# GAME THEORY

INTRODUCTION

# Game (dictionary definition)

- A competitive activity in which players contend (i.e. compete) with each other according to a set of rules

Ex: chess, football, tic-tac-toe

# Our usage of the term is more general

A game is any situation where

(i) there are decision-makers (agents, players)

(ii) they interact

That means, an agent's welfare depends on others' actions as well as her own

# Some applications

- Firms competing for business
- Political candidates competing for votes
- Jury members deciding on a verdict
- Animals fighting over prey
- Bidders competing in an auction
- Evolution of siblings' behavior towards each other
- Competing experts' incentives to correctly diagnose a problem
- Legislators' voting behavior under pressure from interest groups
- The role of threats and punishments in long-term relationships

# Game theory consists of a collection of models

Model: an abstraction that we use to understand our observations and experiences

1. Perceiving relationships
2. Isolating principles that apply to a range of problems

Example: Modeling the flight of a tennis ball in Newtonian physics

the relationship between the ball's trajectory and gravity

same model applies to footballs, stones, guns, etc.

# What is a good model?

Constructing a model involves simplifying reality through assumptions

Good model / good assumptions:

- simplicity

Example: We assume that the earth's force of gravity is independent of the ball's distance from the earth.

- capturing the essence of the situation you want to analyze, not irrelevant details

Example: We abstract away the color of the ball, whether it is a tennis ball or a football, etc.

# Testing a Model

- Run experiments to check if the model's conclusions fit the reality

- This is THE check for a model's success. You could have a model whose assumptions do not fit the reality but whose conclusions do.

  Ex: Modeling animal behavior with the theory of rational choice

# The language of game theory

Game theoretic models are precise expressions of ideas that can be presented verbally.

Verbal descriptions tend to be long and imprecise.

We thus use the language of mathematics

# Mathematics

- A great language and like any language may be used to describe emotions as well as ideas - truths and philosophies - facts and fiction.
  www.allnewuniverse.com/glossary.html
- Richard Feynman (Nobel Prize in Physics, 1965): Mathematics is not just another language. Mathematics is a language plus reasoning; it is like a language plus logic. ··· By mathematics it is possible to connect one statement to another.
  core.ecu.edu/phys/flurchickk/Classes/PH4226/Introduction/Lecture0-2.html
- Mathematics is the investigation of axiomatically defined abstract structures using symbolic logic and mathematical notation. It is commonly defined as the study of patterns of structure, change, and space. Because it is not empirical, it is not a science.
  en.wikipedia.org/wiki/Mathematics
- Mathematics uses very specific, rigorous methods of proof that philosophers sometimes (only rarely) try to emulate. Most philosophy is written in ordinary prose, and while it strives to be precise it does not usually attain anything like mathematical clarity. As a result, mathematicians hardly ever disagree about results, while philosophers of course do disagree about their results, as well as their methods.
  www.kidsseek.com/encyclopedia-wiki/ph/Philosophy
- The origin of the word mathematics is in the Greek word manthanein, to learn. The meaning is preserved today in the word polymath, for a person of great or varied learning. The word is seldom used, but that could be because there are so few people to whom it actually applies.
  www.pballew.net/arithme2.html

# CONSTRUCTING A GAME-THEORETIC MODEL

**1.** How to model an agent making decisions?

# The theory of rational choice

- A component of many models in game theory

- Requires that each agent chooses the best action according to her preferences among all the actions available to her

