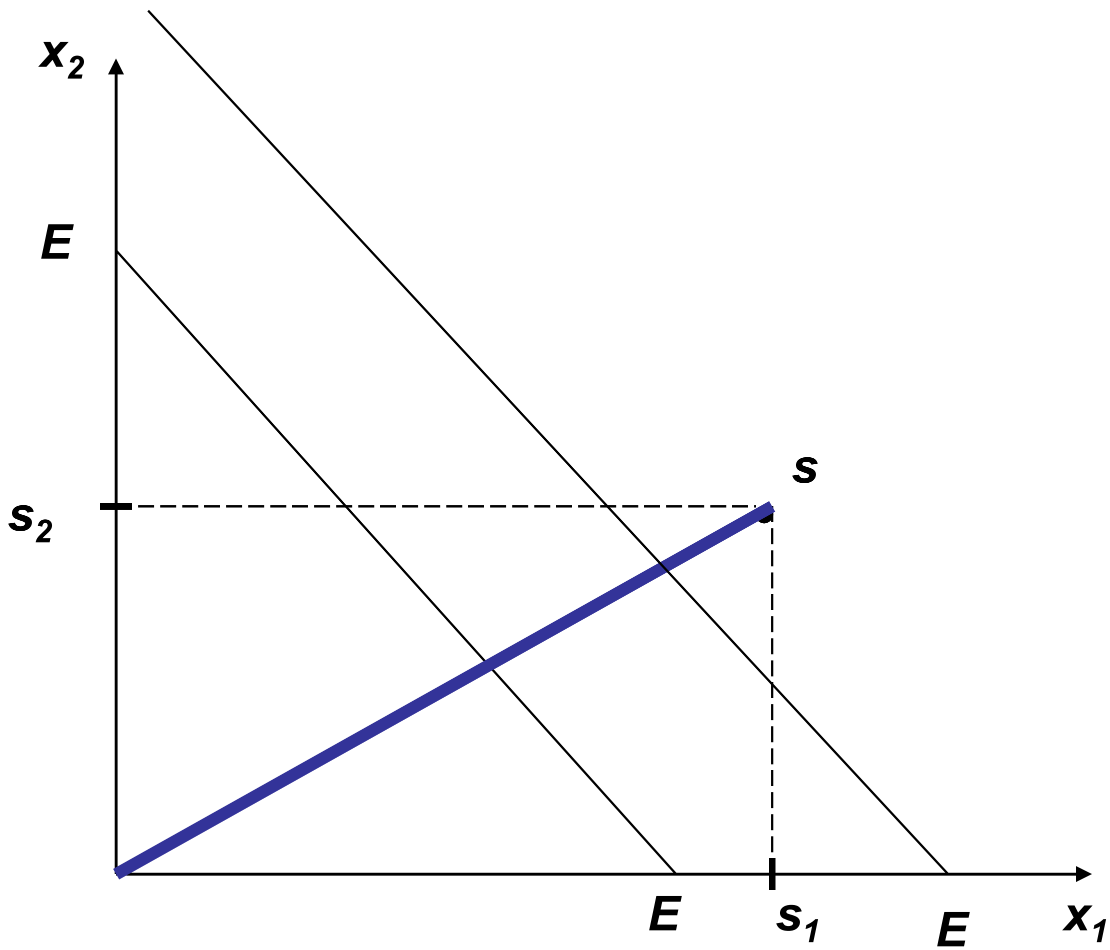
RESULTS I

CALCULATING EQUILIBRIUM

INVESTMENT LEVELS UNDER

ALTERNATIVE BANKRUPTCY RULES

Proportionality (PRO)



