On the Dynamics of Extremist Violence

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Abstract

Many modern armed conflicts contain more than two fighting parties, or armed opposition groups that have factions within them. It is the moderates in an armed opposition that governments negotiate with. But the agreement's fate depends on the approval of all other significant actors within the opposition. We construct a dynamic model of conflict in which such an actor is to decide whether to accept a peace agreement signed by the moderates or not. Using this model we analyze the behavior of our decision maker, focusing on outcomes like the optimal settlement strategy, expected duration of the conflict, and the decision maker's expected payoff from conflict. We then determine how these outcomes are affected by changes in the conflict environment. Finally, we extend our model to analyze the implications of commitment problems, and the possibility that the conflict ends with military victory of either side.

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1 Introduction

In the summer of 2014 the world witnessed yet another episode of the "world's most intractable conflict" (Page Fortna 2004) when the kidnapping and killing of three Israeli students by Hamas members escalated into a full blown battle between Hamas and Israeli forces. The episode lasted for seven weeks during which, according to the Israeli Defense Forces (IDF), 4564 rockets and mortars were launched from Gaza into Israel. In response Israel attacked 5263 sites in Gaza through air strikes and naval bombardment, and conducted a massive ground offensive to destroy Gaza's tunnel system. The international community's desperate attempts to broker a ceasefire agreement finally bore fruit on the last days of August. Unfortunately the death toll was already over 2200 then along with nearly 12000 injured.

The Israeli-Palestinian conflict has been going on since the mid 20th century despite efforts on both sides and on the part of the international community to find a peaceful solution. The high hopes of the early 90s following the Oslo Peace Accords signed between the PLO and the Israeli government were shattered thanks to the relentless efforts of the extremists on both sides to sabotage the peace process. It was then the world first heard about Hamas. Hamas was established in 1987, and has its origins in Egypt's Muslim Brotherhood movement. Its military branch was founded in 1991. The first series of Hamas attacks against Israel began in October 1993, three months after Yasser Arafat, then leader of the Palestine Liberation Organization (PLO), and Yitzhak Rabin, then prime minister of Israel, signed the Oslo Accords which Hamas condemned subsequently. The second series of attacks came a month before Arafat and Rabin signed the Cairo agreement in May 1994. The third set of attacks clustered around the signing of the Israeli-Jordan peace treaty of October 1994. In 1998, when the Wye accord was signed between Netanyahu and Arafat, Hamas resorted to violence again. In 2006 Hamas won a large majority of the seats in the Palestinian Parliament defeating the ruling party Fatah, and since 2007, the territory officially recognized as the State of Palestine is split between Fatah in the West Bank, and Hamas in the Gaza Strip. The latest round of peace negotiations between Israel and the Palestinian National Authority began in July 2013, and were suspended in April 2014 when Fatah and Hamas announced that they had reconciled. And today, after more than twenty years of negotiations and several peace agreements, we are back to square one again talking about casualties, ground offensives, rocket fires and aerial bombardments. A peaceful solution is yet to be found.

Clearly, the problem is not about the lack of negotiations or about an inability to draft agreements. In fact, just as it is the case in the Israeli-Palestinian conflict, peace negotiations are commonly observed in armed conflicts, and agreements do get signed. Harbom et al. (2006) identify 144 accords between warring parties in the 121 armed conflicts active since 1989. Similarly, Walter (1997) argues that between 1940 and 1990, 42 percent of civil wars experienced some form of formal peace negotiation, and in 94 percent of these cases at least a cease-fire accord was drafted. Nonetheless armed conflicts, especially terrorist campaigns and civil conflicts are difficult to end in negotiated settlements. In fact, Kreutz (2010) reveals that a great majority of them end without a decisive outcome. It seems it is at least as complicated and difficult to implement a peace agreement as to draft and sign it. And implementation failures are often followed by the recurrence of the conflict. In some cases, the agreements break down even before key provisions are implemented (DeRouen 2010; Murshed 2002). Researchers have come up with several explanations for why this is so. The most commonly argued ones are power and informational asymmetries, indivisible stakes, bargaining difficulties, opposing identities, and commitment problems (Blattman and Miguel 2010; Fearon 2004; Walter 1997; Walter 2009).

