POSITIVE POLITICAL THEORY

- SOME IMPORTANT THEOREMS

- GAME THEORY IN POLITICAL SCIENCE

Mirror mirror on the wall

which is the fairest of them all ?????

Galatasaray   Fenerbahce   Besiktas

Turkcell   Telsim   Aria

DSP   DP   CHP   DTP   AKP   MHP   XYZ

Strategy and Voting

• implications of strategic behavior on voting situations

• different voting procedures:

   different outcomes

    which voting procedure to choose

• how to manipulate the outcome through strategic voting

Model

A: a set of alternatives that you have to choose from

Ex: political parties

candidates for a committee

social projects (where to spend the tax money?)

N: a set of voters

each voter has an individual **ranking** of the alternatives

i.e. first best, second best, third best, etc.

Denoted by a binary relation, P

a P b means a is ranked <u>higher</u> than b

P is **transitive**: if a P b and b P c,

then a P c

How will the people in N choose from the alternatives in A?

They vote.   (But how?)

Voting rules and procedures

When there are two alternatives

Majority rule:

     the alternative with the majority of votes (i.e. > 50%) wins

     Ex: vote between Fenerbahce and Besiktas

When there are more alternatives

A. Binary methods (pairwise voting):

majority voting between pairs of alternatives in a given order

1. Condorcet method   (Jean Antoine Nicholas Caritat)

Condorcet winner : beats everything else in majority voting

2. Amendment procedure (when there is a status-quo alternative)

First, vote between a and b   (a, b two new proposals)

then, vote between the winner and c   (c   status-quo)

B. Plurative methods:

Voting on all the alternatives at once !

1. Plurality rule

The alternative with the most number of votes wins

Ex: voting between Gsaray, Fbahce, and Besiktas

2. Borda count

Each agent ranks alternatives   Ex: a P b P c

Points assigned   a gets 3, b gets 2, c gets 1

Add up points, highest wins

Ex: Eurovision song contest (not exactly?),

biri bizi gözetliyor

B. Plurative methods:

Voting on all the alternatives at once !

3. Approval voting

Each voter chooses the <u>alternative**s**</u> that she approves

The alternative with the highest approval votes wins or

can choose a set by setting a threshold

Ex: Gsaray, Fbahce, Besiktas (which ones do you approve?)

C. Mixed Methods:

Mixtures of the previous two types!

1. Majority runoff

Each voter chooses one alternative that she wants chosen

If an alternative is the majority winner, it wins

otherwise, majority voting between the first and the second.

2. Voting in rounds

Use a single vote or a ranking (e.g. Borda) in each round

At the end of each round, eliminate the worst-performing alt.

C. Mixed Methods:

Mixtures of the previous two types!

3. Proportional representation

When <u>choosing a set</u> of alternatives (e.g. senators)

The chosen set must mirror the voters' votes

Ex: If votes are 40% AKP, 35% CHP, 25% DP

the parliament is 40% AKP, 35% CHP, 25% DP

4. Single transferable vote (Hare procedure)

Voters declare ranking and vote for the highest ranked alt.

Bottom alternatives eliminated: their votes are transferred

Can choose any one of these rules for your society

The outcome will depend on the voting procedure used

Can choose one strategically

Also: can manipulate each

Voting Paradoxes

Some voting procedures lead to curious outcomes

Condorcet Paradox: (with majority voting)

What is the social ranking between alternatives **G, A**, and **L**?

LEFT	CENTER	RIGHT
Generous	Average	Limited
Average	Limited	Generous
Limited	Generous	Average

G beats **A** beats **L** beats **G**   (with majority voting)

An **intransitive ranking**   (each voter has transitive ranking)

Who is the winner?