$x_i = q\ s_i$

EQUILIBRIUM UNDER PRO

$$F_i^P(s) = \beta s_i$$

Proportional shares in case of bankruptcy

$$U_i^P(s) = -pe^{-a_i r s_i} - (1-p)e^{a_i s_i (1-\beta)}$$

- Independent of agent j's strategy
- Well-behaved => unique best response

$$s_i^* = \frac{1}{a_i\,(r+1-\beta)} \ln\left(\frac{pr}{(1-p)(1-\beta)}\right)$$

Common term for nonnegative investment

EQUILIBRIUM UNDER PRO

The investment game under PRO

unique dominant strategy equilibrium

Equilibrium investment level is

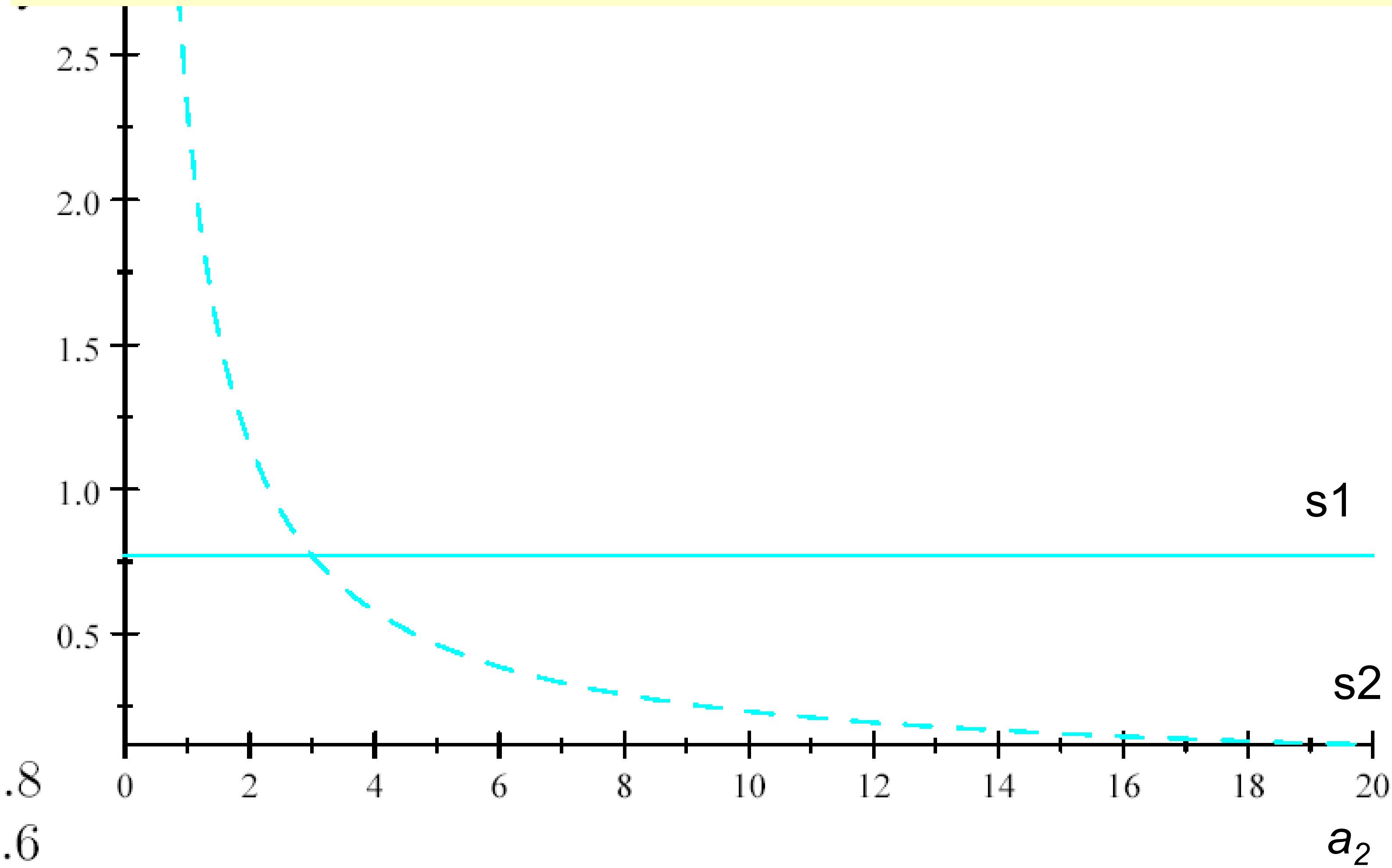
increasing in p and β

decreasing in own risk aversion

independent of other's risk aversion

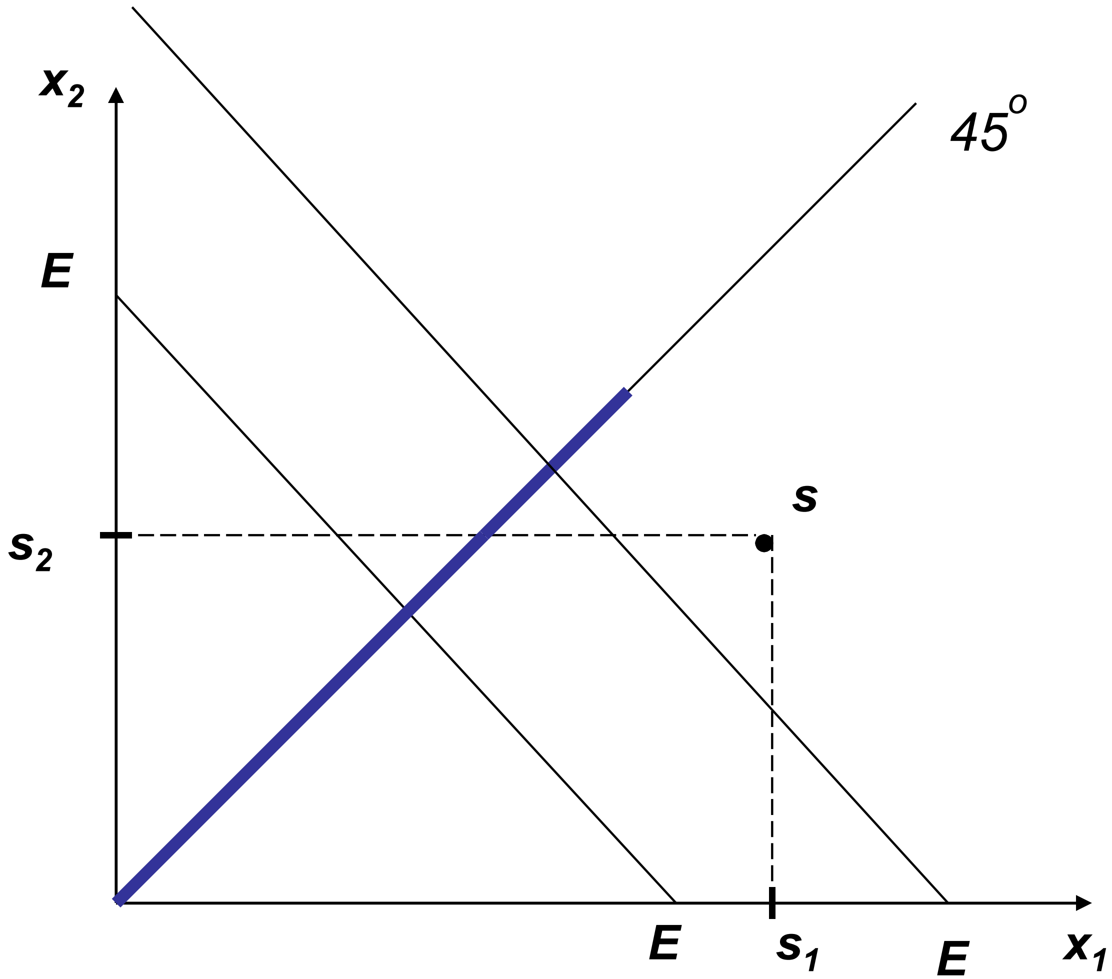
Numerical example:

Equilibrium investment levels under PRO



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Equal Awards (EA)



$x_i = E / 2$

EQUILIBRIUM UNDER EA

- Agents are awarded equal shares in case of bankruptcy

$$EA_i(s) = \frac{\beta}{n} \sum_N s_i$$

- Well-behaved payoff functions
- Unique best response
- Unique NE always exists

MIXTURES OF PRO and EA

Agents receive a convex combination of PRO and EA in case of bankruptcy

$$AP[\alpha]_i(s) \quad = \quad \alpha PRO_i(s) + (1-\alpha)\, EA_i(s)$$

α = 1 *is* PRO *α = 0* *is* EA

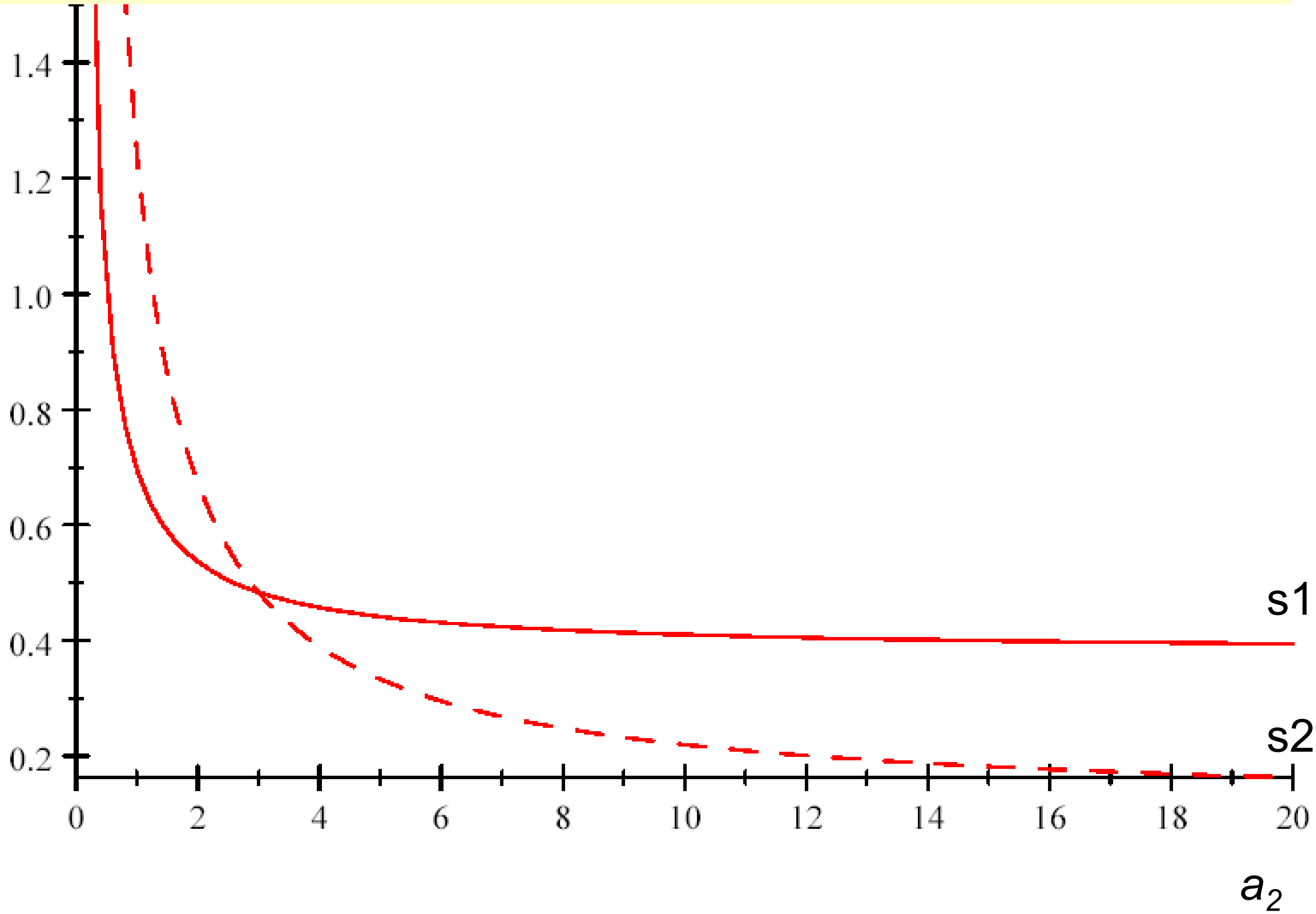
- **Unique NE:**

$$s_i^* = \frac{n(1+r-\beta) + \beta(1-\alpha) + \beta(1-\alpha)\, a_i \sum_{N-i} \frac{1}{a_j}}{a_i n (1+r-\beta)(1+r-\alpha\beta)} \ln\left(\frac{npr}{(1-p)(n-\beta-(n-1)\alpha\beta)}\right)$$