In this article we argue that the heterogenous nature of armed oppositions is another important obstacle against peaceful resolution of conflicts. With very few exceptions, the literature conceptualizes armed conflict as a two-party phenomenon waged between a government and a unitary opposition group, and studies the strategic interaction between this unitary agent and the government it is fighting against. Consequently, whether a conflict ends with a negotiated settlement or not depends on whether the opposition group can come to an agreement with the government. While the unitary agent assumption enables researchers to study conflicts in a parsimonious way and derive important results, it does not match the empirical phenomenon of contemporary conflicts, and thus, risks ignoring important conflict dynamics. Many modern conflicts such as those in Afghanistan, Colombia, Somalia and the former Yugoslavia have contained more than two fighting parties. The Peace Research Institute of Oslo/Uppsala Armed Conflict Database identifies 288 internal conflicts of which 90 contain multiple armed opposition groups at the same time. Moreover, armed opposition groups, be it a terrorist or an insurgent organization, may have (or in response to certain actions by the leadership, may develop) factions within them. And leaders may not always be able to persuade all members that what they have decided is the best course of action for the organization and for the cause.

The long running Turkish-Kurdish conflict presents an interesting case in point. On the last days of 2012, the Turkish Prime Minister revealed that his government had been conducting peace talks with Ocalan, the jailed leader of the Kurdish rebel organization PKK, in an attempt to end the civil conflict that has been going on since 1984. Shortly after, Ocalan was allowed to meet with Kurdish political leaders with whom he sent out letters to the PKK commanders in Turkey and in Europe outlining the peace process he agreed upon with the Turkish government. The process had three steps: a ceasefire accompanied by the withdrawal of PKK guerillas from Turkish soil into Northern Iraq; policy changes to establish a more decentralized government, and to grant certain political and cultural rights to the Kurdish minority in Turkey; and finally, amnesty for PKK members. Upon receiving the letters, PKK leaders announced a ceasefire in March 2013, nevertheless Ocalan's directions for withdrawal was met with less enthusiasm. In an interview conducted late that month, the military leader of the rebels, Karayilan, confessed the resistance in the organization against the deal Ocalan had cut: "We are trying to understand our leader. To tell the truth, we are having difficulty to do so in some respects....While the letter he sent has convinced us to a certain degree, there are issues that we still need to think about. It is not easy for us to decide....The Kurdish Freedom Movement is, for the first time, faced with a conjuncture that allows it to succeed and force resolution on its own. From 2003 to 2011, Turkey, Iran, and Syria formed an anti-Kurdish alliance against us. Now this has changed, the alliance has collapsed. The Kurdish Freedom Movement now has higher maneuverability and a more comfortable base. It is not easy for the PKK to change position and to talk about peace just when the conditions have become such. If it was not for our leader's firm stance no one can make me give up the resistance and withdraw. Nevertheless, as management it is beyond us to persuade everybody to do so". Another high rank PKK commander told in a separate interview: "We need to persuade the guerilla. There is a system in place, a work, a fight that has been going on for years. People came to the mountains for a purpose. It is not easy to undo this. They need to understand. It can not be done with just an order. Of course they are soldiers, but they fight for a purpose, it is not mandatory service. They need to be convinced....Withdrawal is being discussed. It is not just leadership that withdraws. Middle ranks, and the guerilla are having difficulty understanding it....They are asking why". And not surprisingly, in the following days, news about some PKK factions refusing to withdraw appeared in the Turkish media (Milliyet March 30, 2013). In late June, the Turkish prime minister claimed that withdrawal was not progressing as planned and that so far only 15% of PKK forces had withdrawn from Turkish soil.