FIGURE 14.1 Councillor Preferences Over Welfare Policies

Reversal Paradox (with the Borda rule):

Sportswriters trying choose among Ibrahim Kutluay, Mirsad Turkcan, Hidayet Turkoglu, and Kerem Tunceri

1	2	3	4	5	6	7	
MT	IK	MT	HT	IK	IK	HT	4
HT	MT	HT	KT	MT	MT	KT	3
KT	HT	KT	IK	HT	HT	IK	2
IK	KT	IK	MT	KT	KT	MT	1

Apply the Borda rule

FIGURE 14.2 Sportswriter Preferences for Award Candidates

Hidayet gets 20 points   (he wins the award)

Ibrahim gets 19 points

Mirsad gets 19 points

Kerem gets 13 points

They discover Kerem can not be a candidate because …?

Should this effect who wins the award?

1	2	3	4	5	6	7	
MT	IK	MT	HT	IK	IK	HT	**3**
HT	MT	HT	IK	MT	MT	IK	**2**
IK	HT	IK	MT	HT	HT	MT	**1**

Ibrahim: 15 points   (the new winner)

Mirsad: 14 points

Hidayet: 13 points

FIGURE 14.3 Sportswriter Preferences Over a Narrowed Field

Agenda paradox (binary voting procedures):

The **chair** decides the order of voting (i.e. **sets the agenda**)

    she can get any outcome she wants

LEFT	CENTER	RIGHT
Generous	Average	Limited
Average	Limited	Generous
Limited	Generous	Average

G beats **A** beats **L** beats **G**   (with majority voting)

Ex: (chair LEFT) L and A => A   G and A => G

The real game is setting the agenda (or choosing the chair)

FIGURE 14.1 Councillor Preferences Over Welfare Policies

Change the voting method, change the outcome:

strategically choosing the voting method

Ex: 100 voters, 40 voters **A** P **B** P **C**

25 voters **B** P **C** P **A**

35 voters **C** P **B** P **A**

Plurality rule :

A wins

Borda rule :

B wins (225 points)   (C 195 points, A 180 points)

Majority runoff:

C wins (A and C move to second round)

Evaluating vote aggregation methods

Preference aggregation method: individual rankings =>social ranking

Arrow's theorem:

If a preference aggregation method satisfies these:

1. All alternatives must be ranked: complete

2. The ranking must be transitive: transitive

3. If everybody ranks **a** higher than **b**, social ranking does the same:
                    Pareto condition

4. Social ranking of **a** and **b** doesn't depend on how people rank other alternatives: independence of irrelevant alternatives

Then it is dictatorial !!!

Very strong result,   very famous,   Arrow's Ph.D. thesis

Ex: Borda violates independence of irrelevant alternatives

Other criteria:

Condorcet: if there is a Condorcet winner, it should be selected

Non-manipulability: by lying about your ranking, you can't get an alternative you like more to be chosen

Gibbard-Satterthwaite Theorem:

> All nondictatorial voting methods are **manipulable**

What happens when people manipulate the voting outcome?

Strategic Voting

Games in which   people lie about their rankings

    or   vote for an alternative they don't rank at top

Plurality rule:

    Two major candidates and a spoiler (divides the votes)

    say spoiler is your top choice   vote for him?

    Spoilers usually get less votes than they would under honesty

    Ex: Britain (two major parties in the parliament)

Proportional rule:

    Does not have this problem Ex: Italy

    More parties in the parliament (but smaller parties)

    Less decisive government, better for minorities

LEFT is the chair

agenda: first vote between Average and Limited

the winner (A) is voted against Generous

LEFT	CENTER	RIGHT
Generous	Average	Limited
Average	Limited	Generous
Limited	Generous	Average

What can CENTER do?

Vote for Limited (it wins)

Everybody votes strategically

we have a game

use rollback

FIGURE 14.1 Councillor Preferences Over Welfare Policies

If second-round is between A and G: truthful voting

(a) A versus G election

Right votes

for A:

		CENTER	
		A	G
LEFT	A	A	A
	G	A	G

for G:

		CENTER	
		A	G
LEFT	A	A	G
	G	G	G

FIGURE 14.4 A Election Outcomes in Two Possible Second-Round Votes

If second-round is between L and G: truthful voting

(b) L versus G election

Right votes

for L:

LEFT		CENTER	
		L	G
	L	L	L
	G	L	G

for G:

LEFT		CENTER	
		L	G
	L	L	G
	G	G	G

FIGURE 14.4 B Election Outcomes in Two Possible Second-Round Votes

The first-round: strategic voting

Right votes

for A:

LEFT		CENTER	
		A	L
	A	G	G
	L	G	L

for L:

LEFT		CENTER	
		A	L
	A	G	L
	L	L	L

FIGURE 14.5 Election Outcomes Based on First-Round Votes

Borda rule:

>   how can you manipulate it

>   rank the most powerful adversary to your top choice as last

>   everybody does the same: prisoners' dilemma

What about games in which the candidates act strategically?

Each candidate's payoff is the number of votes she gets.

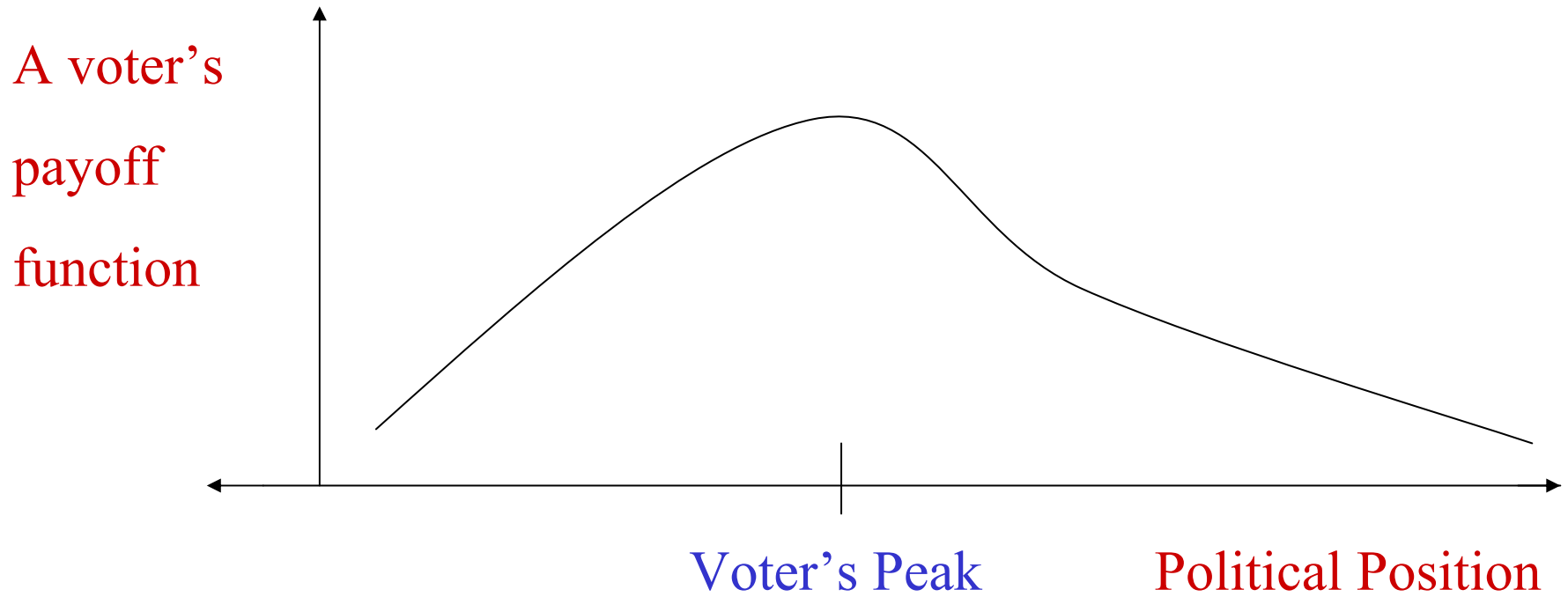Ex: Politicians strategically choosing their political position