Common term for nonnegative investment

Numerical example:

Equilibrium investment levels under EA



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Problematic Parameter Values

- Want to rule out cases where

  equilibrium investment < share in case of bankruptcy

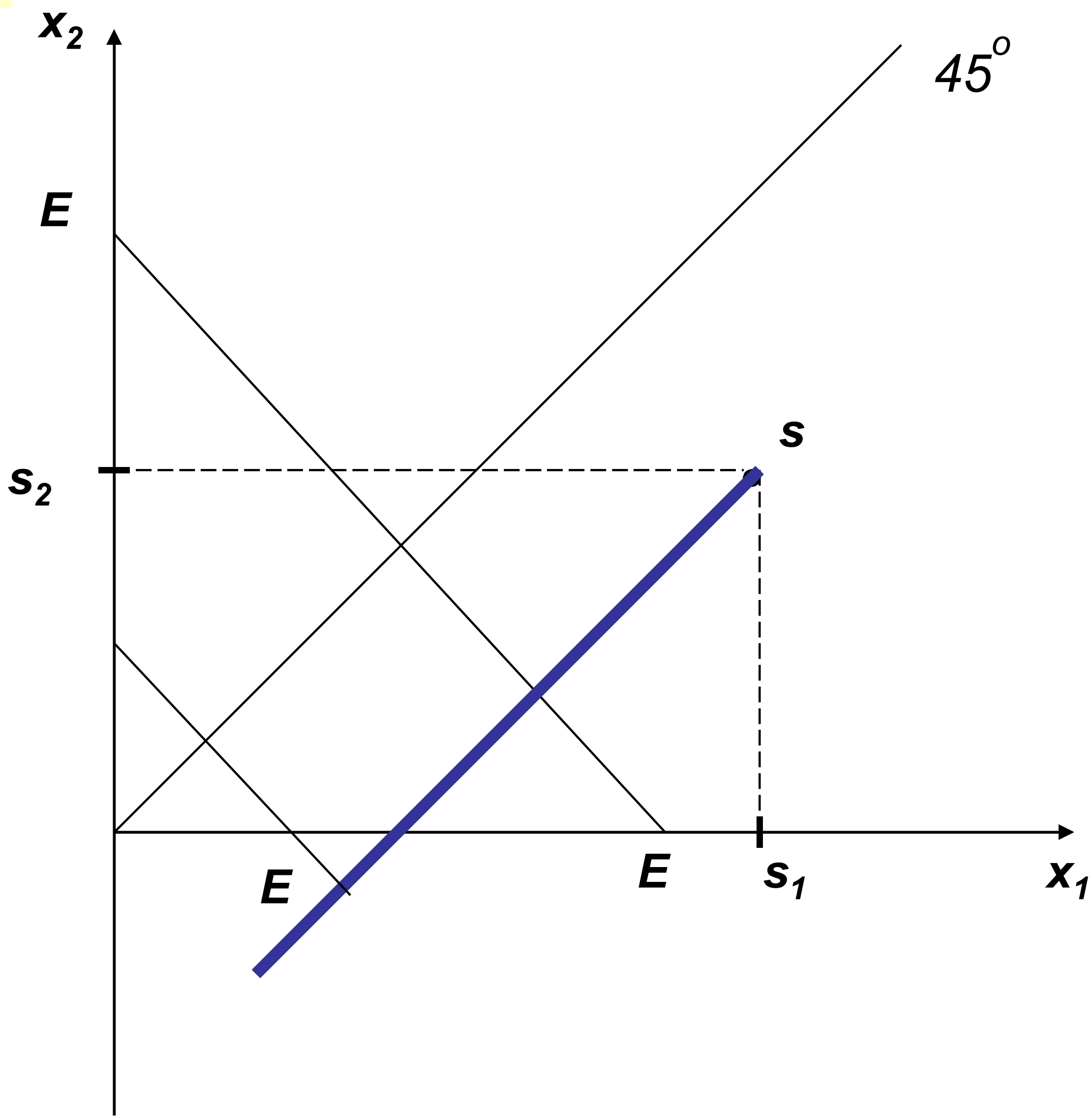
- This implies:

$$\frac{\frac{1}{a_n}}{\sum_N \frac{1}{a_j}} \geq \frac{r\beta\left(1-\alpha\right)}{n\left(1-\alpha\beta\right)\left(1+r-\beta\right)}$$

- Alternatively: use CEA instead of EA

Equal Losses (EL)

$$x_i = s_i - q$$



EQUILIBRIUM UNDER EL

- Agents forego equal shares in case of bankruptcy

$$EL_i(s) = s_i - \tfrac{1-\beta}{n} \sum_N s_j$$

- Well-behaved payoff functions
- Unique best response
- Unique NE always exists

MIXTURES OF PRO and EL

- Agents receive a convex combination of PRO and EL in case of bankruptcy

$$LP[\alpha]_i(s) = \alpha PRO_i(s) + (1-\alpha) EL_i(s)$$

α=1 is PRO and *α=0 is* EL

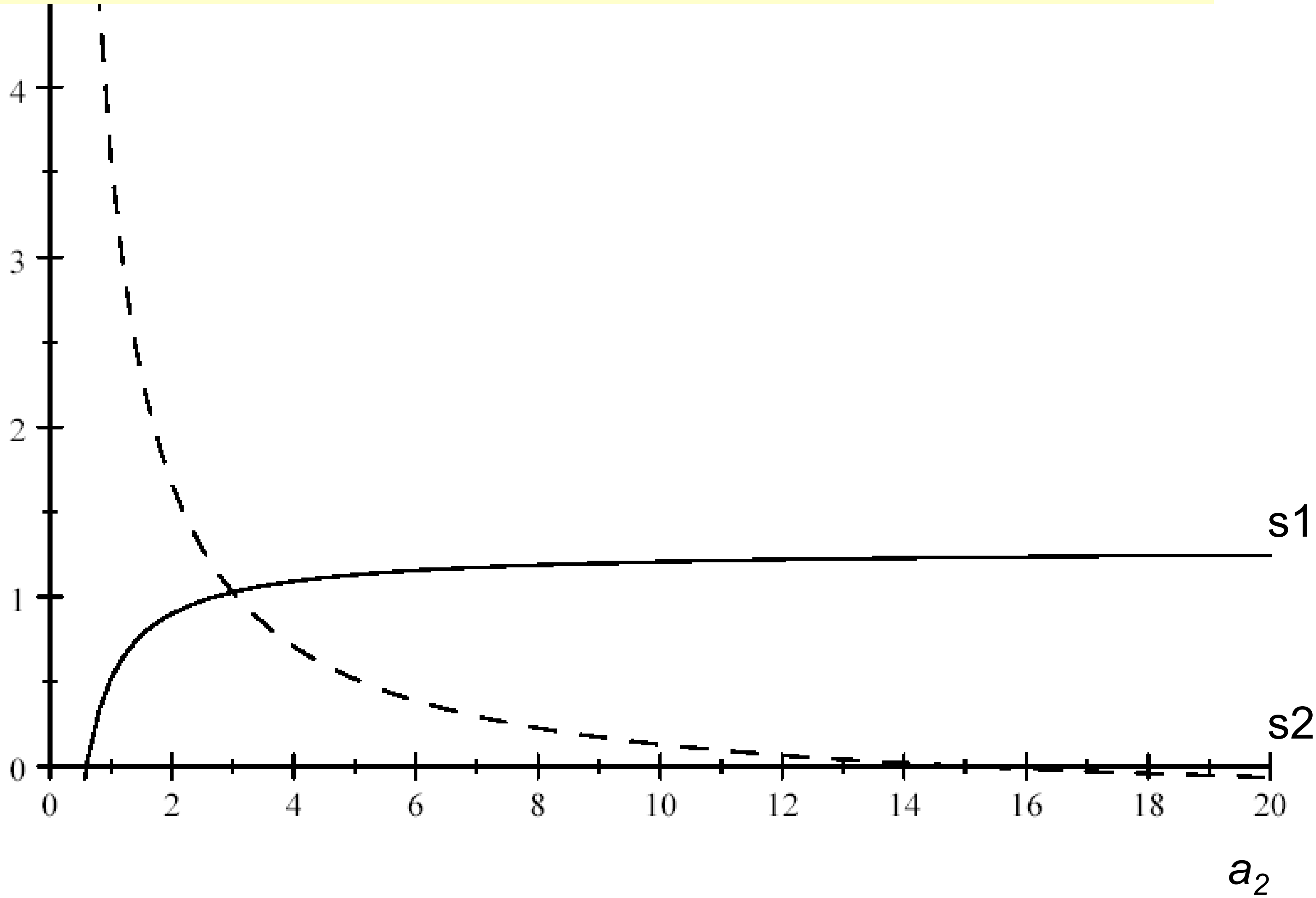
- **Unique NE:**

$$s_i^* = \left(\frac{1}{a_i} - \frac{(1-\alpha)(1-\beta)}{n(1+r-\beta)} \sum_N \frac{1}{a_j}\right) \frac{\ln\left(\frac{npr}{(1-\beta)(1-p)(1+(n-1)\alpha)}\right)}{r + \alpha(1-\beta)}$$