The fate of a peace agreement depends on the approval of all significant actors in the conflict. In the context of civil conflicts, Licklider (1995) argues that "the commitment of the rebels makes them equally unwilling to abandon the struggle, which has become their life, even if they could get government agreement to their demands and some guarantees that the agreements will be carried out after they disarm". In other words, as the PKK commander argues, "rebels have to be convinced", and this may not always be possible. Some groups within the organization may find the terms of a peace agreement insufficient and decide to continue fighting in the hopes of a better deal. Several important works in the literature acknowledge this heterogenous nature of armed groups, and refer to those factions whom the leadership can not convince as the "extremists". Kydd and Walter (2002) point out how succesful "extremists" are in bringing down peace processes if they so desire, and present Hamas as their leading example.

Mesquita (2005) makes a similar heterogeneity argument for terrorist groups emphasizing that they are not unitary actors. Moderates within a terrorist movement are more likely than extremists are to compromise with the government in exchange for concessions. Mesquita argues that concessions to moderates lead to peace only if extremists lose access to resources after moderates have ceased their participation in violence. The conflict in Northern Ireland is among the examples Mesquita offers. It bears a pattern in which dissident groups split away from mainstream Republican organizations to maintain their dream of a united Ireland. When Michael Collins negotiated and signed the Anglo-Irish treaty in 1921, allowing six counties in the North to remain with Britain as Northern Ireland, those who opposed splitted and continued to fight. Then the Provisional IRA split in 1969 when it broke away from what became known as the Official IRA. In 1986 the Provisionals decided to allow Sinn Fein, their political wing, to run in Parliamentary elections. Some in the organization viewed this decision as an unacceptable compromise. They responded by creating a small, more militant organization named the Continuity IRA. The Real IRA came in 1997 as a response to the Provisional's acceptance of the Good Friday Peace Accords.

Similarly, Cunningham applies a veto-player approach to analyze the effect of the fragmented nature of armed oppositions on the duration of civil conflicts. He defines as veto players those armed groups with divergent preferences and the ability to continue the conflict unilaterally when other parties to the conflict reach an agreement. These groups can be original groups who emerge independently and pursue a separate agenda from other original groups in the conflict, or they may be splinter factions who emerge due to a split within a existing group. Cunningham argues that in the case of peace negotiations veto players will have the incentive to hold out to be the last to sign any agreement in order to get a better deal. As an example he points out the Burundian civil war, in which the splinter factions Palipehutu-FNL and CNDD-FDD refused to participate in peace negotiations unless the government acceded to consider more extreme demands than any of the other parties who signed cease-fire agreements made. Similarly, the Abu Sayyaf group refused to participate in the negotiations held in the 1990s between the Philippians government and the other main insurgent groups.

Following in the footsteps of Mesquita (2009), Kydd and Walter (2012), and Cunningham (2009) we start with arguing that a peace agreement is the result of negotiations between the government and the moderates in an armed opposition. Yet, in order to be implemented, the agreement should be accepted by others in the opposition as well. Stedman (1997) argues that groups will spoil peace agreements when they stand to lose out from them or when they have extreme preferences that do not allow compromise. Kydd and Walter (2012) take a more deterministic stand and build their model on the assumption that extremists will always be against negotiated settlements. In this article, we enrich these arguments by looking into the decision problem of an "extremist" group, or using Cunningham's terminology, a "veto player" who has held out from, and not participated in the peace negotiations. We argue that these groups are decision makers who solve a complicated decision problem rather than automatons who oppose any agreement. Whether extremists accept peace agreements or not depends upon a multitude of parameters including the content of the agreement; their assessments about environmental factors, their opponent's and their own strength; their expectations about the future, and as Mesquita (2009) argues, the extent of resources available to them. In this article, we try to model and understand this decision problem faced by an armed opposition group or faction when they are handed a peace agreement. Given how successful extremist groups are in derailing peace processes, we believe such an understanding is crucial in enabling peaceful resolution of armed conflicts.