# The theory of rational choice

- $X$ : a universal set of choices

  Ex: $X = \{ GS, FB, BJK \}$

- $c$ : a choice function

  This is what we observe in experiments

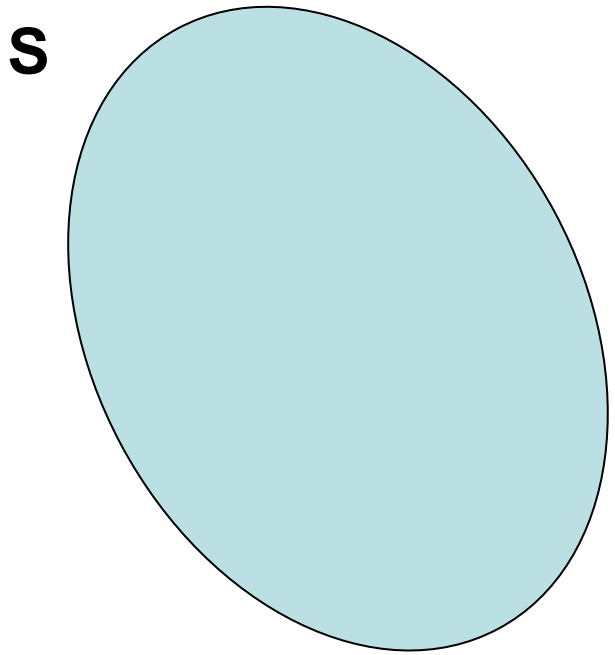
  *Ex:* $c(\{GS,FB\})=\{FB\}$

- Rationality = Consistent choices
  - Sen's property $\alpha$
  - Sen's property $\beta$

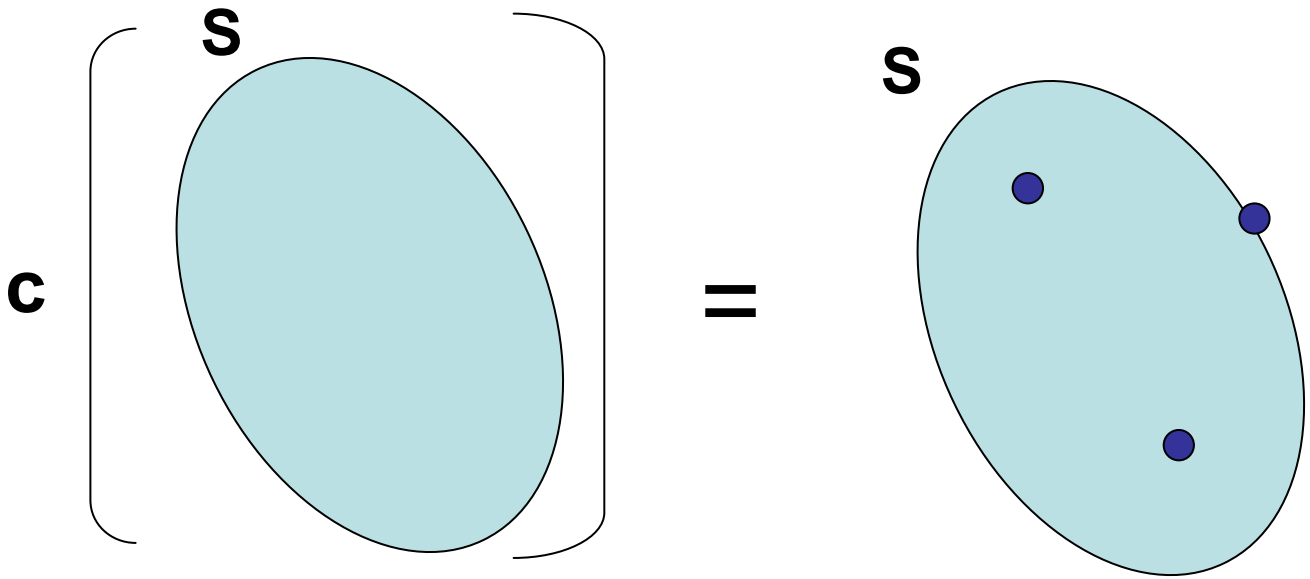
Amartya Sen

Nobel Prize in Economics (1998)