Common term for nonnegative investment

Numerical example:

Equilibrium investment levels under EL



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Problematic Parameter Values

- Want to rule out cases where

  equilibrium share in case of bankruptcy < 0

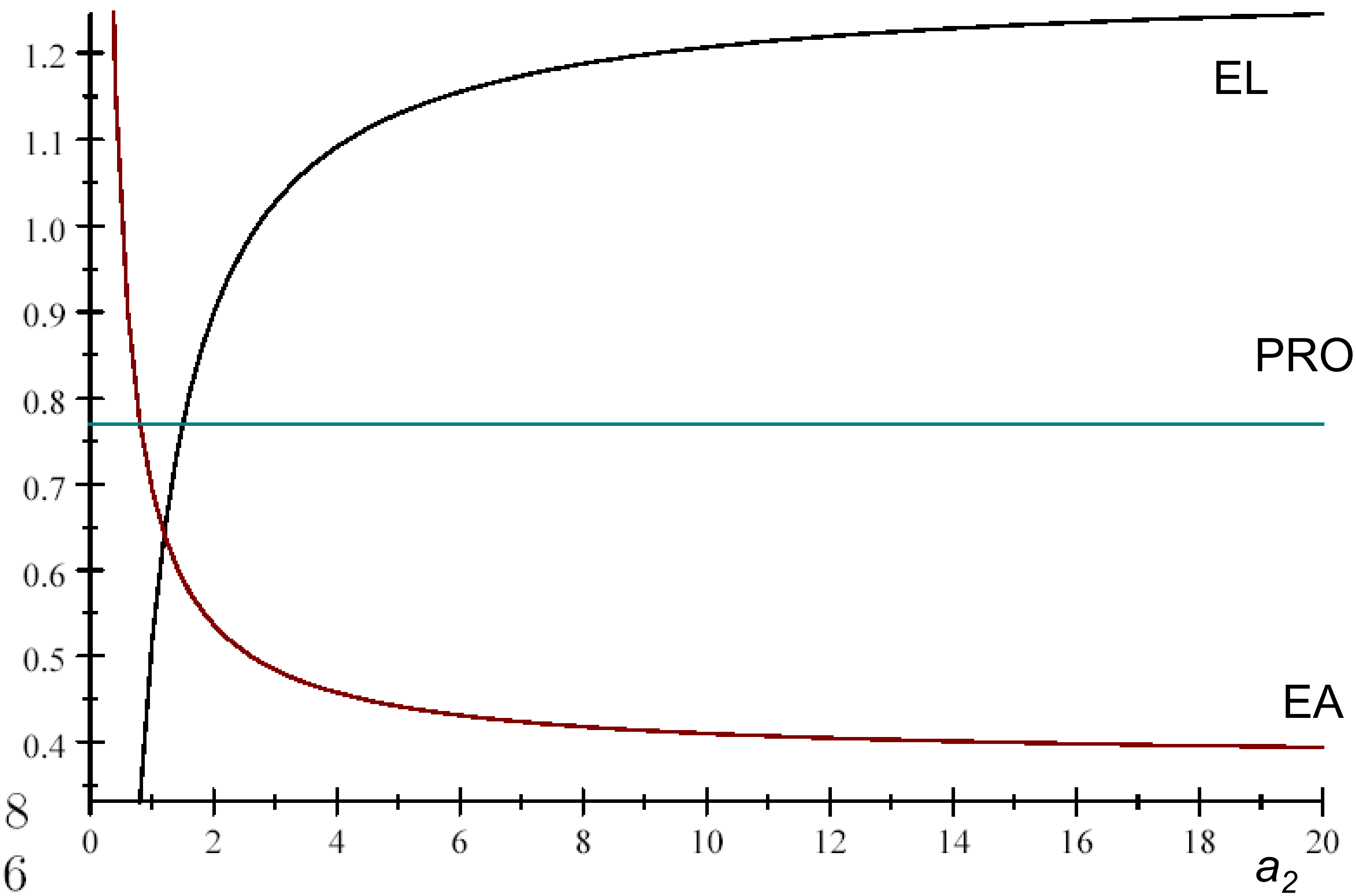
- This implies:

$$\frac{\frac{1}{a_n}}{\sum_N \frac{1}{a_j}} \geq \frac{(r+1)(1-\alpha)(1-\beta)}{n(1-\beta+r)(1-\alpha+\alpha\beta)}$$