Technically, the model is a stochastic dynamic optimization problem where a decision-maker (such as a faction of an armed opposition group) sequentially receives offers on a peace agreement. By accepting the offer, the decision-maker ends the costly conflict but foregoes the chance to receive better offers in the future, since the peace agreement stops the sequential offer process. Rejecting the offer, on the other hand, means extending the costly conflict until the arrival of the next peace agreement. Due to uncertainties that characterize conflict environments and affect the armed group's future bargaining position (such as the outcomes of armed confrontations, how external factors will affect the group's domestic or foreign support, or how the public or the international community will respond to observed conflict outcomes in that period), the decision-maker is uncertain about the nature of future offers. Thus at each period t, the decision to accept an offer involves comparing a sure outcome today to a sequence of uncertain outcomes in the future.

Using this model, we analyze the behavior of the decision maker, focusing on outcomes like the optimal settlement strategy, expected duration of the conflict, and the decision maker's expected payoff from conflict. We then determine how these outcomes are affected by changes in the decision maker's environment, such as its cost of staying in conflict, the frequency of agreements it receives and its uncertainty about how the conflict process will affect future agreements.

Our modeling choice can be seen as complementary to the game theoretic approaches to conflict. Most of the works in this literature, which is reviewed in the next section, treat armed opposition groups as unitary agents. Consequently, once signed by the government and the opposition, peace agreements are assumed to end the game, an assumption which we know does not hold in many real life cases. In this article, we relax this unitary agent assumption to bring an explanation as to why in many cases peace agreements fail to get implemented and end the conflict.

Note that, even though we analyze the behavior of a single decision maker, our model can also be seen as part of a larger two-level game set up in the first stage of which a bargain is strike between the government and the moderates. Then, in the second stage the extremist faction decides whether to accept the peace agreement or to continue fighting. The crucial question here is whether and how the moderates can sign an agreement that would be rejected by the extremist faction. Real life cases, such as the ones we mentioned above, demonstrate that moderates can and do sign agreements that fail to please the extremist factions. As for the why question Kıbrıs (2012) offers an informational asymmetry explanation. She analyzes a very similar setup in the context of domestic ratification of international agreements where two leaders shake hands on an agreement between their states, and then take this agreement home for legislative ratification. Her results demonstrate that if the preferences of the ratifier is private information, and if the ratifier is more extremist than the leader, incentives to misrepresent private information on the part of the ratifier render all communication between the leader and the ratifier ineffective, and create a positive risk that the agreement will not be ratified. In this article we focus on the ratification part of this story in an armed conflict context. The ratifier in our story is an extremist group in an armed opposition who has been handed a peace agreement signed between the moderates in the opposition and the government. We analyze in detail the behavior of this group, and explore the parameters that go into the decision between accepting the offer or continuing with the fight. Note that, in order to correctly anticipate the decision of the extremists, moderates and the government need to be fully informed about the values of all of these parameters. Given the multitude and the nature of the parameters that go into the extremists' decision, we argue that this is a requirement that is quite unlikely to be fulfilled. And Kıbrıs' (2012) result tells us that communication will not help in providing the necessary information either. Israel's dismissal of the 10-year truce in return for the establishment of a Palestinian state in pre-1967 borders proposal by Hamas in 2004 as "insincere and a smokescreen for military preparations" is a good example of the ineffectiveness of such communications.

The paper is organized as follows. In the rest of this section, we discuss the related literature. In Section 2, we present our model. In Section 3, we analyze the benchmark case. We show that the decision-maker's optimal settlement strategy is in the form of an ex-ante fixed minimum acceptable offer, all offers above which are accepted and all below rejected. We use this expression to calculate the average duration of the conflict and the decision maker's expected payoff from conflict. We then analyze how the cost of conflict, the legitimacy of the decision maker as representative of its region and people, and the uncertainties shaping the conflict environment affect the minimum acceptable offer, the average duration of the conflict, and the decision maker's expected payoff from conflict.

