A Mechanism Design Approach to Allocating Central Government Funds Among Regional Development Agencies*

Özgür Kıbrıs† and İpek Gürsel Tapkı‡

February 1, 2013

*We would like to thank Kevin Hasker, Onur Kesten, Arzu Kıbrıs, Levent Koçkesen, and William Thomson as well as seminar participants at University of Rochester, Boston College, Duke University, Boğaziçi University, Bilkent University, Kadir Has University, and GAMES 2012 for comments and suggestions.

†Corresponding author: Faculty of Arts and Social Sciences, Sabanci University, 34956, Istanbul, Turkey. E-mail: ozgur@sabanciuniv.edu Tel: +90-216-483-9267 Fax: +90-216-483-9250

‡Faculty of Economics and Administrative Sciences, Kadir Has University, Cibali, İstanbul, 34083 Turkey. E-mail: ipek.tapki@khas.edu.tr. Tel: +90-212-533-6532.

Abstract. To allocate central government funds among *regional development agencies*, we look for mechanisms that satisfy three important criteria: *efficiency*, (individual and coalitional) *strategy proofness* (*a.k.a.* dominant strategy incentive compatibility), and *fairness*. We show that only a *uniform mechanism* satisfies all three. We also show that all efficient and strategy proof mechanisms *must* function by assigning budget sets to the agencies and letting them freely choose their optimal bundle. In choosing these budget sets, the agencies' private information has to be taken into account in a particular way. The only way to additionally satisfy a weak fairness requirement (regions with identical preferences should be treated equally) is by assigning all agencies the same budget set, as does the *uniform mechanism*. Finally and maybe more importantly, we show that the central government should not impose constraints on how much to fund an activity (e.g. by reserving some funds only for a particular activity): otherwise, there are no *efficient*, *strategy proof* and *fair* mechanisms, no matter how small these constraints are. Our model is an application of a production economy with a linear technology and generalized single peaked preferences. All our results are true for other possible applications as well as a simpler model which allows free disposal.

JEL Classification numbers: D02, D04, D60, O2, R58

Keywords. regional development agencies, single peakedness, efficiency, strategy proofness, coalitional strategy proofness, equal treatment of equals, no envy, production, uniform mechanism.

1 Introduction

In the last century, regional development agencies (hereafter, RDAs) have played an important role in implementing regional development policies. An RDA can be defined as a regionally based, publicly financed institution outside the mainstream of central and local government administration, designed to promote economic development (Halkier and Danson, 1996).[1] Earliest examples date back to 1930s, like the Tennessee Valley Authority in US which was founded to provide flood control, electricity generation, fertilizer manufacturing, and economic development in the Tennessee Valley, a region particularly affected by the Great Depression. Today, RDAs exist in many countries. They have a wide range of functions and responsibilities which vary from country to country but typically include activities such as providing (business, legal, technical) consulting services, carrying out vocational training programs, financing research and development projects or entrepreneurial activities, and promoting the region to outside investors.

To achieve their objectives, RDAs typically use funds allocated to them every year by central governments or international organizations. The total amount of resources allocated among RDAs is quite large. For example, in the 2000-2006 fiscal framework, EU allocated 213 billion Euros (around one-third of the EU budget) among its 271 regions (Funck, Pizzati and Bruncko, 2003). Between 1999 and 2007, England allocated 15.1 billion Pounds among its 9 regions (Daily Telegraph, 2009).[2] Turkey, though its 26 RDAs were created only in 2006 as part of its EU accession process, projects to allocate around 1 billion TL per year among them (Baş Uçar, 2011).

Given the magnitude of funds involved and their potential effects on the development of different regions, it is essential to use a good mechanism to allocate central government funds among RDAs. The existing literature on regional development, discussed at the end of this section, has however remained silent about this issue. In practice, the methods and procedures used to allocate these funds can be quite complicated, their design might seem rather arbitrary, and the criteria on which the allocation is based can be quite vague.

[1] RDAs show a high degree of diversification in remits, organization, finance and activities. However, they typically operate at arm's length from sponsoring authorities, where the latter only interfere at the level of overall resources allocation and broad policy guidelines. (Halkier *et al.,* 1998; Hughes, 1998).

[2] The money is well-spent. In 2009, a study by PriceWaterhouseCoopers stated that when all their job creation, protection and infrastructure projects mature, RDAs will generate £4.50 for regional economies for every £1 of public spending (Department for Business, Enterprise and Regulatory Reform, 2009). It is also interesting to note that the UK government announced plans to abolish RDAs by 2012, with a view to future economic development being undertaken by "local enterprise partnerships", which will not receive funding from central government.

For example, Turkey allocates funds among its RDAs as follows.[3] First, a "central planning committee" made up of the prime minister and several ministers determines a budget cap for each RDA, based on data about each region's population and development levels as well as the RDAs' past performance measures. Next, the general secretary of each RDA is informed of this budget cap and is asked to prepare a "feasible" plan that determines how much will be spent on different activities. The submitted plans are evaluated by the ministry of development which frequently demands the RDAs to revise their regional plans according to the ministry's countrywide objectives.[4] Once the plans are accepted by the ministry, the funds are released for the coming fiscal year.

The existing literature on regional development does not provide guidelines to evaluate such procedural designs. Nor does it discuss properties of a good mechanism. In **this paper**, we try to fill this gap. We argue that, in case of the RDA allocation problem, there are three central issues: **strategic considerations**, **efficiency**, and **fairness**. In the form of essential properties every good mechanism should satisfy, all three have played an important role in welfare economics, in public economics (particularly, in the fiscal federalism literature) and in real-life applications of mechanism-design, each cited at the end of this section. In this paper, using a theoretical model of resource allocation among RDAs, we discuss the design of mechanisms that satisfy these properties.

Strategic considerations exist due to asymmetric information and a misalignment of incentives between RDAs and the central government. Each RDA, both by location and organization, is in direct contact with the region it serves. A significant fraction of its board members, and in some cases its general secretary, come from the representatives of the region. As a result, the RDA is much better informed about the needs and preferences of its region than the central government. For similar reasons, an RDA is most likely to give first priority to the development of its own region. This is a source of tension between the RDA and the funding central government institution which, possibly, has different policy preferences than the agency.[5] Asymmetric information, coupled with

[3] More details (in Turkish) can be found at "Kalkınma Ajansları Bütçe ve Muhasebe Yönetmeliği", *Resmi Gazete*, September 28, 2006.

[4] Until very recently, the ministry of development was called the State Planning Organization (DPT). Since the 1960s until recently, DPT has been solely responsible for making and implementing regional development plans in Turkey. Such plans were generally latched on sectoral plans serving the priority goal of national industrialization (Loewendahl-Ertugal, 2005). The ministry of development seems to be reluctant to give up this central planning tradition. Its meddling in the RDAs' regional development plans is currently a topic of controversy in Turkey (Filiztekin *et al,* 2011).

[5] For a real-life example, see Footnote 4. A similar point is argued regarding local and central governments in the

this conflict of interest gives RDAs reason to be strategic when informing the central government about their needs and preferences. Due to these reasons, it is important to design a mechanism that guarantees truthful revelation of the RDAs' private information about the regions. In mechanism design, this objective is embodied in a well-known property called **strategy proofness** (also called *dominant strategy incentive compatibility*) which roughly means that individual agencies can not benefit from "gaming" the mechanism. We will also consider a stronger property, *coalitional strategy proofness,* which additionally prevents coalitions of agencies to manipulate the mechanism in a coordinated manner.

Another important concern in allocating funds is *efficiency* (*a.k.a. Pareto optimality*). This central notion requires that no alternative allocation of resources makes some regions better-off without making any region worse-off. Given that there are a multitude of activities for which the funds can be used, two aspects of the RDA allocation problem complicate the design of an efficient mechanism. To discuss them, it is useful to think of each RDA as having a production technology that transforms resources spent on different activities to some measure of regional development such as future increases in the region's per capita GNP. Then, the first complication is that different RDAs have potentially different production functions. That is, resources spent on an activity (such as providing legal services to businesses) can potentially affect the development of different regions in different ways. Therefore, efficiency requires specifying not only the amount of funds allocated to each region, but also how much resource each activity should receive in a way that takes this information into account. The second complication is that an RDA does not necessarily have monotone preferences on its budget. As is standard in producer theory, given its production technology and the relative (intertemporal) prices, an RDA has a (possibly finite) optimal demand for resources to spend on different activities.[6] In the next section, we will use a generalized class of *single-peaked binary relations (preferences)* to model these features of agencies. Efficiency then requires this "preference" information to be taken into account (while strategy proofness ensures its truthful revelation).

Our third criterion, *fairness* has always been a central concern in regional development as well as in development economics. Aside from its philosophical appeal, by reducing disparities among

fiscal federalism literature (*e.g.* see Oates, 1999, 2005).

[6]Another possible reason for finite resource demand is the fact that the central government evaluates the RDA's performance by comparing the inputs it uses to the services it produces. Therefore, it is suboptimal for an RDA to demand an inefficiently high level of resources.

regions fairness ensures economic and social stability.[7] Thus, "reducing disparities between the regions' levels of development" is stated as one of the main objectives of regional development policies of many countries such as UK (DTI, 2006) or the EU (Article 158 of the EU Treaty). In this paper, we analyze two formulations of fairness that have been central in the mechanism design literature. The first one, *no-envy* (Foley, 1967) requires that every RDA weakly prefers its share to the shares of others. The second property, called *equal treatment of equals*, is significantly weaker. It applies only in the very unlikely case of two regions having identical "preferences" and requires for them that each should be indifferent between its share and the share of the other.

In Section 3, we propose and discuss a **uniform mechanism** which *satisfies all of the above properties*. In Section 4, we then proceed to show that it is in fact *the only mechanism to do so*. In that section, we also characterize the classes of mechanisms that satisfy (*i*) *efficiency* as well as (*ii*) *efficiency* and *strategy proofness*. We then discuss some common properties of these mechanisms and use them to evaluate real-life designs, such as the one in Turkey.

All efficient and strategy proof mechanisms (including the uniform mechanism) are based on a simple idea: the central government uses preference information to introduce budget caps to the agencies (choosing the same budget caps for all in case of the uniform mechanism) but gives the agencies full discretion in allocating their budget among activities. The idea is similar to what, in the fiscal federalism literature, Oates (1999) calls "unconditional grants" or Levaggi (2002) calls "the classical solution for fiscal federalism". Using such rules (particularly giving the agencies full reign in allocating their budgets) might seem an easy way out of our design problem. But, we would like to emphasize our finding that there is *no other way* to simultaneously satisfy efficiency and strategy proofness. Even a tiny modification of the above mechanisms is bound to create either an inefficient or a manipulable mechanism.

In Section 5, we analyze the importance for our findings of whether there are constraints on how the mechanism allocates funds among activities. This is a very important issue for regional development where it is quite common that governments or international organizations provide some funds only for certain activities (such as the "European Agricultural Guidance and Guarantee Fund" which only supports rural development measures). Surprisingly, we show that existence of

[7] Consider the following quote from the European Commission Regional Policy Institute, Inforegio webpage: "The purpose of EU regional policy is to reduce the significant economic, social and territorial disparities that still exist between Europe's regions. Leaving these disparities in place would undermine some of the cornerstones of the EU, including its large single market and its currency, the Euro."

such constraints lead to an impossibility: there are *no efficient, strategy proof and fair mechanisms* in applications where there is a lower bound on how much the mechanism can allocate to an activity, *no matter how small this lower bound is.*

The "RDA allocation model" that we analyze is an application of "production economies with a linear technology and generalized single-peaked preferences". All our results apply to this domain, and of course, to its other possible applications.

This paper contributes to the literatures on mechanism design in resource allocation, regional development, and fiscal federalism. The mechanism design approach has recently been very fruitful in many real-life resource allocation problems. Important examples include the design of FCC spectrum auctions (*e.g.* see Milgrom, 2004), the re-design of American hospital-intern market (*e.g.* see Roth, 2002), assigning students to public schools (*e.g.* see Abdulkadiroğlu and Sönmez, 2003), kidneys to patients (*e.g.* see Roth, Sönmez, Ünver, 2004), or military cadets to branches (Sönmez and Switzer, 2011). This paper, to the best of our knowledge, is the first to approach the RDA allocation problem from a mechanism-design perspective.