- Alternatively: use CEL instead of EL

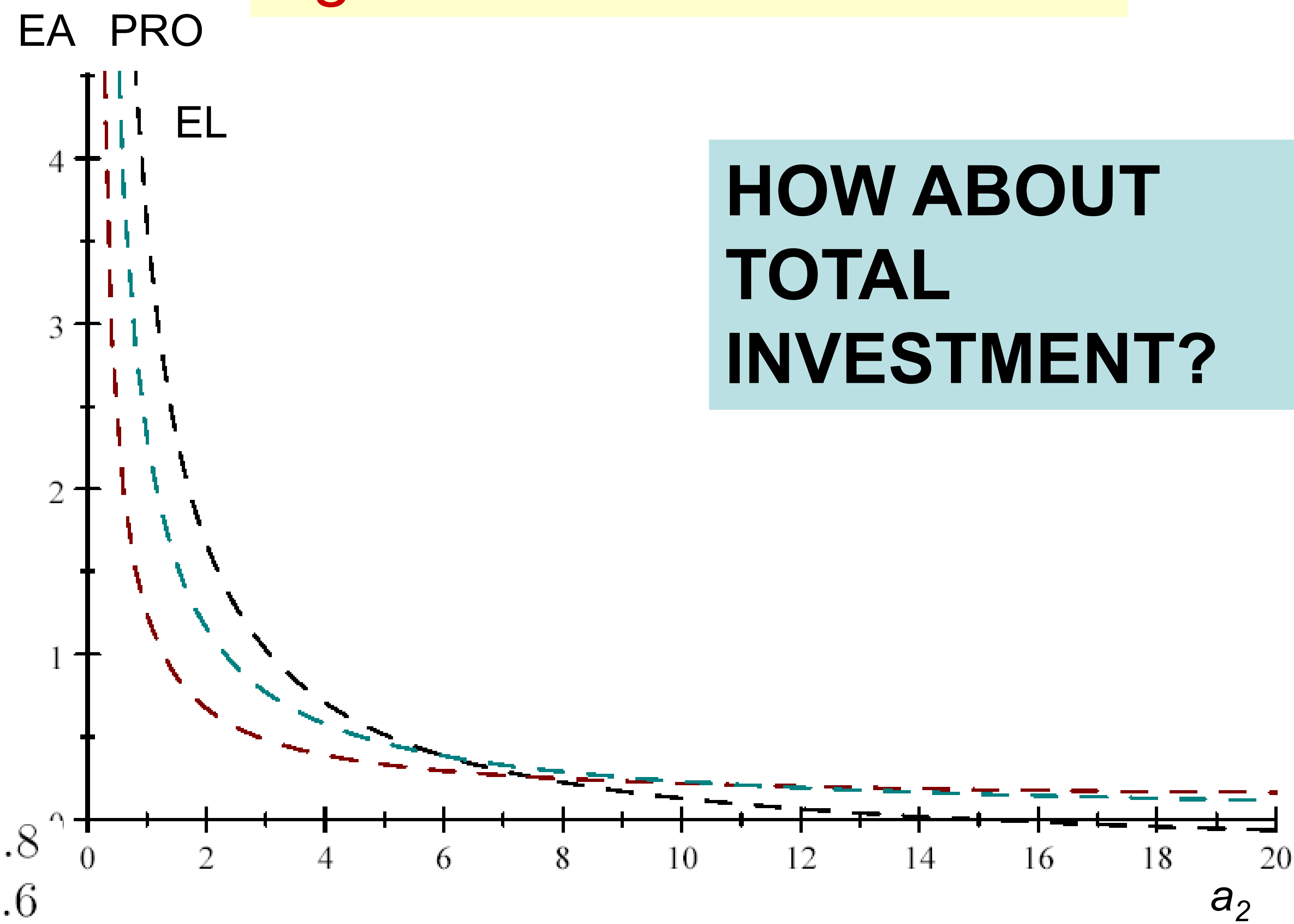
Summary of Part I

Agent 1's investment levels



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Agent 2's investment levels



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

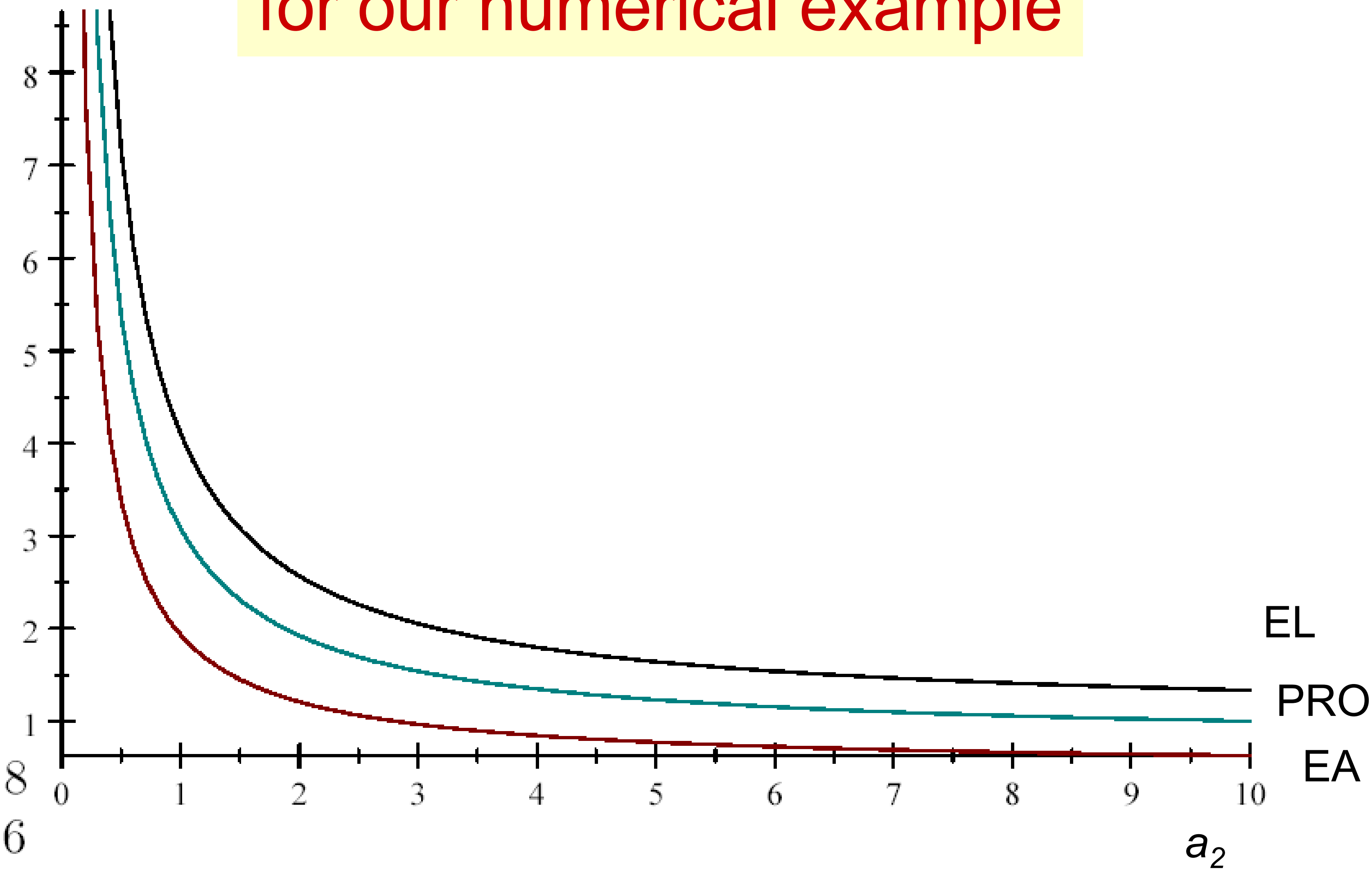
HOW ABOUT TOTAL INVESTMENT?

RESULTS II

COMPARING TOTAL INVESTMENT LEVELS

UNDER ALTERNATIVE BANKRUPTCY RULES

Total investment levels for our numerical example

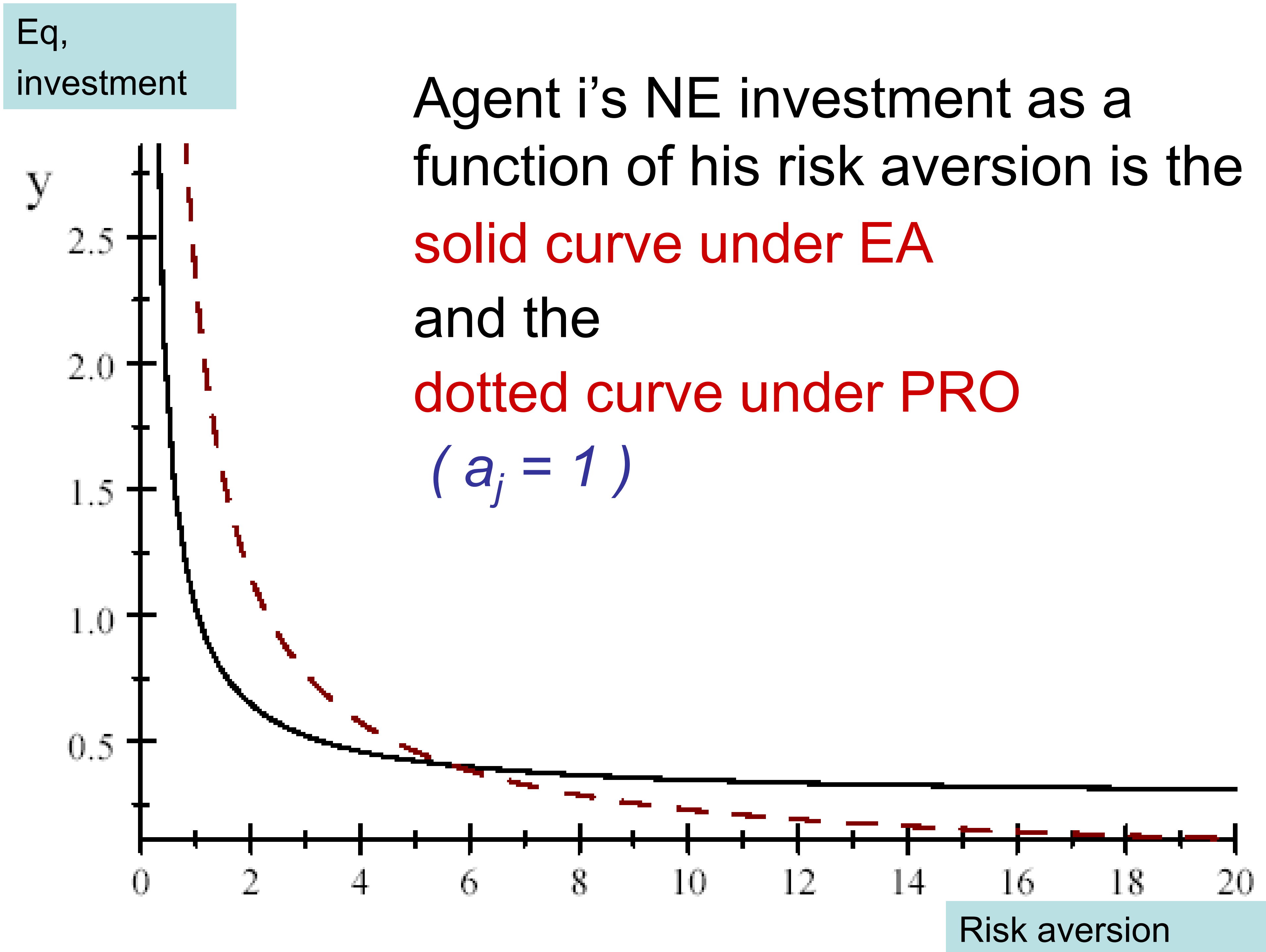


$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

PRO vs. EA

- An agent's equilibrium investment level
  - Decreasing in risk aversion
  - Cutoff risk aversion level
    - Below cutoff: invests more under PRO
    - Above cutoff: invests more under EA