# Choice Problem:

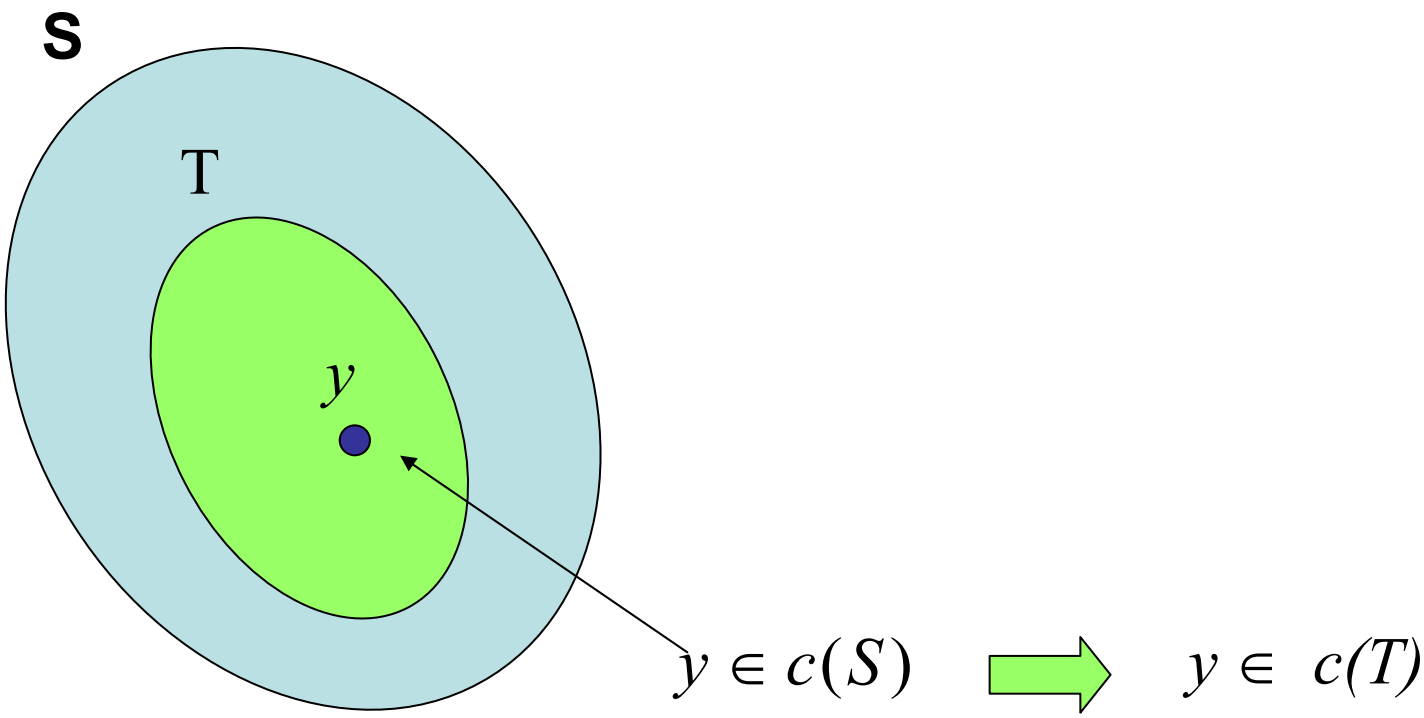


# Choice Correspondence c :



# Properties of a choice correspondence:

- **Property** $\alpha$ **:**



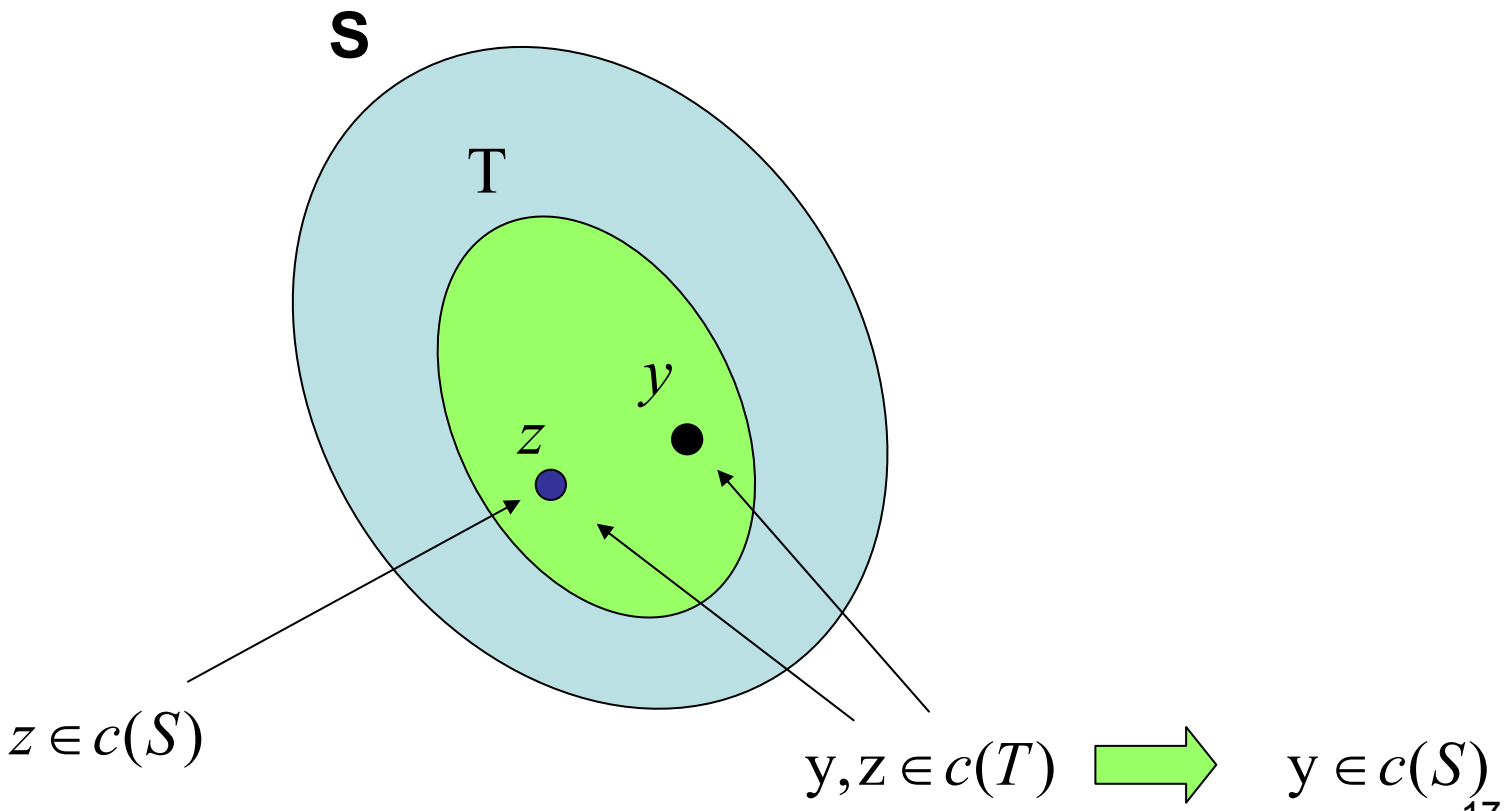
# Property $\alpha$ :

Sen's Property $\alpha$: for all $x \in X$, $S \subseteq X$, and $T \subseteq X$,

if $x \in T$, $T \subseteq S$, and $x \in c(S)$, then $x \in c(T)$.

# Properties of a choice correspondence:

- **Property** $\beta$ **:**



# Property $\beta$ :

**Sen's Property** $\beta$ : For all $x \in X, y \in X, S \subseteq X$ and $T \subseteq X$,

if $x \in T, y \in T, T \subseteq S,\ x, y \in c(T)$, and $y \in c(S)$, then $x \in c(S)$.

# Modeling choices: preferences

*R* : a preference relation on *X*

a ranking of alternatives in *X*

*Ex.* X = { GS , FB , BJK }

| ***R*** | ***or*** | ***R'*** |
|---|---|---|
| GS | | BJK GS |
| FB | | FB |
| BJK | | |

equivalently GS ***R*** FB ***R*** BJK or {BJK , GS} ***R'*** FB

# Very nice theorem

Theorem: If *c* satisfies Sen's properties $\alpha$ and $\beta$ then there is a preference relation *R* such that for each *S*, *c(S)* are the highest ranked alternatives in *S* according to *R*

*i.e.* the agent always chooses the maximizers of *R*

*i.e.* the agent is rational

# Representing preferences with payoff functions (a.k.a. utility functions)

- $u : X \rightarrow \mathbf{R}$ is a payoff function
  $u(x)$ is the payoff from $x$

- u represents R if for each x,y in X,
  $xRy$ if and only if $u(x) > u(y)$

- Rationality: for each S,
  $c(S) = argmax_{x \text{ in } S}\, u(x)$

# Ordinality of payoff functions

- The number *u(x)* is not meaningful in itself
- Only the ordering (i.e. the ordinality) of the numbers is important

# Rationality vs. Preferences

- Rationality puts no restriction on preferences, it only requires some consistency in choices

- E.g. altruistic preferences are perfectly consistent with rationality)

  (Osborne, pg. 5) Person 1 cares about both her income and Person 2's income. Precisely, the value she attaches to each unit of her own income is the same as the value she attaches to any two units of Person 2's income.