The theoretical literature related to our model starts with Hurwicz (1972), who shows for *pure exchange economies* with two agents and two commodities that no individually rational mechanism is both *efficient* and *strategy proof.*[8] This striking negative result is later extended in several dimensions (more agents and commodities, public goods, and production economies with strictly convex production sets), both for monotone preferences (Dasgupta, Hammond and Maskin, 1979; Hurwicz and Walker, 1990; Zhou, 1991; Schummer, 1997 and 1999; Serizawa, 1999 and 2002; Ju, 2003; Serizawa and Weymark, 2003; Leroux, 2004; Goswami, Mitra, and Sen, 2011) and its superdomain of single-peaked preferences (Amorós, 2002; Morimoto, Serizawa and Ching, 2009; Adachi, 2010; Anno and Sasaki, 2010).

The only positive result in the literature is obtained by Maniquet and Sprumont (1999) who analyze production economies with monotone preferences and a linear technology. They show that an "equal budget free choice mechanism" uniquely satisfies *efficiency*, *strategy proofness*, and anonymity. Our RDA allocation model has a significantly larger class of preferences than the Maniquet-Sprumont model. This domain extension makes strategy proofness a stronger requirement since, intuitively, more "lies" are now possible. Existence of desirable solutions to the RDA

[8] This result is closely related to the well-known Gibbard-Satterthwaite Theorem which states that a social choice function defined over an unrestricted domain with at least three alternatives is strategy-proof if and only if it is dictatorial.

problem is, therefore, not guaranteed by their study. However, our results are naturally consistent in the sense that the uniform mechanism that we propose boils down to the Maniquet-Sprumont proposal in case of monotone preferences. Additionally, even though these authors do not discuss the implications of allocation constraints, our impossibility result (Theorem 8) applies to their setting as well, since its proof only uses monotone preferences.

The uniform mechanism is also related to Sprumont (1991) and the following literature (summarized by Thomson, 2012), which shows *in case of a single activity* that, a "uniform rule" satisfies efficiency and strategy proofness as well as many other desirable properties, including no-envy, and equal treatment of equals.

Our paper also contributes to a more applied literature on RDAs and particularly, their organizational structures and responsibilities as a function of the country's political preferences or level of development (Bennett *et al.*, 2001; Syrett and Silva, 2001). This literature shows that, especially in less developed areas such as Turkey, Portugal, Hungary, and the Czech Republic, national governments retain considerable discretion and filtering capacity to determine what RDAs should (or should not) do (*e.g.* see Benneworth, 2001; Gualini, 2004; De Bruijn and Lagendijk, 2005; McMaster, 2006; Lagendijk, Kayasu and Yaşar, 2009). This is a source of tension between the agencies and the central government organizations (*e.g.* see Filiztekin *et al*, 2011). In the confines of our model, this practice seems problematic. As will be seen in our results, efficiency and strategy proofness require that the central government should not attempt to affect allocation of funds among activities, even in the form of lower bounds on money to be spent on a particular activity; instead, it should only interfere (say for equity purposes or to favor some regions) when determining the monetary budget of the agencies and then, only by introducing bounds on the agencies' budget choices.

Finally, our paper also speaks to the public finance literature on fiscal federalism (reviewed in Oates 1999 and 2005). A recent strand of this literature has utilized asymmetric information models to discuss issues such as decentralization of government (Seabright, 1996; Tommasi and Weinschelbaum, 2007), determination of taxes (Bucovetsky, Marchand, and Pestieau, 1998) and redistributive policies (Bordignon, Manasse, and Tabellini, 2001), or the use of random audit mechanisms to induce a regional government to truthfully reveal local parameters (Gilbert and Rocaboy, 2004). The regional governments in some of these models differ from RDAs in important aspects though. For example, they typically collect taxes and are governed by election-motivated politicians. A more related work to ours is Levaggi (2002) who analyzes the trade-offs a central government faces between imposing its own preferences on a regional government and optimally allocating the regional budget among services. Levaggi argues that a "double budget constraint" (where the central government limits not only the regional budget but also its allocation among activities) might be an optimal compromise and notes that it is used in some countries such as Italy. In case of multiple regions, however, we show that double budget constraints have strong negative implications, both for an efficient and a fair allocation of resources.

The rest of the paper is organized as follows. Section 2 presents the RDA allocation model. Section 3 introduces the uniform mechanism and discusses its properties. Section 4 presents the uniqueness result. Section 5 discusses the implications of constraints on how to allocate funds. Section 6 concludes and Section 7 contains all proofs.

2 The RDA Allocation Model

The RDA allocation model, as will be discussed below, is a production economy with a linear technology and generalized single peaked preferences.

The set of regional development **agencies** (RDAs) is $N = \{1, ..., n\}$ and the set of **activities** they can carry out is $L = \{1, ..., l\}$. In case of Turkey, N contains its 26 agencies and L contains a union of all the activities that the RDAs can undertake (such as providing consulting services or carrying out vocational training programs, *etc.*).

Each agency's consumption set is thus $\mathbb{R}^L_+$, with a typical element $x \in \mathbb{R}^L_+$ interpreted as a vector of monetary spendings on different activities.[9]

The central government has **funds** E that it will allocate among agencies as well as different activities. A more versatile interpretation of E, that can be applied to other allocation problems is as follows: the society (or say, the central government) is endowed with a linear production technology which it uses to produce l commodities (which correspond to the l activities in the RDA model) to be allocated among n agents. The (constant) rate of transformation between any two commodities is normalized to 1. Therefore, the maximum amount of each commodity that can be produced by the linear technology is the same, say $E \in \mathbb{R}_+$. It is also useful to think of E as the total endowment of a factor that is used to produce the l commodities.[10]

[9] Vector inequalities are defined as follows: $x \leqq y$ if and only if $x_k \leqq y_k$ for each $k \in L$; $x \leq y$ if and only if $x \leqq y$ and $x \neq y$; $x < y$ if and only if $x_k < y_k$ for each $k \in L$.

[10] In the RDA allocation model, the government technology (which transforms uncommitted funds into funds for particular activities) is linear since a dollar spent for activity A can be equivalently transformed into a dollar for

A **feasible allocation** $z = \left(z^1, ..., z^n\right)$ assigns a share $z^i = \left(z^i_1, ..., z^i_l\right) \in \mathbb{R}^L_+$ to each agency i (z^i_k being money allocated to agency i only to be used on activity k) such that $\sum\limits_N \sum\limits_L z^i_k = E$. Let Z be the set of feasible allocations.

We represent the private information held by each agency i with a binary relation R^i on the agency's input (or consumption) space $\mathbb{R}^L_+$. For lack of better terminology, we refer to R^i as the **preference** relation of agency i. Our interpretation is that each agency is endowed with a production technology that transforms money spent on different activities, $x \in \mathbb{R}^L_+$, into some measure of development for its region (see Figure 1). The binary relation R^i is defined on the inputs of this technology and xR^iy means that, given the agency's production technology and the relative intertemporal prices, the input vector x produces at least as high "profits" for agency i as does y.[11] In what follows, we will make assumptions about R^i and relate them to this interpretation.[12]

For $a, b \in \mathbb{R}_+$ such that $a \leq b$, the **choice set between a and b** is

$$Y\left([a, b]\right) = \left\{ y \in \mathbb{R}^L_+ \mid a \leqq \sum_L y_k \leqq b \right\}.$$

If $a = b$, we will write $Y(a)$ instead of $Y([a, a])$. We denote the set of **maximizers** of R^i on a set $A \subset \mathbb{R}^L_+$ as

$$m\left(R^i, A\right) = \left\{x \in A \mid \text{for each } y \in A,\ xR^iy\right\}.$$

We define the **path of $\mathbf{R^i}$** as the set of its maximizers on all choice sets:

$$\Pi\left(R^i\right) = \bigcup_{a \in [0,E]} m\left(R^i, Y(a)\right).$$

Note that $\Pi\left(R^i\right)$ need not be monotone or continuous (*e.g.* see Figure 2).

another activity B.

[11] The term "profit" here refers to the present value of future increases in the region's income minus the cost of the input vector x. Such comparisons are not uncommon in measuring RDAs' performance, as presented in Footnote 2 for UK. Given this interpretation, the level curves of R^i are then, isoprofit lines for the agency. Unlike in standard producer theory, we use "money spent on an input" instead of the "quantity of an input". Since our analysis is not concerned with changes in input prices, this is without loss of generality.

[12] Since the mechanism we design will only need ordinal information about the agencies, we use binary relations rather than functions to represent the agencies' private information. The two make a difference only for "cardinal" mechanisms that make intensity comparisons across agencies. Such information is significantly more difficult to collect in practice (whereas, for the mechanism we propose, it will be sufficient for each agency to declare a finite set of optimal choices) and it's not clear to us if its intensity comparison across regions is meaningful.

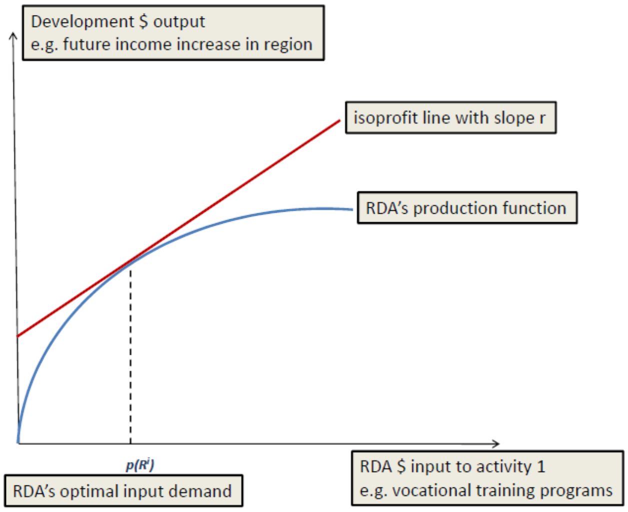

Figure 1: The production function of an agency in case of a single activity. Money spent on activity 1 is transformed into a stream of future returns through development. The interest rate r captures time value of money.

For a tractable model, we make the following *assumptions* about R^i. *First*, as is standard, we assume that R^i is complete and transitive. *Second*, we assume that for each $a \in [0, E]$, R^i has a unique maximizer on $Y(a)$, that is, $m\left(R^i, Y(a)\right)$ is a singleton (see Figure 2). *Third*, we assume that R^i is single-peaked on its path $\Pi\left(R^i\right)$, that is, there is a most preferred bundle $p\left(R^i\right) \in \Pi\left(R^i\right)$ such that for each $x, y \in \Pi\left(R^i\right)$, $\sum_L y_k > \sum_L x_k \geq \sum_L p_k\left(R^i\right)$ or $\sum_L y_k < \sum_L x_k \leq \sum_L p_k\left(R^i\right)$ implies xP^iy.[13] The bundle $p\left(R^i\right)$ is called the **peak of $\mathbf{R^i}$** and represents the optimal input bundle for agency i (see Figure 1). Let $\mathcal{R}$ be the class of all such binary relations, which we call **generalized single peaked preferences.**[14]

The first assumption is standard. The second assumption means that the agency has a unique optimal way of allocating each monetary budget $a \in [0, E]$ among its l activities. The third assumption means that the agency has single-peaked preferences on the amount of budget it receives. Specifically, it has an optimal budget which it (again, optimally) allocates as $p\left(R^i\right)$. These assumptions are satisfied by a large class of preferences, including "continuous, strictly convex preferences

[13] We would like to emphasize that this assumption is significantly weaker than requiring R_i to be single-peaked, since our assumption only requires single-peakedness on a one-dimensional subspace of $\mathbb{R}^L_+$.

[14] This class of preferences is significantly larger than the single-peaked domains analyzed by the earlier literature. On pure exchange economies, it is not possible to construct desirable mechanisms on such a large domain (Cho and Thomson, 2011).