Agent i’s NE investment as a function of his risk aversion is the
solid curve under EA
and the
dotted curve under PRO
($a_j = 1$)



Investment Under PRO vs. EA

- Small investors: invest more under EA
- Big investors: invest more under PRO
- How about total investment?
  - Independent of the parameters,

    the following is always true:

PRO vs. EA

THM: In terms of total investment,

PRO > EA

GENERALIZE IT FURTHER?

Mixtures of PRO and EA

Total investment is an increasing function of α

THM:

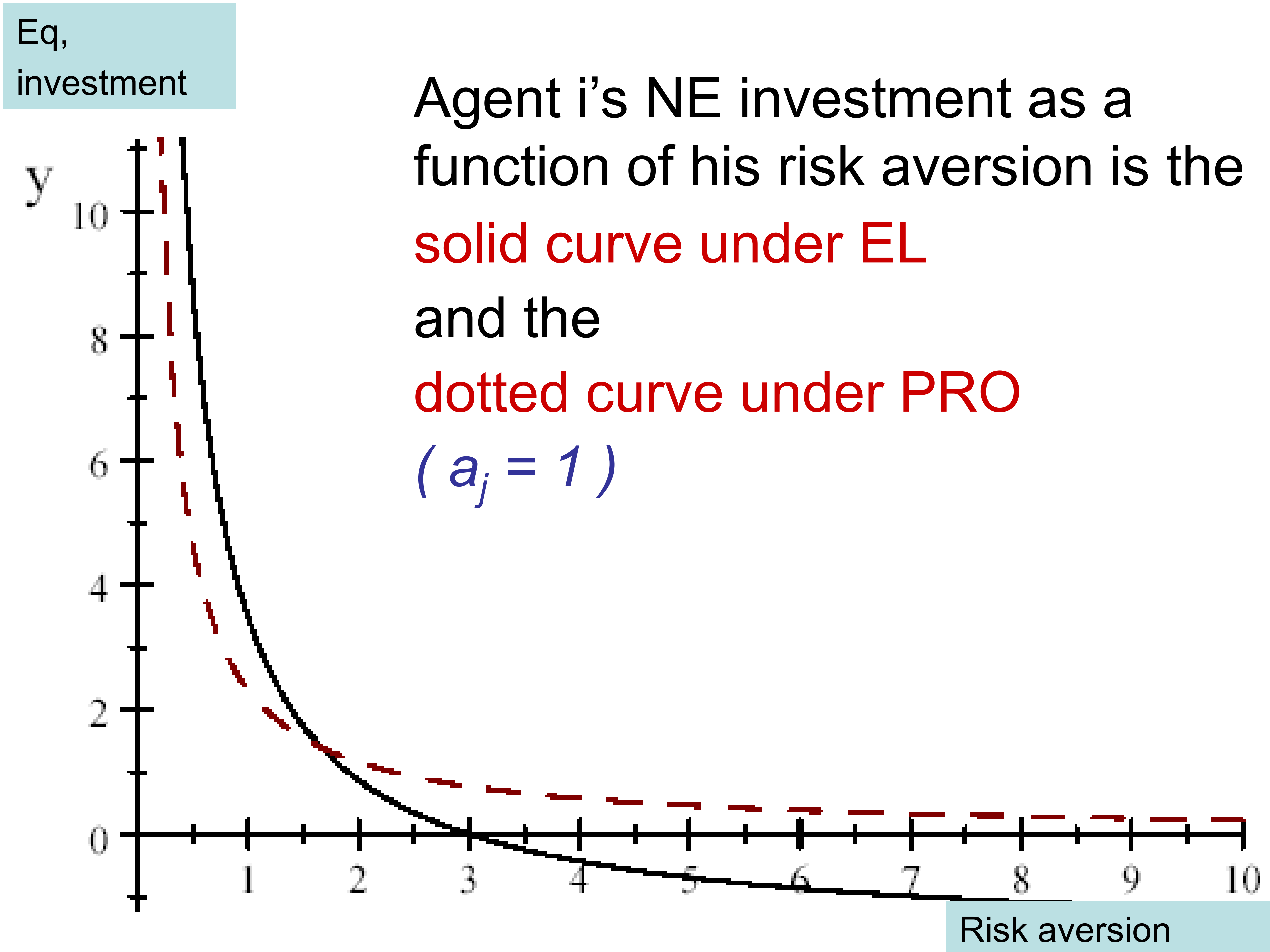
$$\alpha > \alpha'$$
$$\text{implies}$$
$$\text{Total Investment under } AP[\alpha] > \text{Total Investment under } AP[\alpha']$$

PRO and EA are the two extremes

PRO vs. EL

- An agent's equilibrium investment level
  - Decreasing in risk aversion
  - Cutoff risk aversion level
    - Below cutoff: invests more under EL
    - Above cutoff: invests more under PRO

Agent i's NE investment as a function of his risk aversion is the solid curve under EL and the dotted curve under PRO ($a_j = 1$)



Investment Under PRO vs. EL

- Small investors: invest more under PRO
- Big investors: invest more under EL
- How about total investment?
  - Independent of the parameters, the following is always true:

PRO vs. EL

THM: In terms of total investment,

EL > PRO

GENERALIZE IT FURTHER?

Mixtures of PRO and EL

Total investment is a decreasing function of α

THM:

$$\alpha > \alpha'$$
$$\text{implies}$$
$$\text{Total Investment under } LP[\alpha] < \text{Total Investment under } LP[\alpha']$$