The rest of the paper extends our benchmark model in two dimensions that we believe are important in conflict processes. In Section 4, we introduce the possibility that the conflict ends with military victory of either side. In Section 5, we discuss the implications of commitment problems, that is, the possibility that someone will renege on the peace agreement. Section 6 concludes. All formal proofs are presented in the (online) appendix.

1.1 Literature review

Jackson and Morelli (2009), Walter (2009) and Blattman and Miguel (2010) present three excellent surveys of the literature, each with a slightly different emphasis.

Jackson and Morelli (2009) present a comprehensive review of the literature on the causes of interstate and civil wars between rational agents. They emphasize the parties' expectations and bargaining failures as the two important prerequisites for war, and they mention commitment problems as probably the most pervasive reason for bargaining failures. The authors also note that the cause of war (such as commitment problems versus asymmetric information) also affects its duration.

Walter (2009) reviews the literature on bargaining failures in civil wars. She notes that civil wars tend to last longer, end at a higher rate with military victory (since bargains are notably harder to attain), and are more frequent than interstate wars. She says wars last longer if (*i*) information revelation is slow (e.g. such as wars fought with guerilla or terrorist tactics) or (*ii*) there are multiple competing factions and a large number of outside actors that make it hard to locate a common bargaining range, or more importantly if (*iii*) there are commitment problems.

Blattman and Miguel's (2010) review of the literature on the causes and economic consequences of civil wars presents a detailed overview of the empirical literature on civil wars as well as the literature on the micro foundations of conflict (e.g. such as collective action problems or group formation).

As Blattman and Miguel (2010) also mention, the theoretical models on civil war can be classified into two. The first type are contest models. Following Haavelmo (1954) and Hirschleifer (1988;1989), contest models of conflict have been very popular. For a review, see Garfinkel and Skaperdas (2007).

The second type are bargaining models. Depending on how they interpret war, these models can additionally be subclassified into two groups. Costly-lottery models (e.g. Fearon 1995; Powell 1999; Schultz 1999, Smith 1998b) analyze bargaining games where war is simply a lottery which determines the winner. War is thus a game-ending move in these models. These papers focus on the conditions under which the bargaining process fails and leads to war. For example, Fearon (1995) emphasizes information asymmetries, commitment problems, and issue indivisibilities as possible reasons for why bargaining could fail between rational leaders that internalize the costs of war but can not avoid war nonetheless.

Alternatively, costly-process models of bargaining (as named by Powell (2004)) treat war as a costly process during which the states can continue to bargain while they fight.[1] Accordingly war is seen as a bargaining process during which the states run the risk of military collapse. Powell claims that costly-process models can also be used to study the factors that determine the duration of war. The literature that adopts the costly-process modelling approach forwards two important factors that determine duration. Asymmetric information (e.g. see Filson and Werner 2002; Powell 2004; Slantchev 2003; Smith and Stam 2004) and commitment problems (e.g. McBride and Skaperdas 2007; Powell 2006; Walter 1997).

Fearon (2004) presents both a theoretical and empirical study of the duration of civil wars. He empirically shows that the duration of a civil war is significantly affected by where it falls among five classes of civil wars. He also analyzes a dynamic game and demonstrates that an increase in the balance of power between the two parties increases the duration of conflict. The model also features commitment problems between the parties.

Bapat (2005) empirically shows that the probability of negotiations in conflicts between governments and insurgencies is a single-peaked function of time and it is maximized at roughly the fourth year of conflict. He then analyzes a game theoretic model to argue that

[1] Powell (2004) argues that costly-lottery models, since they treat war as a game-ending move, are not appropriate to be used to study intrawar conflict and bargaining.

at initial stages of a conflict, the insurgents are more willing to negotiate than the government and, as conflict evolves and the support for the insurgency increases, the insurgents' willingness to negotiate decreases while the government's willingness to negotiate increases.