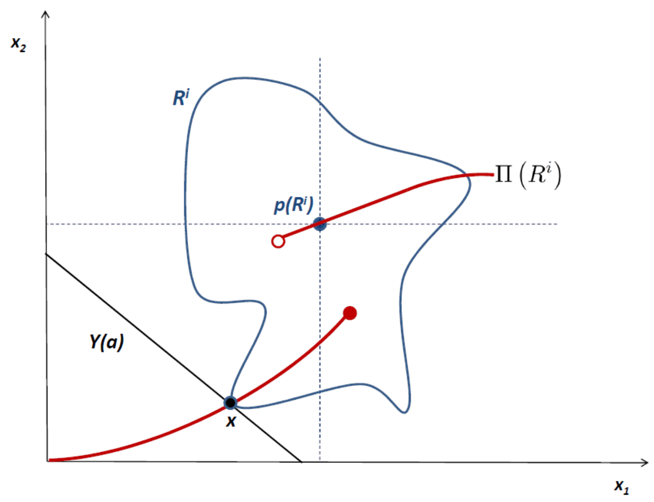

Figure 2: The preference relation R^i satisfies our assumptions. Its unique maximizer on the choice set $Y(a)$ is $x = m\left(R^i, Y(a)\right)$, and its path (drawn in red and containing its peak $p\left(R^i\right)$) is $\Pi\left(R^i\right)$.

(which contains the "classical domain" with the additional assumption of monotonicity)" as well as the standard "single-peaked preferences". Even the convexity and continuity assumptions can be dropped as long as a unique choice from each choice set is possible (as shown in Figure 2).[15]

An **RDA allocation problem** is then

1. a finite set of agencies $N = \{1, ..., n\}$,
2. a finite set of activities $L = \{1, ..., l\}$,
3. a fixed amount of government funds E, and
4. a list of agency preferences $R = \left(R^1, ..., R^n\right)$.

Throughout the paper, we will fix N, L, E and represent an RDA allocation problem with a preference profile $R \in \mathcal{R}^N$. We will say that R exhibits **excess demand** when $\sum\limits_N \sum\limits_L p_k\left(R^i\right) > E$, that R exhibits **excess supply** when $\sum\limits_N \sum\limits_L p_k\left(R^i\right) < E$, and that R exhibits **no excess** when $\sum\limits_N \sum\limits_L p_k\left(R^i\right) = E$.

Our feasibility condition in Z does not allow free disposal of funds in case of excess supply. Even though free-disposal is a natural assumption for the RDA allocation problem, it need not be so in other allocation problems, some very similar to the RDA allocation problem. For example, in case

[15] This is not much of a restriction since all choice sets have the unit normal vector.

of controversial investments with negative local externalities, such as the construction of a nuclear power plant (or a prison or a waste-management factory) which no region prefers to host, the "total demand" can fall short of the central funds to be allocated. In such cases free-disposal is not sensible since the central funds must be fully allocated for the investment to be carried out somewhere in the country. To accommodate such possible applications, and for maximum possible generality of our model, we do not allow free disposal. However, as will be discussed in the conclusion, *all our results trivially apply to a simpler model which allows free disposal.*

We will evaluate allocations according to the following criteria. An allocation $z \in Z$ is **efficient** with respect to a preference profile $R \in \mathcal{R}^N$ if there is no alternative allocation that makes all agencies weakly better-off and some agencies strictly better-off, that is, if there is no $\overline{z} \in Z$ such that for each $i \in N$, $\overline{z}^i R^i z^i$ and for some $j \in N$, $\overline{z}^j P^j z^j$.

The following is a notion of fairness which has played a central role in the mechanism design literature. An allocation $z \in Z$ is **envy-free** (satisfies *no-envy*) with respect to a preference profile $R \in \mathcal{R}^N$ if every agency prefers its share to that of another, that is, if $z^i R^i z^j$ for each $i, j \in N$. *No-envy* is based on the notion that *all agencies have equal rights* and an allocation which favors an agency over another, by creating envy of the latter agency for the former's share, will violate fairness.

In regional development, there are cases where not all regions are considered equals. For example, EU divides its 271 regions into four categories from poorest to richest as (i) convergence regions, (ii) phasing out regions, (iii) phasing in regions, and (iv) competitiveness and employment regions. Convergence regions, being the poorest, receive much higher funds than the others. For such cases, the envy-free notion should only prevent the envy of poorer regions to richer ones, similar to the notion of *hierarchical no envy* proposed and analyzed in Kıbrıs (2003). In this paper, we assume all regions to have equal priority. A thorough analysis of the asymmetric case requires a separate study. We, however, present a discussion in Section 4.3.

For the reader who finds the *no-envy* requirement too demanding, the literature offers a significantly weaker notion of fairness which only compares agencies with identical preferences. **Equal treatment of equals** requires two agencies with identical preferences not to envy each other. An allocation $z \in Z$ satisfies *equal treatment of equals* if for each $i, j \in N$ with $R^i = R^j$, we have $z^i R^i z^j$. As in the case of no-envy, this property can be weakened only to apply to regions in the same development category.

A **mechanism** is a strategy space S^i for each agency i along with an outcome function F :

$(S^1 \times S^2 \times ... \times S^n) \to Z$ that selects an allocation for each strategy vector $(s^1, s^2, ..., s^n) \in (S^1 \times S^2 \times ... \times S^n)$. Given an agency i and strategy profile $s \in S$, let s^{-i} denote the strategy of all agencies except agency i. A mechanism satisfies **efficiency**, **no-envy**, or **equal treatment of equals** if it always picks allocations that satisfy these properties.

A **direct mechanism** is a mechanism where the strategy space is simply the set of preferences $\mathcal{R}$ for each agency i. Hence a direct mechanism is simply a function $F : \mathcal{R}^N \to Z$ that selects an allocation for each preference profile. In what follows, we will restrict ourselves to direct mechanisms. By the famous revelation principle (*e.g.* see Myerson, 1979), this is without loss of generality.

A highly desirable property of a direct mechanism is that it is always in the agencies' best interests to be entirely truthful about their preferences. Hence, the agencies can never benefit from "gaming" such mechanisms. A direct mechanism F is **strategy proof** if

$$F^i\left(R^i, R^{-i}\right) \quad R^i \quad F^i\left(\overline{R}^i, R^{-i}\right)$$

for each $i \in N$, $R^i, \overline{R}^i \in \mathcal{R}$ and $R^{-i} \in \mathcal{R}^{N \setminus i}$. That is, no agency i can be strictly better off by misrepresenting its preferences. This makes revealing the true preferences a weakly dominant strategy.

A stronger requirement takes into account coalitional manipulations as well. A mechanism F is **coalitional strategy proof** if under no preference profile can a coalition of agencies be better-off by misrepresenting their preferences in a coordinated manner, that is, for each $R \in \mathcal{R}^N$, $M \subset N$, and $\overline{R}^M \in \mathcal{R}^M$, if there is $i \in M$ such that $F^i\left(\overline{R}^M, R^{-M}\right) P^i F^i(R)$, then there is $j \in M$ such that $F^j(R) P^j F^j\left(\overline{R}^M, R^{-M}\right)$.[16]

3 The Uniform Mechanism

In this section, we propose a direct mechanism to allocate central government funds among agencies and activities.

Formally, the **uniform mechanism**, U is defined as follows: for each profile of declared preferences $R \in \mathcal{R}^N$,

(i) (no excess) if $\sum_N \sum_L p_k\left(R^i\right) = E$, then for each $i \in N$, $U^i(R) = p\left(R^i\right)$,

(ii) (excess demand) if $\sum_N \sum_L p_k\left(R^i\right) > E$, then for each $i \in N$, $U^i(R) = m\left(R^i, Y([0, \lambda])\right)$

[16] Note that ours is the stronger formulation of the property. A weaker version considers only coalitional manipulations that make all agencies in the coalition strictly better-off.

where $\lambda \in [0, E]$ satisfies $\sum\limits_N \sum\limits_L m_k \left(R^i, Y\left([0,\lambda]\right)\right) = E$ and,
(iii) (excess supply) if $\sum\limits_N \sum\limits_L p_k \left(R^i\right) < E$, then for each $i \in N$, $U^i\left(R\right) = m\left(R^i, Y\left([\lambda, E]\right)\right)$
where $\lambda \in [0, E]$ satisfies $\sum\limits_N \sum\limits_L m_k \left(R^i, Y\left([\lambda, E]\right)\right) = E$.

Item (i) is trivial. If there are just sufficient funds to award each agency i its ideal bundle $p\left(R^i\right)$, the uniform mechanism does that.

Item (ii) is about how the uniform mechanism rations the agencies when the government funds are insufficient. In this case, all agencies are offered a uniform budget cap λ. An agency i whose ideal bundle $p\left(R^i\right)$ does not require a higher budget (i.e. $\sum\limits_L p_k\left(R^i\right) \leqq \lambda$) receives precisely its ideal bundle $p\left(R^i\right)$ (which, in this case, coincides with the $m\left(R^i, Y\left([0,\lambda]\right)\right)$ in the formula). If on the other hand, the ideal bundle of agency i requires a higher budget than λ (i.e. $\sum\limits_L p_k\left(R^i\right) > \lambda$), agency i is rationed and receives precisely a budget of λ which is allocated among the l activities so as to maximize its declared preferences. That is, $U^i\left(R\right) = m\left(R^i, Y\left([0,\lambda]\right)\right)$.

Item (iii) is about how the uniform mechanism allocates the excess when the government funds exceed the aggregate demand of the agencies. In this case, all agencies are offered a uniform lower bound λ. An agency i whose ideal bundle $p\left(R^i\right)$ does not require a lower budget (i.e. $\sum\limits_L p_k\left(R^i\right) \geqq \lambda$) receives precisely its ideal bundle $p\left(R^i\right)$ (which, in this case, coincides with the $m\left(R^i, Y\left([\lambda, E]\right)\right)$ in the formula). If on the other hand, the ideal bundle of agency i requires a lower budget than λ (i.e. $\sum\limits_L p_k\left(R^i\right) < \lambda$), agency i receives precisely this minimum budget of λ which is allocated among the l activities so as to maximize its declared preferences. That is, $U^i\left(R\right) = m\left(R^i, Y\left([\lambda, E]\right)\right)$.

Item (iii) is not necessary in a simplified model that allows free-disposal. Then, excess supply is equivalent to no excess and the uniform mechanism is made up of only the first two items.

Note that, in cases (ii) and (iii), λ is chosen so as to allocate all available funds. The above formulation, however, is not explicit about how the "market clearing" λ is determined. The following algorithm is explicit about this choice. And it provides an *alternative but equivalent definition of the uniform mechanism.*

The Uniform Algorithm

Step 0.

Determine if the problem exhibits (i) no excess, (ii) excess demand, or (iii) excess supply.

If no excess.

Assign each agency its ideal bundle $p\left(R^i\right)$.

If excess demand

Step 1. Determine the set of agencies whose ideal bundle, $p\left(R^i\right)$, requires strictly less budget than in equal division of E, that is $\sum\limits_{k \in L} p_k\left(R^i\right) < \frac{E}{|N|}$. If no such agency exists, let each agency optimally allocate this equal budget among activities, that is, let $U^i(R) = m\left(R^i, Y\left(\frac{E}{|N|}\right)\right)$ and terminate the algorithm. Otherwise, assign each such agency its ideal bundle $p\left(R^i\right)$ and move to the next step.

Step 2. Determine the remaining agencies (say N') to be allotted and the remaining funds to be allotted (say E'). If N' is nonempty, repeat Step 1 by replacing N with N' and E with E'. Otherwise, terminate the algorithm.

If excess supply

Step 1. Determine the set of agencies whose ideal bundle, $p\left(R^i\right)$, requires strictly more budget than in equal division of E, that is $\sum\limits_{k \in L} p_k\left(R^i\right) > \frac{E}{|N|}$. If no such agency exists, let each agency optimally allocate this equal budget among activities, that is, let $U^i(R) = m\left(R^i, Y\left(\frac{E}{|N|}\right)\right)$ and terminate the algorithm. Otherwise, assign each such agency its ideal bundle $p\left(R^i\right)$ and move to the next step.