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Median voter theorem

One dimensional policy space

   Ex: from left to right or government's budget for education

    extreme left   center   extreme right

Each voter has **single-peaked preferences**

A voter's payoff function

Voter's Peak   Political Position

The game: 1. 2 candidates simultaneously choose their policies

2. Voters vote (majority voting)

NOTE: with 2 candidates, voting honestly is the best

NOTE: the voters' top choices are distributed on the policy space

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     |        |              |              |
─────┼────────┼──────────────┼──────────────┼─────
  1 voter   1 voter      2  voters        1 voter
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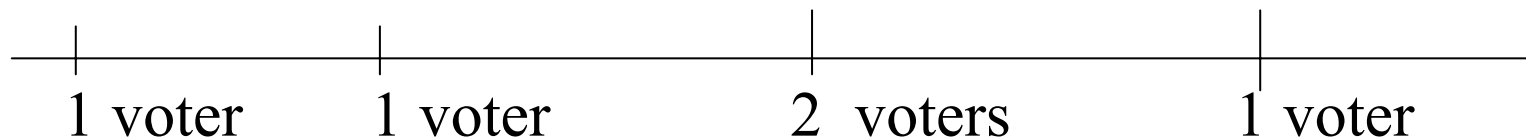
Median: the midpoint(s) of a distribution

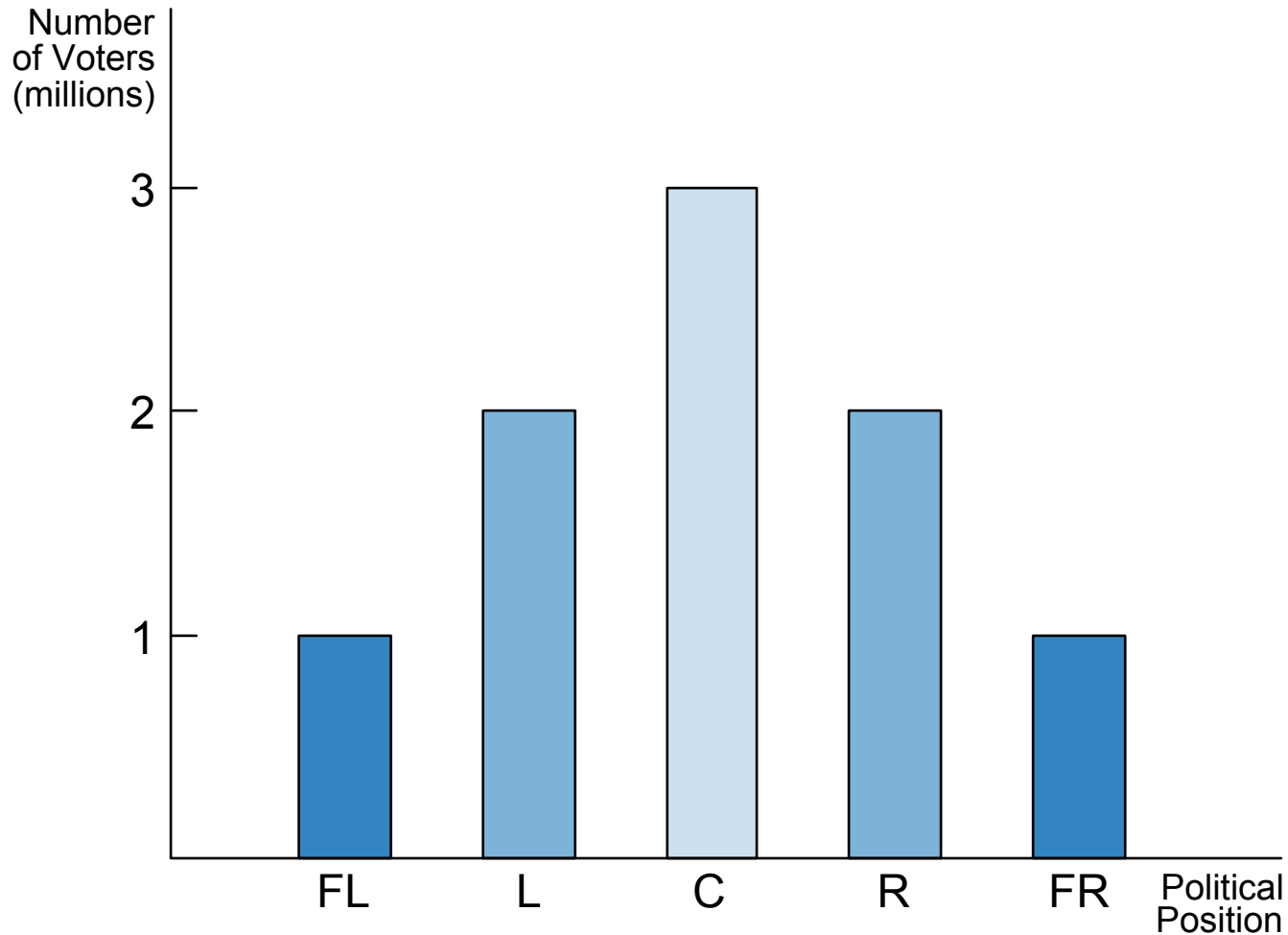min. 50% of the points to the left and min 50% of the points to the right