# Examples

## 1. Government's choices

X = { Military attack , Political campaign , No Action }

## 2. Major choices

X = { Econ , SPS , Cult }

# CONSTRUCTING A GAME-THEORETIC MODEL

**2.** How to model the interaction between the agents?

COMING SOON TO A CLASS NEAR YOU

# To specify a game you need to define:

1. The list of players

**Strategy profile:** a list that contains exactly one strategy for each player.

2. The possible strategies of each player

3. How each player ranks strategy profiles

   To define this, you first need to define

   a. A rule that maps strategy profiles to outcomes of the game

   b. How each player ranks the outcomes

# Examples

1. Drivers meeting at a crossroads

2. Firms competing in an auction (TÜPRAŞ ihalesi için teklif veren firma ve konsorsiyumlar, TÜPRAŞ Acquisition Cons.-Petrol Ofisi, PKN ORLEN SA (Polonya)-Zorlu Holding, Koç Holding-Aygaz-Opet-Shell, Indian Oil Corp. (Hindistan)-Çalık Enerji, OYAK, OMV Aktiengesellschaft (Avusturya), ENI Refining and Market Division (İtalya), Anadolu Taşıma, MOL (Macaristan) olarak belirlendi.

3. Chess (how to define a strategy?)

# The Tüpraş Auction

| Not: | %51 hisse için teklif | Teklif veren grup | Elenen/ çekilen |
|---|---|---|---|
| First price sealed bid auction with no elimination | 2,780 | | |
| First price sealed bid auction with elimination | 3,025 | | Zorlu elendi |
| First price sealed bid auction with elimination | 3,225 | | ENI elendi |
| Starting bid | 3,360 | | MOL elendi |
| The auction: phase II | 3,380 | OMV | |
| | 3,400 | OYAK | |
| | 3,420 | ÇALIK | |
| | 3,440 | ANADOLU | |
| | 3,460 | KOÇ | |
| | 3,480 | POAŞ | |
| | 3,500 | OMV | |
| | 3,520 | OYAK | |
| | 3,540 | ÇALIK | |
| | 3,560 | ANADOLU | |
| | 3,580 | KOÇ | |
| | 3,640 | POAŞ | OMV çekildi |
| | 3,660 | OYAK | |
| | 3,680 | ÇALIK | Anadolu çekildi |
| | 3,700 | KOÇ | |
| | 3,720 | POAŞ | |

# The Tüpraş Auction

| %51 hisse için teklif | Teklif veren grup | Elenen/ çekilen |
|---|---|---|
| 3,740 | ÇALIK | |
| 3,760 | ÇALIK | |
| 3,780 | KOÇ | |
| 3,800 | OYAK | POAŞ çekildi |
| 3,820 | ÇALIK | |
| 3,860 | KOÇ | |
| 3,880 | OYAK | |
| 3,900 | ÇALIK | |
| 3,920 | KOÇ | |
| 3,940 | OYAK | |
| 3,960 | ÇALIK | |
| 3,980 | KOÇ | |
| 4,000 | OYAK | |
| 4,020 | ÇALIK | |
| 4,060 | KOÇ | |
| 4,080 | ÇALIK | OYAK çekildi |
| 4,100 | KOÇ | |
| 4,120 | ÇALIK | |
| 4,140 | KOÇ | |

# CLASSIFYING GAMES

- Are agreements to cooperate enforceable?
- Are the moves sequential or simultaneous?
- Are the players' interests in total conflict or is there some commonality?
- Is the game played once or repeatedly? If second, is it always played with the same opponent?
- Do the players have full or equal information?

# WE START WITH GAMES WHERE

- Agreements are not enforceable
- Moves are simultaneous
- The game is played once.
- Players both have full information.