Step 2. Determine the remaining agencies (say N') to be allotted and the remaining funds to be allotted (say E'). If N' is nonempty, repeat Step 1 by replacing N with N' and E with E'. Otherwise, terminate the algorithm.

The equal division amount in the last step of the algorithm gives the "market clearing" λ in our first definition. The following example demonstrates how the uniform algorithm works in case of excess demand.

Example 1 *(Uniform algorithm in excess demand) Let* $N = \{1,2,3,4\}$, $L = \{1,2\}$, $E = 100$, $p\left(R^1\right) = (7,3)$, $p\left(R^2\right) = (5,21)$, $p\left(R^3\right) = (20,12)$, *and* $p\left(R^4\right) = (18,18)$. *Note that the four agencies' ideal budget requirements are* 10, 26, 32, *and* 36. *We will also need to specify the paths for the agencies' preferences. For simplicity, assume they are all linear, that is,* $\Pi\left(R^i\right) = \left\{\alpha p\left(R^i\right) \mid \alpha \in \mathbb{R}_+\right\}$ *for each agency* i. *The uniform algorithm, applied to this problem, works as follows. In the first step of the algorithm, each agency is offered a budget of* $\frac{100}{4} = 25$. *Since agency* 1 *requires a smaller budget than that, it is awarded its ideal bundle:* $x^1 = (7,3)$. *N and E are updated as* $N' = \{2,3,4\}$ *and* $E' = 90$. *Equal division now gives each agency in* N' *a budget of* $\frac{90}{3} = 30$. *Since agency 2 requires a smaller budget than that, it is awarded its ideal bundle:* $x^2 = (5,21)$. *Once again, we update* $N'' = \{3,4\}$ *and* $E'' = 64$. *Equal division now gives each agency in* N'' *a budget of* $\frac{64}{2} = 32$. *Since no agency in* N'' *requires a smaller budget than that, we let each agency in* N'' *to optimally allocate this budget among activities. Agency 3 thus receives* $x^3 = p\left(R^3\right)$ *and agency 4 receives* $x^4 = m\left(R^4, Y(32)\right) = (16,16)$. *The uniform mechanism thus picks the allocation* $U(R) = x$ *for this problem. The equal division in the final step is what the previous definition picks for* λ*, that is,* $\frac{E''}{|N''|} = 32 = \lambda$.

The following example demonstrates how the uniform algorithm works in case of excess supply.

Example 2 *(Uniform algorithm in excess supply) In the above example, let* $E = 120$. *The uniform algorithm, applied to this problem, works as follows. In the first step of the algorithm, each agency is offered a budget of* $\frac{120}{4} = 30$. *Since agencies 3 and 4 require a higher budget than that, they are awarded their ideal bundles:* $x^3 = (20,12)$, $x^4 = (18,18)$. *N and E are updated as* $N' = \{1,2\}$ *and* $E' = 52$. *Equal division now gives each agency in* N' *a budget of* $\frac{52}{2} = 26$. *Since no agency in* N' *requires a larger budget than that, we let each agency in* N' *to optimally allocate this budget among activities. Agency 2 thus receives* $x^2 = p\left(R^2\right)$ *and agency 1 receives* $x^1 = m\left(R^1, Y(26)\right) = \frac{13}{5}p\left(R^1\right) = (18.2, 7.8)$. *The uniform mechanism thus picks the allocation* $U(R) = x$ *for this problem. The equal division in the final step is what the previous definition picks for* λ*, that is,* $\frac{E'}{|N'|} = 26 = \lambda$.

The following proposition shows that the uniform mechanism satisfies all our criteria.

Proposition 3 The uniform mechanism satisfies *efficiency, coalitional strategy proofness,* and *no envy.*

Due to Proposition 3, the uniform mechanism also satisfies the weaker requirements of *strategy proofness* and *equal treatment of equals.*

Another desirable property of the uniform mechanism, not stated in the above proposition is as follows. Even though it is a centralized mechanism (*i.e.* it is the central government that determines the share of each agency), the uniform mechanism has a market-like interpretation. According to this interpretation, the central government (much like the Walrasian auctioneer) determines the "budget set" of each agency (i.e. the λ above). Then, each agency chooses its optimal bundle from its budget set. The budget sets are chosen to clear the market, that is, to equate aggregate supply to aggregate demand.

As noted at the end of Section 1, the right to choose the agencies' final consumptions is a source of conflict in applications. The uniform mechanism, with its two alternative interpretations, each interpretation giving the right to choose to one side, reconciles this tension.[17]

4 Characterization Results

In this section, we characterize the classes of mechanisms that satisfy our criteria, as we introduce them one by one. We also discuss the policy implications of each criterion.

4.1 Efficiency

The following lemma characterizes *efficient* allocations by two properties that point to an interesting "duality" in the RDA allocation problem. A mechanism can either fix a budget for each agency and then allocate each agency's budget among its activities, or alternatively it can fix a budget for each activity and then allocate each activity-budget among agencies. The two properties below require either allocation be done optimally. Every efficient mechanism, including the *uniform mechanism* satisfies them.

The **first property** (item (i) below) requires that for each agency, the total amount of funds assigned to it need to be optimally allocated among activities. If it is violated, a Pareto improvement can be obtained by switching to an alternative allocation of the same amount of funds among activities.

The **second property** (item (ii) below) requires that all agencies' shares fall on to the same side of their peaks. If it is violated, the aggregate funds determined for each activity can be reallocated among the agencies so as to lead to a Pareto improvement. This second property is equivalent to efficiency in the one-dimensional model of Sprumont (1991). There, it is typically referred to as *same-sidedness*. For multiple commodities, however, Amorós (2002) shows that this property is weaker than efficiency and analyzes its implications.

[17] Independent of who makes the choice, however, the uniform mechanism explicitly states that the choice should maximize the preferences of the agencies. It therefore takes position against any distortion of the agencies' choice by the central government agencies, such as the ministry of development in the Turkish example.

Lemma 4 *An allocation* $z \in Z$ *is* efficient *with respect to* $R \in \mathcal{R}^N$ *if and only if*
(i) *for each* $i \in N$, $z^i \in \Pi\left(R^i\right)$ *and*
(ii) *if* $\sum\limits_N \sum\limits_L p_k\left(R^i\right) \geqq E$, *then for each* $i \in N$, $\sum\limits_L z_k^i \leqq \sum\limits_L p_k\left(R^i\right)$ *and if* $\sum\limits_N \sum\limits_L p_k\left(R^i\right) \leqq E$, *then for each* $i \in N$, $\sum\limits_L z_k^i \geqq \sum\limits_L p_k\left(R^i\right)$.

Lemma 4 has two important policy implications. First, every efficient solution to the RDA allocation problem must give the agencies full discretion in allocating their budget among activities. Central government's intervention in budget allocation, as exemplified in the case of Turkey in Section 1, leads to inefficiencies.

Second, the central government needs to take into account agencies' declared preferences to make sure that all consume at the same side of their ideal budgets. Ignoring this information, again as in the case of Turkey, can lead to violations of efficiency by forcing an agency to spend too much while rationing another.

4.2 Efficiency and Strategy Proofness

The following proposition characterizes the class of mechanisms that are efficient and strategy proof. It states that every efficient and strategy proof mechanism must employ budget upper and lower bounds (respectively, the b^i and a^i functions below) and must allow each agency to maximize its preferences in choice sets defined by these bounds (as stated in item (i)). The choice of such bounds needs to be consistent to satisfy feasibility (as stated in item (ii)). Their construction is therefore not trivial. Finally, the bounds imposed on an agency can only depend on the others' declarations, not that of agency i.

Proposition 5 A mechanism F is *efficient* and *strategy proof* if and only if for each $i \in N$, there is $a^i : \mathcal{R}^{N\setminus\{i\}} \to \mathbb{R}_+$ and $b^i : \mathcal{R}^{N\setminus\{i\}} \to \mathbb{R}_+$ such that for each $R \in \mathcal{R}^N$

(i) $$F^i(R) = \begin{cases} m(R^i, Y([0, b^i(R^{-i})])) & if \quad \sum\limits_N \sum\limits_L p_k(R^i) \geqq E, \\ m(R^i, Y([a^i(R^{-i}), E])) & if \quad \sum\limits_N \sum\limits_L p_k(R^i) < E \end{cases}.$$

(ii)

$$\text{if } \sum_N \sum_L p_k\left(R^i\right) \geqq E, \text{ then } \sum_N \sum_L m_k(R^i, Y([0, b^i(R^{-i})])) = E \text{ and}$$
$$\text{if } \sum_N \sum_L p_k\left(R^i\right) < E, \text{ then } \sum_N \sum_L m_k(R^i, Y([a^i(R^{-i}), E])) = E.$$

The *uniform mechanism* is *efficient* and *strategy proof.* Thus, it is a member of the above class. Also, the uniform algorithm presented in Section 3 can easily be generalized to construct other examples of efficient and strategy proof mechanisms.

Proposition 5 has three important policy implications. First, it states that every efficient and strategy proof mechanism needs to use budget caps. Second, it states that an agency's budget cap can not depend on the agency's private (preference) information. Both the uniform mechanism and the Turkish mechanism presented in Section 1 satisfy this property. The Turkish mechanism, however, errs on the side of caution by making the choice of an agency's budget cap also independent of other agencies' declarations. As a result, the Turkish mechanism can end up wasting resources by introducing caps that do not clear the market.

A third implication of Proposition 5 is that every agency should be able to choose its most preferred bundle from its assigned choice set. For example, the uniform mechanism gives the agencies full discretion in choosing their bundles. The Turkish mechanism again violates this property since, as discussed in the introduction, the agencies' choices need to be ratified by the ministry of development. This potentially incentivizes the agencies to manipulate the Turkish mechanism by misrepresenting their private information.

Proposition 5 generalizes a characterization presented by Barberà *et al* (1997) for a single activity. This relationship is further discussed in the appendix.

4.3 Fairness

As a member of the class of mechanisms described in Proposition 5, the *uniform mechanism* has an interesting property: it offers the same choice set (i.e. the same a_i and b_i) to every agency. Thus, it is no surprise that the *uniform mechanism* additionally satisfies several fairness properties. More surprisingly, the following theorem shows that even a very weak fairness requirement like *equal treatment of equals* is satisfied by no other mechanism.

Theorem 6 The uniform mechanism is the only mechanism to satisfy *efficiency, strategy proofness,* and *equal treatment of equals.*

The uniform mechanism satisfies *no envy* and *coalitional strategy proofness* which are stronger than *equal treatment of equals* and *strategy proofness* respectively. Thus, the latter properties can be replaced with the former ones in the statement of Theorem 6. This theorem generalizes a characterization presented by Ching (1994) for a single activity. The relationship is further discussed in the appendix.

Given the importance of fairness in regional development, Theorem 6 has important policy implications. For a country where all regions have equal priority, it singles out the uniform mechanism as the only mechanism to satisfy our criteria.

As discussed in Section 2, there are examples in regional development where policy makers favor some regions over others. For example, EU territories are classified into four development categories as convergence regions, phasing-out regions, phasing-in regions, and competitiveness and employment regions. When allocating resources, EU only requires fairness among regions in the same category and gives priority to poorer regions over richer ones. We next present a straightforward extension of the uniform mechanism that allows asymmetric treatment of different groups.