Median voter: the voter whose top choice is the

median of the distribution of the top choices

Median voter theorem:

Both candidates will place themselves on

the top choice of the median voter

Discrete political spectrum (9 million voters)

FIGURE 14.6 Discrete Distribution of Voters

	EX-ACTOR				
	FL	L	C	R	FR
EX-GOVERNOR FL	4.5, 4.5	1, 8	2, 7	3, 6	4.5, 4.5
L	8, 1	4.5, 4.5	3, 6	4.5, 4.5	6, 3
C	7, 2	6, 3	4.5, 4.5	6, 3	7, 2
R	6, 3	4.5, 4.5	3, 6	4.5, 4.5	8, 1
FR	4.5, 4.5	3, 6	2, 7	1, 8	4.5, 4.5

FIGURE 14.7 Election Results: Symmetric Voter Distribution

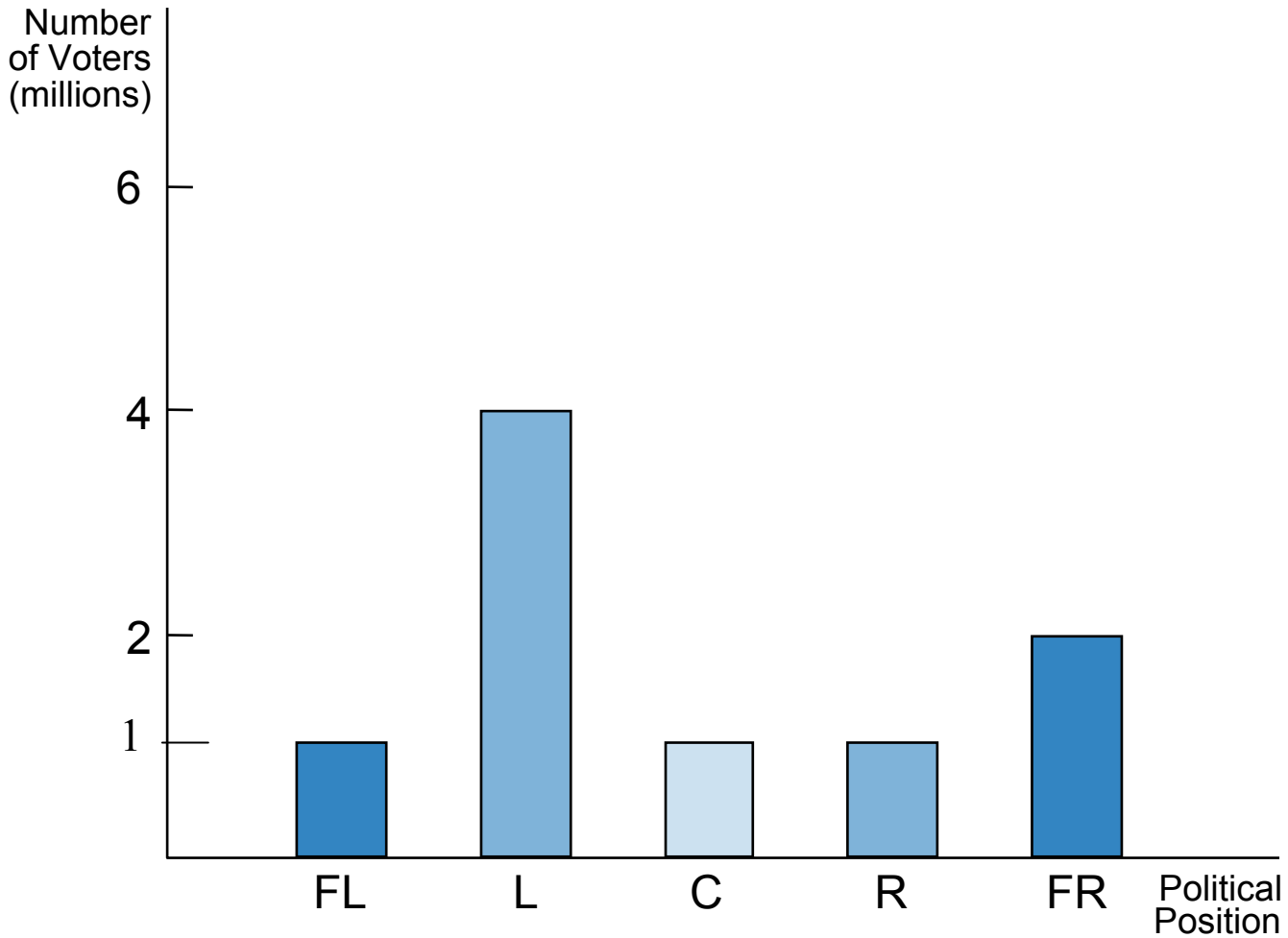# Discrete political spectrum with asymmetric dist.   (9 million voters)