PRO and EL are the two extremes

OVERALL

In terms of total investment

EL > PRO > EA

Mixtures of EL and PRO

Mixtures of EA and PRO

RESULTS: III

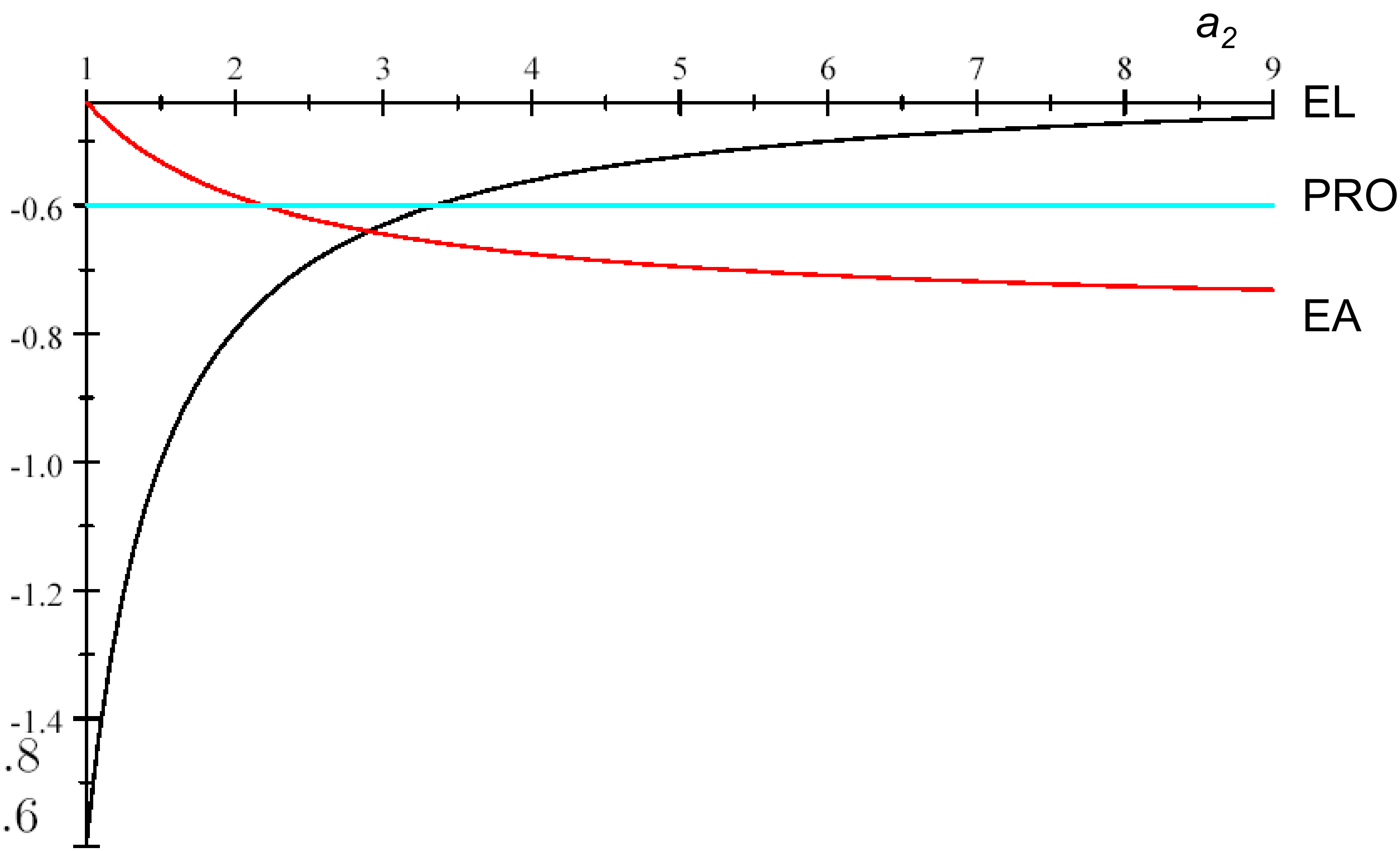
COMPARING SOCIAL WELFARE UNDER

THE THREE MAIN RULES

Welfare Calculation

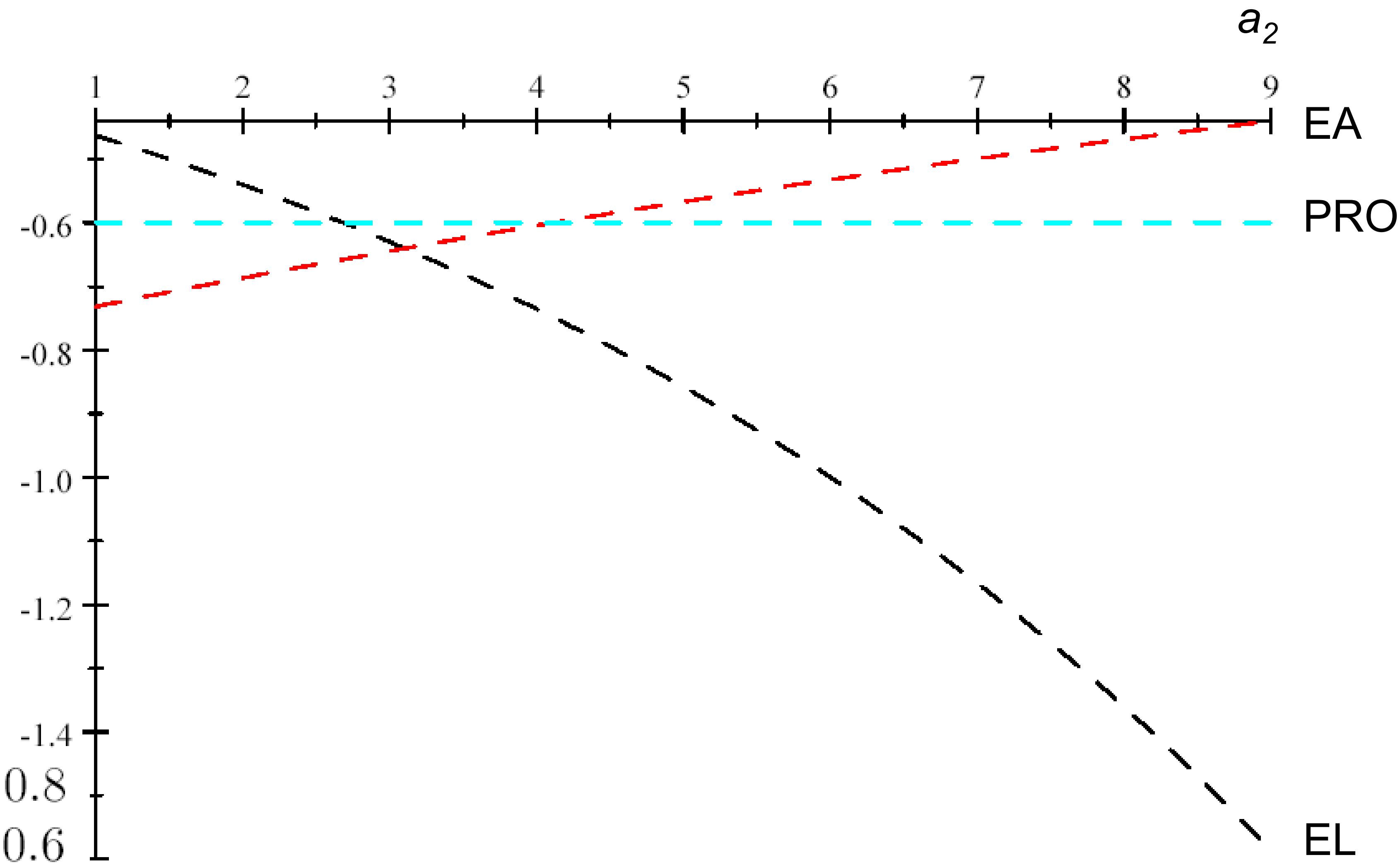
- Messy expressions
- Restrict analysis to
  - Three main rules:
    - PRO
    - EA
    - EL
  - Two agents

Example: Agent 1's welfare levels



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Example: Agent 2's welfare levels