Let N be partitioned into $N_1, ..., N_m$ where N_1 has priority over others, N_2 has priority over $N_3, ..., N_m$, and so on. Let $\rho = (\rho_1, ..., \rho_m) \in (0,1]^m$ be such that $1 = \rho_1 \geqq \rho_2 \geqq ... \geqq \rho_m > 0$. Then, the **$\boldsymbol{\rho}$-weighted uniform mechanism** assigns groups budget sets of proportional size, using the vector ρ. It can be formally defined as follows: for each profile of declared preferences $R \in \mathcal{R}^N$,

(i) *(no excess)* if $\sum\limits_N \sum\limits_L p_k(R^i) = E$, then for each $i \in N$, $U^i(R) = p(R^i)$,

(ii) *(excess demand)* if $\sum\limits_N \sum\limits_L p_k(R^i) > E$, then for each $j \in \{1, ..., m\}$ and $i \in N_j$, $U^i(R) = m(R^i, Y([0, \lambda\rho_j]))$

where $\lambda \in \mathbb{R}_+$ satisfies $\sum\limits_{j=1}^m \sum\limits_{i \in N_j} \sum\limits_{k \in L} m_k(R^i, Y([0, \lambda\rho_j])) = E$ and,

(iii) *(excess supply)* if $\sum\limits_N \sum\limits_L p_k(R^i) < E$, then for each $j \in \{1, ..., m\}$ and $i \in N_j$, $U^i(R) = m\left(R^i, Y\left(\left[\lambda(\rho_j)^{-1}, E\right]\right)\right)$

where $\lambda \in \mathbb{R}_+$ satisfies $\sum\limits_{j=1}^m \sum\limits_{i \in N_j} \sum\limits_{k \in L} m_k\left(R^i, Y\left(\left[\lambda(\rho_j)^{-1}, E\right]\right)\right) = E$.

These mechanisms are both *efficient* and *strategy proof.* If $\rho = (1, ..., 1)$, they boil down to the uniform mechanism. For different ρ, however, the ρ-weighted uniform mechanism allows preferred treatment of groups with higher priority. As a result it violates *no envy* and *equal treatment of equals.* All ρ-weighted uniform mechanisms however satisfy the following weakening of no envy which requires that an agency prefer its share to that of another with equal or lower priority: a mechanism F satisfies **hierarchical no envy** if $h, h' \in \{1, ..., m\}$ such that $h \leqq h'$, $i \in N_h$, $j \in N_{h'}$, and $R \in \mathcal{R}^N$ imply $F^i(R) R^i F^j(R)$. This property is similar to the "hierarchical no envy" property of Kıbrıs (2003). It also resembles the "fairness" property used in matching markets (*e.g.* see Balinski and Sönmez, 1999 or Sönmez and Switzer, 2011).

The proportional treatment of different groups proposed in the ρ-weighted uniform mechanisms is, of course, one of the very many ways of asymmetric treatment. Choosing the best method among alternative proposals requires a formal axiomatic analysis of an extended model. This is left for future research.

5 Implications of Constraints on the Allocation of Funds

In this section, we discuss the implications of constraining the way central government funds can be allocated among different activities, that is, constraining Z. Such constraints commonly appear in applications since it is quite frequent that some funds are specifically awarded for certain activities. As an example, consider the "European Agricultural Guidance and Guarantee Fund" which only supports rural development measures.

If ϵ dollars out of the total E is reserved for an activity k, at least this much needs to be spent on that activity. This imposes a lower bound of ϵ on the amount of money that can be spent on activity k. For $k \in L$ and $\epsilon \in (0, E)$, we define the k, ϵ constrained feasible set as:

$$Z_{k,\epsilon} = \left\{ z \in Z \mid \sum_{i \in N} z_k^i \geqq \epsilon \right\}.$$

A $\mathbf{k}, \boldsymbol{\epsilon}$ **constrained direct mechanism** is then a function $F : \mathcal{R}^N \to Z_{k,\epsilon}$ that selects a k, ϵ constrained allocation for each preference profile and thus, by definition, can not allocate less than ϵ on activity k.

The uniform mechanism does not directly apply to such problems due to cases where there is excess demand for the central government funds to be allocated but at the same time insufficient demand for activity k (that is, $\sum_N \sum_L p_m \left(R^i\right) \geqq E$ and $\sum_N p_k \left(R^i\right) < \epsilon$). Due to such problems, the uniform mechanism which would normally impose only a uniform upper bound on the total budget each agency can allocate among activities, needs to be augmented with an additional constraint, a uniform lower bound on the amount each agency can allocate on activity k. This "augmented uniform mechanism" is both *strategy proof* and *fair*. The efficiency requirement on k, ϵ constrained mechanisms should of course be weakened accordingly: an allocation $z \in Z_{k,\epsilon}$ is $\mathbf{k}, \boldsymbol{\epsilon}$ **constrained efficient** if there is no $\overline{z} \in Z_{k,\epsilon}$ such that for each $i \in N$, $\overline{z}^i R^i z^i$ and for some $j \in N$, $\overline{z}^j P^j z^j$. The augmented uniform mechanism is not k, ϵ *constrained efficient* as demonstrated in the following example.

Example 7 *Let* $N = \{1, 2\}$*,* $L = \{1, 2\}$ *,* $E = 20$*,* $k = 2$*, and* $\epsilon = 10$*. Let* R_1 *and* R_2 *be increasing linear preferences with marginal rates of substitution of* -2 *and* -4*, respectively. For this problem, a uniform allocation of both* E *and* ϵ *gives both agencies the same choice set*

$$\left\{x \in \mathbb{R}^2_+ \mid x_1 + x_2 \leqq 10 \text{ and } x_2 \geqq 5\right\}.$$

The preferences of both agencies are maximized at the bundle $(5, 5)$*. The "augmented uniform allocation" is then,* $z^1 = z^2 = (5, 5)$ *. However, this allocation is not* k, ϵ *constrained efficient since the allocation* $z_1 = (3, 10)$ *,* $z_2 = (7, 0)$ *in* $Z_{k,\epsilon}$ *makes both agents better-off.*

There might of course be other ways of augmenting the uniform mechanism to take into account the k, ϵ constraint. We, however, find that no matter how clever these mechanisms are, and no matter how small ϵ is, they will never satisfy our three criteria simultaneously.

Theorem 8 No k, ϵ constrained direct mechanism simultaneously satisfies k, ϵ *constrained efficiency, strategy proofness,* and *equal treatment of equals.*

As part of its proof, this theorem adapts an argument developed by Serizawa (2002). The relationship is discussed further in the appendix.

The proof of Theorem 8 gives us a hint as to the reach of its policy implications. The proof makes specific use of problems where there is excess demand for the central government funds, but insufficient demand for activity-specific funds on an activity k, as in Example 7. Thus, in applications where the agency preferences make this configuration a possibility, it warns the policy maker about the impossibility of simultaneously meeting our three criteria.

6 Conclusion

In this paper, we presented a *uniform mechanism* to allocate central government funds among regional development agencies. We showed that it is the *only* mechanism to satisfy three essential criteria: *strategy proofness, efficiency*, and *fairness.* We also provided a *uniform algorithm* to be used in practice. To calculate the uniform allocation, this simple algorithm only uses a very limited number of optimal choices from the agencies.

Our analysis also produced some interesting policy recommendations for the RDA allocation problem: First, it is good practice to assign budget sets to the agencies and let them freely choose their optimal bundle from their budget sets. Meddling in the bundle choice gives agencies incentive to misrepresent private information and potentially violates efficiency. Second, in choosing these budget sets, the agencies' private information has to be taken into account in a particular way. Other ad hoc methods, such as the one in the Turkish example, can create efficiency losses. Third, fairness concerns need to apply only to the choice of these budget sets, not to the choice of the final bundle. In the simplest case where one aspires to assign all regions equal priority, the unique fair way, as shown in the uniform mechanism, is to assign all agencies the same budget. Finally but maybe most importantly, constraints on how central government funds can be allocated among activities rule out the possibility of constructing desirable mechanisms, no matter how small these constraints are.

It is useful to re-iterate that, though the paper is centered around an application to the RDA allocation problem, the analysis applies to all production economies with a linear production technology and quite a large class of preferences that we call the generalized single peaked preferences. This is mainly the reason why we do not simplify the model by assuming free disposal. There might be other applications of our analysis where cases of excess supply need to be addressed.

Free disposal is however quite a natural assumption in case of the RDA allocation problem. As noted earlier, all our results continue to hold under this assumption, where additionally the uniform mechanism simplifies as follows: (i) in case of no excess and excess supply, give all agencies their peaks and (ii) in case of excess demand, follow the original definition in Section 3.

It is again useful to note that if there is a single activity, the *uniform mechanism* coincides with the well-known uniform rule of Sprumont (1991). Thus, Theorem 6 extends the characterization of Ching (1994) from allocation of a single commodity to production economies with multiple commodities.[18] If, on the other hand, we restrict the analysis to the subdomain of monotone preferences, the *uniform mechanism* coincides with the "equal budget free choice mechanism" proposed by Maniquet and Sprumont (1999) and shown to uniquely satisfy *efficiency*, *strategy proofness* and *equal treatment of equals*. Theorem 6 extends this result to the domain of generalized single-peaked preferences. Although Maniquet and Sprumont (1999) do not discuss the implications of production constraints, the negative findings of our Theorem 8 also apply to their domain, since our proof only uses monotone preferences.

[18] One important difference between the two domains is that in case of a single commodity, *efficiency* and *strategy proofness* imply that the share of an agency only depends on its peak. This is not the case on our domain where such rules, including the uniform mechanism, make use of more preference information than the agencies' peaks. Yet, these rules are sufficiently insensitive to preference information, as shown in *lemmas 9* and *10* in the appendix.

References

[1] Abdulkadiroğlu, A. and T. Sönmez (2003), "School Choice: A Mechanism Design Approach", *American Economic Review*, 93, 729-747.

[2] Adachi, T. (2010), "The uniform rule with several commodities: A generalization of Sprumont's characterization", *Journal of Mathematical Economics*, 46:6, 952-964.

[3] Amorós, P. (2002), "Single-peaked preferences with several commodities", *Social Choice and Welfare*, 19, 57-67.

[4] Anno, H. and Sasaki, H. (2010), "Second-best Efficiency of Allocation Rules: Strategy-proofness and Single-peaked Preferences with Multiple Commodities", mimeo.

[5] Balinski, M. and Sönmez, T. (1999), "A Tale of Two Mechanisms: Student Placement", *Journal of Economic Theory*, 84, 73-94.

[6] Barberà, S., Jackson, M. O., Neme, A. (1997), "Strategy-proof Allotment Rules", *Games and Economic Behavior*, 18, 1-21.

[7] Baş Uçar, R. (2011), "Kalkınma ajanslarından 250 milyon TL'lik destek", *Para Dergisi*, 279, 24-38.

[8] Bennett, R.J., Robson, P.J.A. and Bratton, W.J.A. (2001) 'The Influence of Location on the Use by SMEs of External Advice and Collaboration', *Urban Studies* 38: 1531–57.

[9] Benneworth, P. (2001) *Regional Development Agencies. The Early Years.* Regional Studies Association: Seaford.

[10] Bordignon, M., P. Manasse and G. Tabellini. (2001), "Optimal Regional Redistribution Under Asymmetric Information", *American Economic Review*, 91, 709–723.

[11] Bucovetsky, S., M. Marchand and P. Pestieau. (1998). "Tax Competition and Revelation of Preferences for Public Expenditure", *Journal of Urban Economics*, 44, 367–390.

[12] Ching, S. (1994), "An alternative characterization of the uniform rule", *Social Choice and Welfare*, 11, 131-136.

[13] Cho, W.J. and Thomson, W. (2011), "On the Extension of the Uniform Rule to More Than One Commodity: Existence and Maximality Results", mimeo.

[14] Daily Telegraph (31-03-2009), "Regional development agencies' deliver value for economy".

[15] Dasgupta, P., Hammond, P., and Maskin, E., (1979), "The implementation of social choice rules: some general results on incentive compatibility", *Review of Economic Studies*, 46, 185-216.

[16] De Bruijn, P. and Lagendijk, A. (2005) "Regional Innovation Systems in the Lisbon Strategy", *European Planning Studies* 13: 1153–72.

[17] Department for Business, Enterprise and Regulatory Reform (2009), "Impact of RDA Spending", National Report, Volume 1, Main Report.

[18] Department of Trade and Industry (2006), "Evaluating the Impact of England's Regional Development Agencies: Developing a Methodology and Evaluation Framework", DTI Occasional Paper No. 2, London.