FIGURE 14.6 Discrete Distribution of Voters

		EX-ACTOR				
		FL	L	C	R	FR
EX-GOVERNOR	FL	4.5, 4.5	1, 8	3, 6	5, 4	5.5, 3.5
	L	8, 1	4.5, 4.5	5, 4	5.5, 3.5	6, 3
	C	6, 3	4, 5	4.5, 4.5	6, 3	6.5, 2.5
	R	4, 5	3.5, 5.5	3, 6	4.5, 4.5	7, 2
	FR	3.5, 5.5	3, 6	2.5, 6.5	2, 7	4.5, 4.5

FIGURE 14.8 Election Results: Asymmetric Voter Distribution

Same conclusion with a continuous distribution of voters

histogram ⟶ distribution function

Ex: uniform distribution

        normal distribution

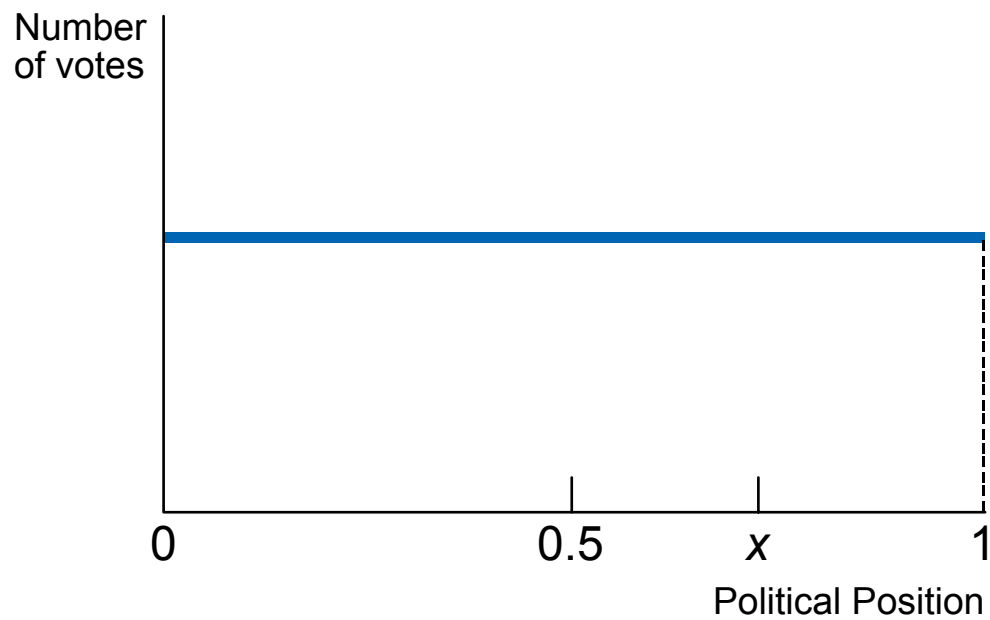The value of the function at a given policy:

        the number of people who ranks that policy first
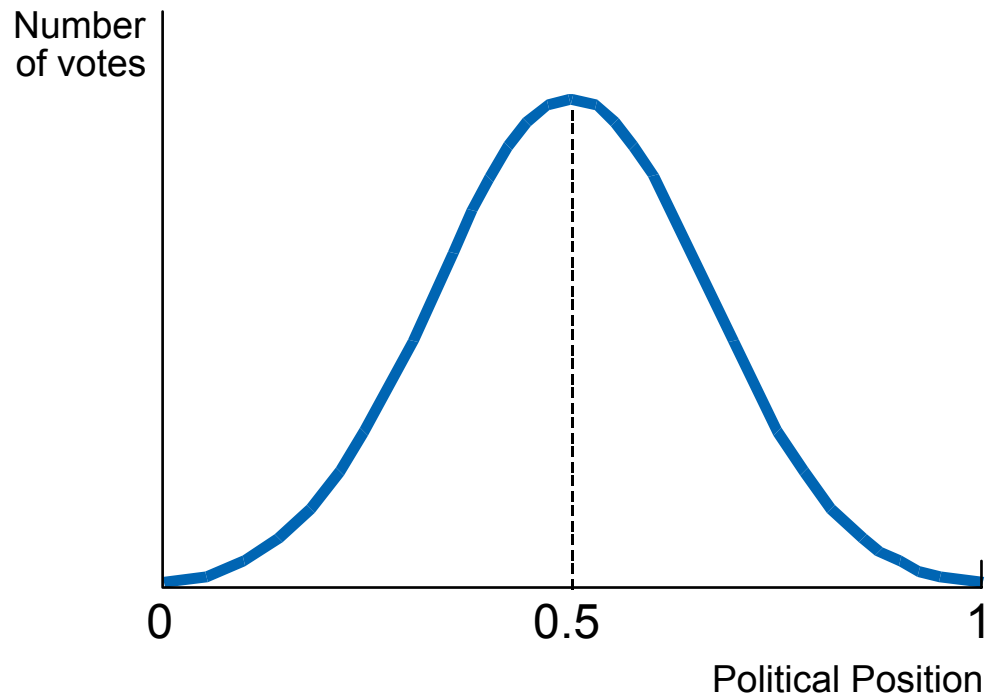
        i.e. their peaks are at that policy

No payoff table

        solve it on the graph

(a) Uniform distribution

Number
of votes

0   0.5 x 1

Political Position

(b) Normal distribution

FIGURE 14.9 B Continuous Voter Distributions

A	B	C
Philosophy	Anthropology	Geology
Geology	Philosophy	Anthropology
Anthropology	Geology	Philosophy

1	2	3	4
A	A	B	C
B	B	C	B
C	C	A	A

RANKING	GROUPS (AND THEIR SIZES)					
	I (18)	II (12)	III (10)	IV (9)	V (4)	VI (2)
1	T	C	B	K	H	H
2	K	H	C	B	C	B
3	H	K	H	H	K	K
4	B	B	K	C	B	C
5	C	T	T	T	T	T