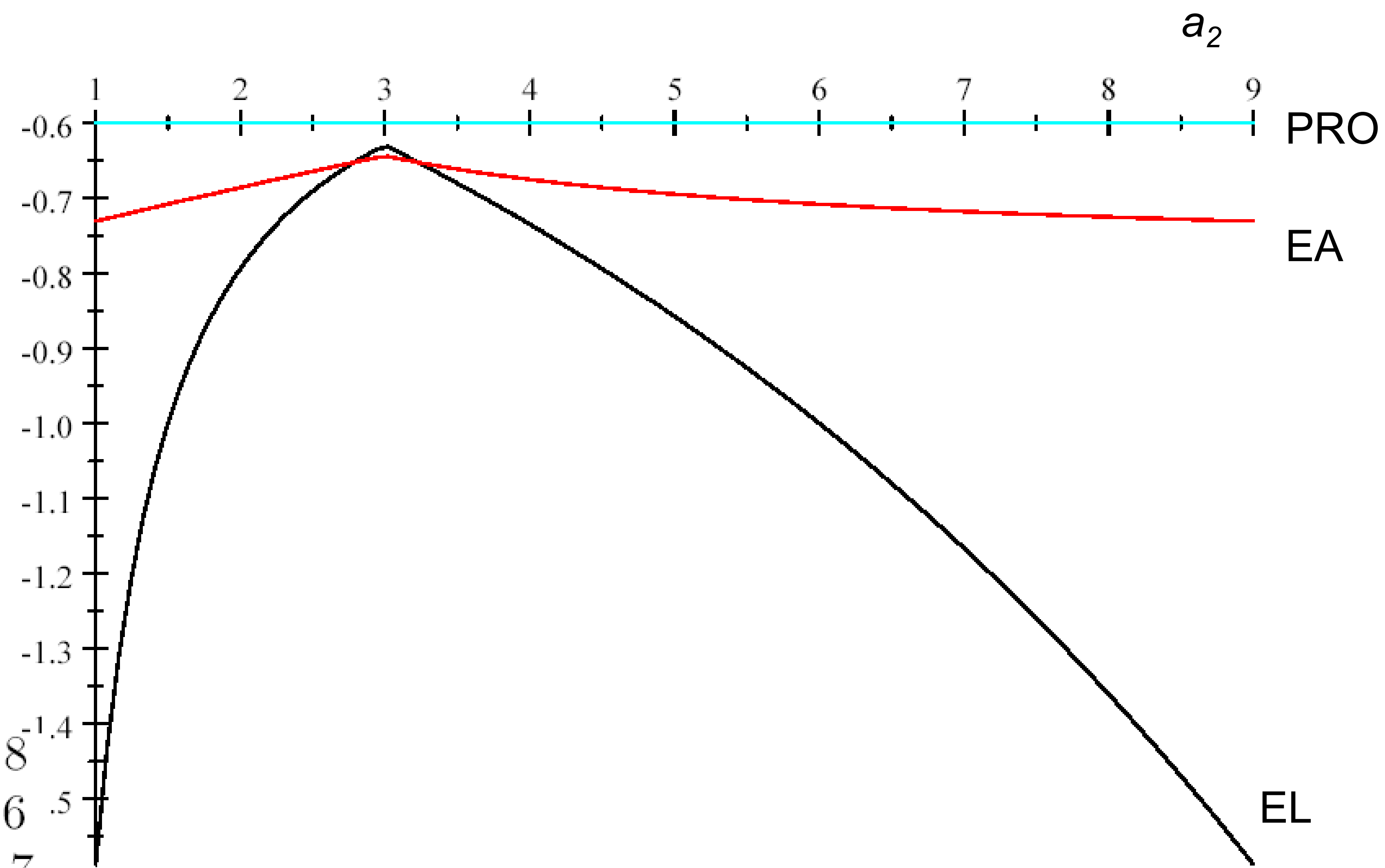


$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

Egalitarian Social Welfare induced by ***F***

$$EG^F\underbrace{(p, r, \beta, a_1, a_2)}_{\text{a function of the parameters}} = \min\left\{\underbrace{U_1^F(\epsilon\left(G^F\right))}_{\text{Agent 1's equilibrium utility}}, \underbrace{U_2^F(\epsilon\left(G^F\right))}_{\text{Agent 2's equilibrium utility}}\right.$$

Egalitarian social welfare for our example



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

THEOREM

Assume parameter values are such that there is an interior equilibrium under all three rules

Then in terms of egalitarian social welfare

PRO > EL and

PRO > EA

Egalitarian Social Welfare EA vs EL

Numerical comparison of interior equilibria

1.3 million parameter combinations

EA > EL on 73% of the parameter space

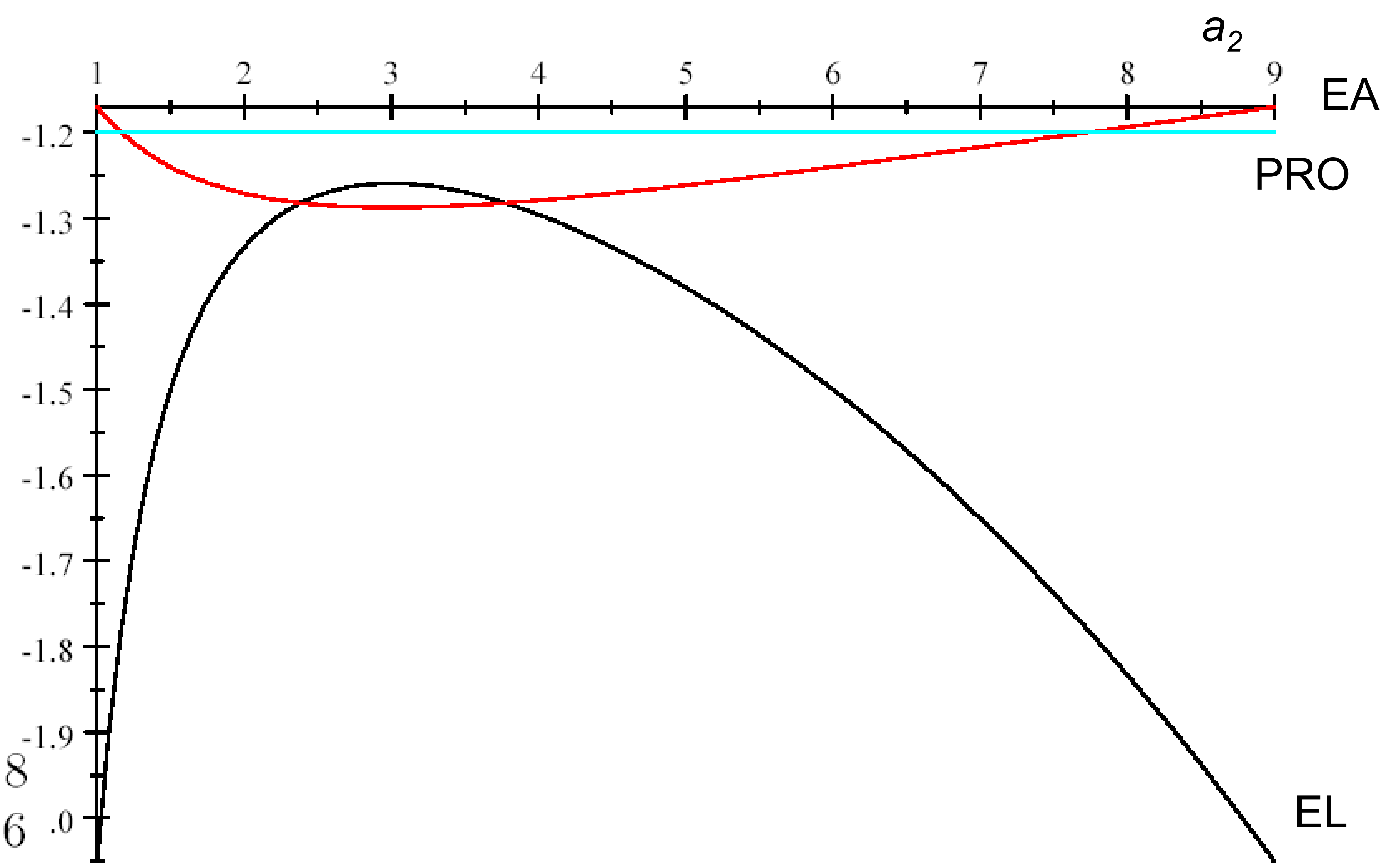
EL > EA on 27% of the parameter space

Never equal

Utilitarian Social Welfare induced by *F*

$$UT^{F}\underbrace{(p, r, \beta, a_1, a_2)}_{\text{a function of the parameters}} = \underbrace{U_1^{F}\left(\epsilon\left(G^{F}\right)\right)}_{\text{Agent 1's equilibrium utility}} + \underbrace{U_2^{F}\left(\epsilon\left(G^{F}\right)\right)}_{\text{Agent 2's equilibrium utility}}$$

Utilitarian social welfare for our example



$p = 0.8$
$r = 0.6$
$\beta = 0.7$
$a_1 = 3$

THEOREM

Assume parameter values are such that
there is an
interior equilibrium under all three rules

Then in terms of utilitarian social welfare

PRO > EL

Utilitarian Social Welfare
PRO vs EA

Proposition:

Assume agents equally risk averse

Then

in terms of utilitarian social welfare

PRO > EA

Utilitarian Social Welfare PRO vs EA

Numerical comparison of interior equilibria

2.7 million parameter combinations

PRO > EA on 61% of the parameter space

EA > PRO on 39% of the parameter space

Never equal

Utilitarian Social Welfare
EA vs EL

Numerical comparison of interior equilibria

1.3 million parameter combinations

EA > EL on 66% of the parameter space

EL > EA on 34% of the parameter space

Never equal

SUMMARY

In terms of total investment

EL > PRO > EA

In terms of egalitarian social welfare

PRO > EL and EA

In terms of utilitarian social welfare

PRO > EL

SUMMARY

1. Switching from PRO to EL

   increases total investment

   but

   decreases social welfare

2. Switching from PRO to EA

   decreases total investment

   decreases egalitarian social welfare

   might increase utilitarian social welfare

THANK YOU!