[19] Filiztekin, A., Barlo, M. and Kıbrıs, Ö. (2011), *Türkiye'de Bölgesel Kalkınma: Farklılıklar, Bağıntılar ve Yeni bir Mekanizma Tasarımı*, Mega Basım, İstanbul. (ISBN: 978-9944-0172-3-7).

[20] Foley, D. (1967), "Resource Allocation and the Public Sector", *Yale Economic Essays*, 7, 45-98.

[21] Funck B., Pizzati L. and Bruncko M. (2003), "Chapter 1: Overview", in *European Integration, Regional Policy, and Growth* (Funck and Pizzati, eds.), The World Bank, Washington DC.

[22] Gilbert, G. and Y. Rocaboy (2004), "The central government grant allocation problem in the presence of misrepresentation and cheating", *Economics of Governance*, 5, 137-147.

[23] Goswami, M. P., Mitra, M., Sen, A. (2011), "Strategy-proofness and Pareto-efficiency in Classical Exchange Economies", Mimeo.

[24] Gualini, E. (2004) *Multi-level Governance and Institutional Change. The Europeanization of Regional Policy in Italy.* Aldershot: Ashgate.

[25] Halkier, H. and Danson, M. (1996), "Regional Development Agencies in Western Europe: a Survey of Key Characteristics and Trends", Occasional Paper 15, European Research Unit, Aalborg.

[26] Halkier, H., Danson, M. and Damborg, C. (1998) *Regional Development Agencies in Europe*. London: Jessica Kingsley.

[27] Hughes, J. T. (1998), "The Role of Development Agencies in Regional Policy", *Urban Studies*, 35:4, 615-626.

[28] Ju, B.-G. (2003), "Strategy-proofness versus efficiency in exchange economies: general domain properties and applications", *Social Choice and Welfare*, 21, 73–93.

[29] Hurwicz, L. (1972), "On informationally decentralized systems", in *Decision and Organization* (McGuire and Radner, Eds.), pp.297-336, North-Holland, Amsterdam.

[30] Hurwicz, L. and Walker, M. (1990), "On the generic nonoptimality of dominant-strategy allocation mechanisms: a general theorem that includes pure exchange economies", *Econometrica*, 58, 683-704.

[31] Kıbrıs, Ö. (2003) "Constrained Allocation Problems with Single-Peaked Preferences: An Axiomatic Analysis", *Social Choice and Welfare*, 20:3, 353-362.

[32] Lagendijk, A., Kayasu, S. and Yaşar, S. (2009), "The Role of Regional Development Agencies in Turkey: From Implementing EU Directives to Support Regional Business Communities", *European Urban and Regional Studies*, 16:4, 383-396.

[33] Leroux, J. (2004), "Strategy-proofness and efficiency are incompatible in production economies", *Economics Letters*, 85, 335-340.

[34] Levaggi, R. (2002), "Decentralized budgeting procedures for public expenditure", *Public Finance Review*, 30: 4, 273-295.

[35] Loewendahl-Ertugal, E. (2005), "Europenisation of Regional Policy and Regional Governance: The Case of Turkey", *European Political Economy Review*, 3:1, 18-53.

[36] Maniquet, F., Sprumont, Y. (1999), "Efficient strategy-proof allocation functions in linear production economies", *Economic Theory*, 14, 583–595.

[37] McMaster, I. (2006) "Czech Regional Development Agencies in a Shifting Institutional Landscape", *Europe-Asia Studies* 58: 347–70.

[38] Milgrom, M. (2004), *Putting Auction theory to Work, Churchill Lectures in Economics.* Cambridge. Cambridge University Press.

[39] Morimoto, S. Serizawa, S. and Ching, S. (2009), "A characterization of the uniform rule with several goods and agents", *ISER Discussion Paper* 0769, Osaka University.

[40] Myerson, R. B. (1979), "Incentive compatibility and the bargaining problem", *Econometrica*, 47:1, 61-73.

[41] Oates, W. E. (1999), "An essay on fiscal federalism", *Journal of Economic Literature*, 37: 3, 1120-1149.

[42] Oates, W. E. (2005), "Toward a second-generation theory of fiscal federalism", *International Tax and Public Finance*, 12, 349-373.

[43] Roth, A. E. (2002), "The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics", *Econometrica*, 70:4, 1341-1378.

[44] Roth, A. E., T. Sönmez, and M. U. Ünver (2004), "Kidney Exchange", *Quarterly Journal of Economics*, 119, 457-488.

[45] Seabright, P. (1996), "Accountability and decentralization in government: an incomplete contracts model", *European Economic Review*, 40, 61-89.

[46] Serizawa, S. (1999), "Strategy-proof and symmetric social choice functions for public good economies", *Econometrica*, 67, 121–145.

[47] Serizawa, S. (2002), "Inefficiency of strategy-proof rules for pure exchange economies," *Journal of Economic Theory*, 106, 219-241.

[48] Serizawa, S. and Weymark, J. A. (2003), "Efficient strategy-proof exchange and minimum consumption guarantees", *Journal of Economic Theory*, 109, 246-263.

[49] Schummer, J. (1997), "Strategy-proofness versus efficiency on restricted domains of exchange economies", *Social Choice and Welfare*, 14, 47–56.

[50] Schummer, J. (1999), "Strategy-proofness versus efficiency for small domains of preferences over public goods", *Economic Theory*, 13, 709-722.

[51] Sönmez, T. and Switzer, T. B. (2011), "Matching with (Branch-of-Choice) Contracts at the United States Military Academy", *Econometrica*, forthcoming.

[52] Sprumont, Y. (1991), "The division problem with single-peaked preferences: A characterization of the uniform allocation rule", *Econometrica*, 59, 509-519.

[53] Syrett, S. and Silva, C.N. (2001) "Regional Development Agencies in Portugal: Recent Development and Future Challenges", *Regional Studies* 35: 174–80.

[54] Thomson, W. (2012), "Allocating a Commodity Among Agents with Single-peaked Preferences", *Social Choice and Welfare*, forthcoming.

[55] Tommasi, M. and F. Weinschelbaum (2007), "Centralization vs. Decentralization: A Principal-Agent Analysis", *Journal of Public Economic Theory*, 9: 369–389.

[56] Zhou, L. (1991),"Inefficiency of strategy-proof allocation mechanisms in pure exchange economies", *Social Choice and Welfare*, 8, 247-257.

7 Appendix

Proof. (Proposition 3) *Efficiency* follows from Lemma 4. For *coalitional strategy proofness*, let $R \in \mathcal{R}^N$, $M \subset N$, and $\overline{R}^M \in \mathcal{R}^M$. Let $\overline{R} = \left(\overline{R}^M, R^{-M}\right)$, $z = U(R)$, and $\overline{z} = U\left(\overline{R}\right)$. Suppose there is $i \in M$ such that $\overline{z}^i P^i z^i$. Then $z^i \neq p\left(R^i\right)$. By *efficiency* of U, this implies $\sum_N \sum_L p_k\left(R^j\right) \neq E$. Assume $\sum_N \sum_L p_k\left(R^j\right) > E$ (the proof for the alternative case is similar). By definition of U, there is $\lambda \in [0, E]$ such that $z^i = m\left(R^i, Y([0,\lambda])\right)$. Thus, $\sum_L z_k^i = \lambda < \sum_L p_k\left(R^i\right)$ and $\sum_L z_k^i < \sum_L \overline{z}_k^i$.
Case 1: $\sum_N \sum_L p_k\left(\overline{R}^j\right) \geqq E$. Then $\overline{z}_k^i = m\left(R^i, Y\left(\left[0, \overline{\lambda}\right]\right)\right)$ for some $\overline{\lambda} \in [0, E]$. Since $\overline{\lambda} \geqq \sum_L \overline{z}_k^i$, we have $\overline{\lambda} > \lambda$. Since $\sum_N \sum_L z_k^i = E = \sum_N \sum_L \overline{z}_k^i$, there is $q \in N$ such that $\sum_L z_k^q > \sum_L \overline{z}_k^q$. Suppose $q \notin M$. Then $R^q = \overline{R}^q$ and thus, $\sum_L \overline{z}_k^q = \min\left\{\overline{\lambda}, \sum_L p_k\left(R^q\right)\right\} \geqq \min\left\{\lambda, \sum_L p_k\left(R^q\right)\right\} = \sum_L z_k^q$, a contradiction. Therefore $j \in M$.
Case 2: $\sum_N \sum_L p_k\left(\overline{R}^j\right) \leqq E$. As in Case 1, there is $q \in N$ such that $\sum_L z_k^q > \sum_L \overline{z}_k^q$. This implies $z^q P^q \overline{z}^q$. Suppose $q \notin M$. Then $R^q = \overline{R}^q$. Thus, $\overline{z}^q = m\left(R^q, Y\left(\left[\overline{\lambda}, E\right]\right)\right)$ for some $\overline{\lambda} \in [0, E]$.

This implies $\sum\limits_L \overline{z}_k^q \geqq \sum\limits_L p_k\left(R^q\right)$. Since $\sum\limits_L z_k^q \leqq \sum\limits_L p_k\left(R^q\right)$, we have $\sum\limits_L z_k^q \leqq \sum\limits_L \overline{z}_k^q$, a contradiction. Therefore, $j \in M$. This proves that U is *coalitional strategy proof.*

No envy and *equal treatment of equals* follow from the fact that U chooses the same choice set for every agency and every agency chooses its most preferred bundle from this set. ■

Proof. (Lemma 4) ($\Rightarrow$) Assume z is *efficient* with respect to R. We will first show (i). Suppose there is $i \in N$ such that $z^i \notin \Pi\left(R^i\right)$. Then, $z^i \neq m\left(R^i, Y\left(\sum\limits_L z_k^i\right)\right)$. Let $x^i = m\left(R^i, Y\left(\sum\limits_L z_k^i\right)\right)$. Then a Pareto improvement can be obtained by switching to $\hat{z} \in Z$ defined as follows: for each $j \in N \setminus \{i\}$, $\hat{z}^j = z^j$ and $\hat{z}^i = x^i$.

Next, we will show (ii). Suppose $\sum\limits_N \sum\limits_L p_k\left(R^i\right) \geqq E$. Suppose for a contradiction there are $i, j \in N$ such that $\sum_L z_k^i < \sum_L p_k(R^i)$ and $\sum_L z_k^j > \sum_L p_k(R^j)$. Let $\delta = \frac{1}{2}\min\{\sum_L p_k(R^i) - \sum_L z_k^i, \sum_L z_k^j - \sum_L p_k(R^j)\}$. Define $\hat{z} \in Z$ as follows:

$$\begin{aligned}
\hat{z}^i &= m(R^i, Y(\sum_L z_k^i + \delta)), \\
\hat{z}^j &= m(R^j, Y(\sum_L z_k^j - \delta)), \text{ and} \\
\hat{z}^q &= z^q, \text{ for each } q \in N \setminus \{i, j\}.
\end{aligned}$$

Then, $\hat{z}^i\ P^i\ z^i$, $\hat{z}^j\ P^j\ z^j$, and $\hat{z}^q\ I^q\ z^q$, contradicting *efficiency.*

($\Leftarrow$) Now assume that $z \in Z$ satisfies properties (i) and (ii). Suppose $\widehat{z} \in Z$ is a Pareto improvement over z. Then there is some $i \in N$ such that $\widehat{z}^i\ P^i\ z^i$ and for each $j \in N \setminus \{i\}$, $\widehat{z}^j\ R^j\ z^j$. Note that by (i), $\sum\limits_L z_k^i \neq \sum\limits_L p_k\left(R^i\right)$. First, assume $\sum\limits_L z_k^i < \sum\limits_L p_k\left(R^i\right)$. If $\sum_L \hat{z}_k^i < \sum_k z_k^i$, consider $x^i = m(R^i, Y(\sum_L \hat{z}_k^i))$. Note that $z^i\ P^i\ x^i$ and $x^i\ R^i\ \hat{z}^i$. Thus, $z^i\ P^i\ \hat{z}^i$, a contradiction. If $\sum_L \hat{z}_k^i > \sum_L z_k^i$, by *feasibility*, there is $j \in N$ such that $\sum_L \hat{z}_k^j < \sum_L z_k^j$. Let $x^j = m(R^j, Y(\sum_L \hat{z}_k^j))$. Note that $z^j\ P^j\ x^j$ and $x^j\ R^j\ \hat{z}^j$. Thus, $z^j\ P^j\ \hat{z}^j$, a contradiction. The proof for the case $\sum\limits_L z_k^i > \sum\limits_L p_k\left(R^i\right)$ is similar. ■

The next two lemmas and the following proof establish Proposition 5. This result generalizes a characterization presented by Barberà *et al* (1997) for a single activity. One can alternatively prove it by using efficiency to establish a relationship between our model and the one dimensional model and then, make reference to the Barberà *et al* (1997) characterization (which they state without proof). We prefer to present an independent proof that, in our opinion, highlights some interesting properties of efficient and strategy proof mechanisms.

Our first lemma shows that changing an agency's preferences without altering its peak has no effect on the choice set it will be offered. If additionally the agency's optimal bundle on this choice set remains unchanged, it receives precisely the same share as before.

Lemma 9 *Assume that* F *is* efficient *and* strategy proof*. Let* $R \in \mathcal{R}^N$*,* $i \in N$*, and* $\hat{R}^i \in \mathcal{R}$ *be such that* $p(R^i) = p(\hat{R}^i)$*. Then,* $\sum_L F_k^i(R^i, R^{-i}) = \sum_L F_k^i(\hat{R}^i, R^{-i})$*. Additionally, if* $m(R^i, Y(\sum_L F_k^i(R^i, R^{-i}))) = m(\hat{R}^i, Y(\sum_L F_k^i(R^i, R^{-i})))$*, then* $F^i(R^i, R^{-i}) = F^i(\hat{R}^i, R^{-i})$*.*

Proof. Without loss of generality, assume that $\sum_L F_k^i\left(R^i, R^{-i}\right) \leqq \sum_L p_k\left(R^i\right)$. Then by Lemma 4, $\sum_L F_k^i\left(\hat{R}^i, R^{-i}\right) \leqq \sum_L p_k\left(R^i\right)$.

Case 1. ($\Pi(R^i) = \Pi(\hat{R}^i)$) Suppose $\sum_L F_k^i\left(R^i, R^{-i}\right) < \sum_L F_k^i\left(\hat{R}^i, R^{-i}\right)$. By Lemma 4, $F^i\left(R^i, R^{-i}\right) \in \Pi(R^i)$ and $F^i\left(\hat{R}^i, R^{-i}\right) \in \Pi(R^i)$. But then, $F^i\left(\hat{R}^i, R^{-i}\right)\ P^i\ F^i\left(R^i, R^{-i}\right)$, a contradiction to *strategy proofness*. Similarly, $\sum_L F_k^i\left(R^i, R^{-i}\right) > \sum_L F_k^i\left(\hat{R}^i, R^{-i}\right)$ creates a contradiction. Thus, $\sum_L F_k^i\left(R^i, R^{-i}\right) = \sum_L F_k^i\left(\hat{R}^i, R^{-i}\right)$. Since

$$m\left(R^i, Y\left(\sum_L F_k^i\left(R^i, R^{-i}\right)\right)\right) = m\left(\hat{R}^i, Y\left(\sum_L F_k^i\left(R^i, R^{-i}\right)\right)\right),$$

by Lemma 4, $F^i\left(R^i, R^{-i}\right) = F^i\left(\hat{R}^i, R^{-i}\right)$.

Case 2. ($\Pi(R^i) \neq \Pi(\hat{R}^i)$) Suppose $\sum_L F_k^i\left(R^i, R^{-i}\right) < \sum_L F_k^i\left(\hat{R}^i, R^{-i}\right)$. Now change R^i to $\overline{R}^i$ so that $p\left(R^i\right) = p\left(\overline{R}^i\right)$ and $\Pi\left(R^i\right) = \Pi\left(\overline{R}^i\right)$ (thus, we have $m(R^i, Y(\sum_L F_k^i(R^i, R^{-i}))) = m(\overline{R}^i, Y(\sum_L F_k^i(R^i, R^{-i}))))$ but according to $\overline{R}^i$, $F^i\left(\hat{R}^i, R^{-i}\right) \overline{P}^i F^i\left(R^i, R^{-i}\right)$. By Case 1,

$$F^i\left(R^i, R^{-i}\right) = F^i\left(\overline{R}^i, R^{-i}\right)$$

and thus, $F^i\left(\hat{R}^i, R^{-i}\right) \overline{P}^i F^i\left(\overline{R}^i, R^{-i}\right)$. This contradicts *strategy proofness.* Similarly,

$$\sum_L F_k^i\left(R^i, R^{-i}\right) > \sum_L F_k^i\left(\hat{R}^i, R^{-i}\right)$$

creates a contradiction (we now change $\hat{R}^i$ instead of R^i). The rest of the argument is the same as in Case 1. ■

The following is an "invariance" lemma. It shows that an agency whose peak is above (below) its assigned choice set can not end up on a higher (lower) choice set by declaring a different preference relation. Define

$$X_i^F\left(R^{-i}\right) = \left\{F^i\left(R^i, R^{-i}\right) \mid R^i \in \mathcal{R}\right\}.$$

Lemma 10 *Assume that* F *is* efficient *and* strategy proof*. Let* $\left(R^i, R^{-i}\right) \in \mathcal{R}^N$ *and let* $z^i = F^i\left(R^i, R^{-i}\right)$*. If* $\sum_L z_k^i < \sum_L p_k\left(R^i\right)$ *and* $y^i \in \mathbb{R}_+^L$ *is such that* $\sum_L y_k^i > \sum_L z_k^i$*, then* $y^i \notin X_i^F\left(R^{-i}\right)$*. Similarly, if* $\sum_L z_k^i > \sum_L p_k\left(R^i\right)$ *and* $y^i \in \mathbb{R}_+^L$ *is such that* $\sum_L y_k^i < \sum_L z_k^i$*, then* $y^i \notin X_i^F\left(R^{-i}\right)$*.*

Proof. First, assume $\sum_L z_k^i < \sum_L p_k\left(R^i\right)$ and $y^i \in \mathbb{R}_+^L$ is such that $\sum_L y_k^i > \sum_L z_k^i$. Suppose for a contradiction, $y^i \in X_i^F(R^{-i})$. Then, there is $\bar{R}^i \in \mathcal{R}$ such that $F^i(\bar{R}^i, R^{-i}) = y^i$. Let $\hat{R}^i \in \mathcal{R}$ be such that $p(\hat{R}^i) = p(R^i)$, $m(\hat{R}^i, Y(\sum_L z_k^i)) = z^i$, and $y^i\ \hat{P}^i\ z^i$. By Lemma 9, $F^i(\hat{R}^i, R^{-i}) = z^i$. Then, we have $F^i(\bar{R}^i, R^{-i})\ \hat{P}^i\ F^i(\hat{R}^i, R^{-i})$, contradicting *strategy proofness.* The proof for the case $\sum_L z_k^i > \sum_L p_k(R^i)$ is similar. ■

We now use the above lemmas to characterize all *efficient* and *strategy proof* mechanisms.

Proof. (Proposition 5) ($\Rightarrow$) Assume that F is *efficient* and *strategy proof.* Let $R \in \mathcal{R}^N$. Let $z^i = F^i(R^i, R^{-i})$. First, assume that $\sum\limits_N \sum\limits_L p_k(R^i) \geqq E$. By Lemma 4, $\sum\limits_L z_k^i \leqq \sum\limits_L p_k(R^i)$. Let $\overline{R}^i \in \mathcal{R}$ be such that $p\left(\overline{R}^i\right) = (E, ..., E)$ and define $b^i\left(R^{-i}\right) \in [0, E]$ as

$$b^i\left(R^{-i}\right) = \sum_L F_k^i\left(\overline{R}^i, R^{-i}\right).$$

If $\sum\limits_L p_k(R^i) < b^i(R^{-i})$, then $z^i = p(R^i)$. Otherwise, $\sum\limits_L z_k^i < \sum\limits_L p_k(R^i) < b^i(R^{-i})$ and by Lemma 10, $F^i(\overline{R}^i, R^{-i}) \notin X_i^F(R^{-i})$, a contradiction. Alternatively, let $\sum\limits_L p_k(R^i) \geqq b^i(R^{-i})$. If $\sum_L z_k^i > b^i(R^{-i})$, by Lemma 10, $z^i \notin X_i^F(R^{-i})$, a contradiction. If $\sum_L z_k^i < b^i(R^{-i})$, let $\hat{R}^i \in \mathcal{R}$ be such that $p(\hat{R}^i) = p(R^i)$, $m(\hat{R}^i, Y(\sum\limits_L z_k^i)) = m(R^i, Y(\sum\limits_L z_k^i))$, and $F^i(\bar{R}^i, R^{-i})\ \hat{P}^i\ z^i$. Then, by Lemma 9, $F^i(\hat{R}^i, R^{-i}) = z^i$ and $F^i(\bar{R}^i, R^{-i})\ \hat{P}^i\ F^i(\hat{R}^i, R^{-i})$, contradicting *strategy proofness.* Thus, $\sum_L z_k^i = b^i(R^{-i})$ and by Lemma 4, $z^i = m(R^i, Y(b^i(R^{-i})))$. Combining the two cases, we can write $z^i = m(R^i, Y([0, b^i(R^{-i})]))$.

Second, assume that $\sum\limits_N \sum\limits_L p_k(R^i) < E$. By Lemma 4, $\sum\limits_L z_k^i \geqq \sum\limits_L p_k(R^i)$. Let $\underline{R}^i \in \mathcal{R}$ be such that $p\left(\underline{R}^i\right) = (0, ..., 0)$ and define $a^i\left(R^{-i}\right) \in [0, E]$ as

$$a^i\left(R^{-i}\right) = \sum_L F_k^i\left(\underline{R}^i, R^{-i}\right).$$

If $a^i(R^{-i}) < \sum\limits_L p_k(R^i)$, then $z^i = p(R^i)$. Otherwise, $\sum\limits_L z_k^i > \sum\limits_L p_k(R^i) > a^i(R^{-i})$ and by Lemma 10, $F^i(\underline{R}^i, R^{-i}) \notin X_i^F(R^{-i})$, a contradiction. Alternatively, let $\sum\limits_L p_k(R^i) \leqq a^i(R^{-i})$. If

$\sum_L z_k^i > a^i(R^{-i})$, by Lemma 10, $F^i(\underline{R}^i, R^{-i}) \notin X_i^F(R^{-i})$, a contradiction. If $\sum_L p_k(R^i) \leqq \sum_L z_k^i < a^i(R^{-i})$, let $\hat{R}^i \in \mathcal{R}$ be such that $p(\hat{R}^i) = p(\underline{R}^i)$, $m(\hat{R}^i, Y(a^i(R^{-i}))) = m(\underline{R}^i, Y(a^i(R^{-i})))$, and $z^i \; \hat{P}^i \; F^i(\underline{R}^i, R^{-i})$. Then, by Lemma 9, $F^i(\hat{R}^i, R^{-i}) = F^i(\underline{R}^i, R^{-i})$. Thus, $F^i(R^i, R^{-i}) \; \hat{P}^i \; F^i(\hat{R}^i, R^{-i})$, contradicting *strategy proofness*. Thus, $\sum_L z_k^i = a^i(R^{-i})$ and by Lemma 4, $z^i = m(R^i, Y(a^i(R^{-i})))$. Combining the two cases, we can write $z^i = m(R^i, Y([a^i(R^{-i}), E]))$.

Condition (ii) follows from *feasibility* of F.

($\Longleftarrow$) For the converse, note that *strategy proofness* follows from the fact that agency i's option set $X_i^F\left(R^{-i}\right) = Y\left(\left[0, b^i\left(R^{-i}\right)\right]\right)$ (or $X_i^F\left(R^{-i}\right) = Y\left(\left[a^i(R^{-i}), E\right]\right)$) is independent of R^i and for each R^i, $F^i\left(R^i, R^{-i}\right) = m\left(R^i, Y\left(\left[0, b^i\left(R^{-i}\right)\right]\right)\right)$ (or $F^i\left(R^i, R^{-i}\right) = m\left(R^i, Y\left(\left[a^i\left(R^{-i}\right), E\right]\right)\right)$. *Efficiency* of F follows from (i). ■

The following proof extends an argument developed by Ching (1994) for a single activity to our domain.

Proof. (Theorem 6) Let F be a mechanism that satisfies *efficiency, strategy proofness*, and *equal treatment of equals*. Let $R \in \mathcal{R}^N$.

Case 1. If $\sum_N \sum_L p_k(R^i) = E$, then by efficiency, $F(R) = U(R)$.

Case 2. Suppose $\sum_N \sum_L p_k(R^i) < E$. Without loss of generality, suppose that $\sum_L p_k(R^1) \leqq \cdots \leqq \sum_L p_k(R^n)$. If $R = (R^1, \cdots, R^1)$, then by *efficiency* and *equal treatment of equals*, $F(R) = U(R)$. Alternatively, suppose $R \neq (R^1, \cdots, R^1)$. Suppose for a contradiction, $F(R) \neq U(R)$.

Step 1. Since $F(R) \neq U(R)$, by *feasibility* and *efficiency*, there is $j \in N$ such that $\sum_L p_k(R^j) \leqq \sum_L F_k^j(R) < \sum_L U_k^j(R)$. Let $\hat{R}^j = R^1$. Then, by the definition of U, $\sum_L U_k^j(R) = \sum_L U_k^j(\hat{R}^j, R^{-j})$. Also, by Proposition 5, $\sum_L F_k^j(\hat{R}^j, R^{-j}) \leqq \sum_L F_k^j(R^j, R^{-j})$. Thus, $\sum_L F_k^j(\hat{R}^j, R^{-j}) < \sum_L U_k^j(\hat{R}^j, R^{-j})$. If $(\hat{R}^j, R^{-j}) = (R^1, \cdots, R^1)$, this contradicts *efficiency* and *equal treatment of equals*. Otherwise, let $\widetilde{R} = (\hat{R}^j, R^{-j})$, define $\overline{S} = \left\{i \in N \mid \widetilde{R}^i = \widetilde{R}^1\right\}$ and note that for each $i \in \overline{S}$, we have $\sum_L F_k^i(\widetilde{R}) < \sum_L U_k^i(\widetilde{R})$.

Step 2. By *feasibility* and *efficiency*, there is $q \in N \setminus \overline{S}$ such that $\sum_L p_k(\widetilde{R}^q) \leqq \sum_L U_k^q(\widetilde{R}) < \sum_L F_k^q(\widetilde{R})$. Let $\hat{R}^q = \widetilde{R}^1$. By definition of U, $\sum_L U_k^q(\hat{R}^q, \widetilde{R}^{-q}) \leqq \sum_L U_k^q(\widetilde{R})$. By Proposition 5, $\sum_L F_k^q(\hat{R}^q, \widetilde{R}^{-q}) = \sum_L F_k^q(\widetilde{R})$. Thus, $\sum_L U_k^q(\hat{R}^q, \widetilde{R}^{-q}) < \sum_L F_k^q(\hat{R}^q, \widetilde{R}^{-q})$. If $(\hat{R}^q, \widetilde{R}^{-q}) = (\widetilde{R}^1, \cdots, \widetilde{R}^1)$, this contradicts *efficiency* and *equal treatment of equals*. Otherwise, we go back to Step 1 by re-defining $R = (\hat{R}^q, \widetilde{R}^{-q})$ and noting that, by *efficiency* and *equal treatment of equals,* for each $i \in \overline{S} \cup \{q\}$, we have $\sum_L U_k^i(R) < \sum_L F_k^i(R)$.

Since there is a finite number of agencies, after a finite number of iterations of steps 1 and 2, we obtain $R = (R^1, \cdots, R^1)$ and $F(R) \neq U(R)$. This contradicts *efficiency* and *equal treatment of* *equals.*

Case 3. Suppose $\sum_N \sum_L p_k(R^i) > E$. Then, the proof is similar except that, now the agencies' preferences are sequentially updated to R^n, the preference with the highest peak. ■

We next prove Theorem 8. The proof involves showing that a consumption lower bound on an activity k, together with preferences that "value" activity k, fixes a unique efficient production vector. On this subclass of preferences, thus, our problem boils down to a pure exchange economy. We then use an argument similar to Serizawa (2002) to show the impossibility of a mechanism that satisfies *efficiency, strategy proofness*, and *equal treatment of equals.* The proof will make use of the following definitions and *Lemma 12* below.

Definition 11 *For each $R \in \mathcal{R}$ and $z \in \mathbb{R}^L_+$, let $UC(R,z) = \{x \in \mathbb{R}^L_+ \mid \ x \ R \ z\}$. For each $R \in \mathcal{R}^N$ and $i \in N$, let $P^i(R) = \{z^i \in \mathbb{R}^L_+ \mid \ z$ is efficient with respect to $R\}$. For each $i \in N$, $R^i, \hat{R}^i \in \mathcal{R}$, $\hat{R}^i$ is a* strict monotonic transformation *of R^i at z^i if (i) $UC(\hat{R}^i, z^i) \subseteq UC(R^i, z^i)$ and (ii) $\hat{z}^i \in UC(\hat{R}^i, z^i)$ and $\hat{z}^i \neq z^i$ together imply that $\hat{z}^i \ P^i \ z^i$. Let $M(R^i; z^i)$ be the set of strict monotonic transformations of u^i at z^i.*

Lemma 12 *Let F be a* strategy proof *rule. For any $R \in \mathcal{R}^N$, $i \in N$, and $\hat{R}^i \in M(R^i; F(R))$, $F^i(\hat{R}^i, R^{-i}) = F^i(R)$.*

Proof. Suppose for a contradiction that $F^i(\hat{R}^i, R^{-i}) \neq F^i(R)$. By *strategy proofness*, $F(\hat{R}^i, R^{-i}) \ \hat{R}^i \ F(R)$. Then, since $\hat{R}^i \in M(R^i; F(R))$, $F(\hat{R}^i, R^{-i}) \ P^i \ F(R)$, contradicting *strategy proofness.* ■

Proof. (Theorem 8) Let $\epsilon \in (0, E)$. Suppose F is a k, ϵ constrained direct mechanism that satisfies *efficiency, strategy proofness* and *equal treatment of equals.* Without loss of generality, let $k = 2$. Let $L' = \{1, 2\}$. Let $\underline{\mathcal{R}}$ be the class of preferences that are (i) indifferent in amounts of commodities in $L \setminus L'$, (ii) continuous, strictly convex, homothetic, and smooth in $\mathbb{R}^{L'}_+$, and (iii) strictly monotonic on the interior of $\mathbb{R}^{L'}_+$. Note that, by homotheticity, for each $R \in \underline{\mathcal{R}}$, $\Pi(R)$ is linear. Let $\mathcal{R}_\epsilon \subset \underline{\mathcal{R}}$ be a class of preferences such that for each $R \in \mathcal{R}_\epsilon$, each $x \in \Pi(R) \setminus \{0\}$ satisfies $\frac{x_2}{x_1} < \frac{\epsilon}{E - \epsilon}$.

Let G be the restriction of F to $\mathcal{R}^N_\epsilon$. Then, G also satisfies *efficiency, strategy proofness,* and *equal treatment of equals.* Let $R^0, R^1 \in \mathcal{R}_\epsilon$ be two distinct Cobb Douglas preferences. Let $w = (E - \epsilon, \epsilon, 0, ..., 0)$ and $d = \frac{w}{n}$. By *efficiency* and *equal treatment of equals*, for each $i \in N$, $G^i(R^0, ..., R^0) = d$. Let $z = G(R^1, R^0, ..., R^0)$. Note that by *efficiency*, $\sum_{i \in N} z^i = w$. Note also that, z^1 is not proportional to w, because otherwise by strong monotonicity, either $z^1 \ P^0 \ d$ or

$d\ P^1\ z^1$ and both cases contradict *strategy proofness.* Since $G^1(R^0,...,R^0)=d$, *strategy proofness* implies $z^1_{L'}\in\mathbb{R}^{L'}_{++}$. Since $z^1\neq w$, by *efficiency* and *equal treatment of equals*, for each $i\in N\setminus\{1\}$, $z^i=\frac{w-z^1}{n-1}\in\mathbb{R}^{L'}_{++}$. Also, z^1 is not proportional to z^i. Let $\hat{R}^0\in\mathcal{R}_\epsilon$ be such that $\hat{R}^0\in M(R^0;z^i)$.

We claim that $G(R^1,\hat{R}^0,...,\hat{R}^0)=z$. To see this, let $G(R^1,\hat{R}^0,R^0,...,R^0)=\hat{z}$. By *efficiency*, $\sum_{i\in N}\hat{z}^i=w$. Since $\hat{R}^0\in M(R^0;z^2)$, by *Lemma 12*, $\hat{z}^2=z^2$. By smoothness, $\hat{R}^0$ and R^0 have the same marginal rate of substitution at z^2. Thus by *efficiency*, the marginal rate of substitution of each $i\in N$ remains constant from z^i to $\hat{z}^i$. Then by homotheticity, for each $i\in N$, $\frac{\hat{z}^i_1}{\hat{z}^i_2}=\frac{z^i_1}{z^i_2}$. Let $\hat{S}=\hat{z}^2+...+\hat{z}^n$ and $S=z^2+...+z^n$. Then, $\frac{\hat{S}_1}{\hat{S}_2}=\frac{S_1}{S_2}$. Also $\hat{S}=S$ since otherwise, $\hat{z}^1=w-\hat{S}$ and $z^1=w-S$ imply $\frac{\hat{z}^1_1}{\hat{z}^1_2}\neq\frac{z^1_1}{z^1_2}$, a contradiction. Then, $\hat{z}^1=z^1$. This, by *equal treatment of equals*, implies that $\hat{z}^i=z^i$ for each $i\in N\setminus\{1\}$. To sum up, we showed $G(R^1,\hat{R}^0,R^0,...,R^0)=G(R^1,R^0,R^0,...,R^0)=z$. Sequentially changing the preferences of agencies $3,\cdots,n$ and applying the above arguments, we obtain $G(R^1,\hat{R}^0,...,\hat{R}^0)=z$.

Now, let $\hat{R}^1\in\mathcal{R}_\epsilon$ be such that $d\ \hat{P}^1\ z^1$. Let $\overline{z}^1\in[z^1,w]$ be such that $\overline{z}^1\widehat{I}^1 d$ and note that $\overline{z}^1>z^1$. If $\hat{R}^0$ was a Leontieff preference relation, we would have $P^1\left(\hat{R}^1,\hat{R}^0,...,\hat{R}^0\right)\cap UC\left(\hat{R}^1,z^1\right)=[z^1,w]$. However, Leontieff preferences are not in our domain. Nevertheless, $\hat{R}^0$ can be chosen arbitrarily close to a Leontieff preference and by doing so, the set $P\left(\hat{R}^1,\hat{R}^0,...,\hat{R}^0\right)\cap UC\left(\hat{R}^1,z^1\right)$ can be approximated to $[z^1,w]$. By *strategy proofness* and $G(\hat{R}^0,...,\hat{R}^0)=d$, we also have $G^1(\hat{R}^1,\hat{R}^0,...,\hat{R}^0)\ \hat{R}^1\ d$. Thus by *efficiency*, $G^1(\hat{R}^1,\hat{R}^0,...,\hat{R}^0)$ is arbitrarily close to a member of $[\bar{z}^1,w]$. But then, by strict monotonicity, $G^1(\hat{R}^1,\hat{R}^0,...,\hat{R}^0)\ P^1\ G^1(R^1,\hat{R}^0,...,\hat{R}^0)=z^1$. This contradicts *strategy proofness* of G.

Since G does not simultaneously satisfy *efficiency, strategy proofness*, and *equal treatment of equals*, so can not F. ■