Acemoğlu, Ticchi, and Vindigni (2010) present a model of agency problems to explain the long average duration of post-World War II civil wars. In their model, a civilian government can successfully defeat armed oppositions only by creating a relatively strong army. But, in weakly institutionalized polities this opens the way for excessive influence or coups by the military. Therefore, civilian governments whose rents are largely unaffected by civil wars then choose small and weak armies that are incapable of ending insurrections, thereby leading to long periods of conflict.

Note that the literature reviewed above is aimed at explaining the inability of unitary, rational actors to "reach a mutually advantageous and enforceable agreement" (Jackson and Morelli 2009, pp.2), and assumes the conflict over once such an agreement is reached. Our contribution to the literature is to point out the possibility of agreement breakdown and conflict recurrence due to the nonunitary, ideologically heterogenous nature of armed opposition groups and to explore the dynamics created by this possibility. We argue that more than often negotiations lead to mutually acceptable peace agreements for those at the table. Nevertheless,whether an agreement puts an end to a conflict depends on whether it gets accepted by all significant factions within the armed opposition.

Our work is closely related to Kydd and Walter's (2012) study on extremist violence in which they present a game theoretic model of the implementation phase of a peace agreement that has already been reached by the moderates in an armed opposition group and the government. In their model the extremists in the opposition prefer to continue with violent acts which, if not effectively suppressed by the moderates, can derail the peace process. Whether or not extremists are successful in doing so depends on the strength of the moderates and the level of trust between the moderates and the government. Note that Kydd and Walter are more interested in understanding the conditions under which extremist violence is likely to derail peace agreements rather than understanding the conditions under which extremists would prefer to do so. Consequently, their model focuses on the interaction and the informational asymmetries between the government and the moderates. The extremists are assumed to prefer sabotaging the peace to accepting the deal, and they act accordingly. We however argue that extremist groups are not automatons that oppose all deals. Whether they do so depends on a multitude of parameters including the content of the agreement; their assessments about environmental factors, their opponent's and their own strength; and their expectations about the future. In that sense, our model complements Kydd and

Walter's work and completes the picture by focusing on the decision problem of an extremist group when handed an already agreed upon peace accord.

2 Model

We consider a group/faction (hereafter referred to as the **decision maker**) within an armed opposition in a conflict situation. Each period, the moderates within the opposition reaches an agreement with the government, and the decision maker receives this agreement ω, a real number which summarizes the decision-maker's period payoff from accepting the conditions in the agreement. In other words, at each period of conflict, the decision-maker receives an offer ω which has been generated through a bargaining process between the moderates in the rebel organization and the government. Note that the agreement might also be produced by a party outside the conflict, such as an international arbitrator. For example, just as we were working on this article, it became headline news that Hamas rejected an Egyptian-brokered Gaza ceasefire agreement (The Guardian July 16, 2014).

The offers are drawn from an **offer distribution** F where $F(\Omega) = prob(\omega \leq \Omega)$ satisfies $F(0) = 0$ and $F(M) = 1$ for some finite maximum offer $M < \infty$. The offer distribution F is meant to capture the uncertainty the decision maker faces. A range of external factors, such as changes in the international politics of the region or unexpected events (such as failed military attacks) can affect the relative bargaining power of parties to the conflict or the legitimacy of their position in public opinion or in the eyes of the outside world. All of these factors have unexpected effects on what the next peace offer will be.[2]

Having received an offer ω, the decision-maker has to choose between two actions. Either it accepts the offer ω and ends the conflict.[3] Or the decision-maker rejects the offer ω and continues with violent tactics. In this case, the conflict flares up again[4] and continues for another period, at the end of which, the decision-maker receives a new offer ω' and once again, faces a choice between accepting it or continuing the conflict for another period.

[2] We do not impose any restriction on the variance of F. Therefore, our analysis covers the rather unrealistic zero-variance case where the agent is fully informed about future offers.

[3] Having accepted a settlement offer does not necessarily end the agent's decision problem forever. For example, when discussing commitment problems we will analyze an extension of the benchmark model where a negotiated peace can be destroyed in the future. In that case, the agent once again faces the same decision problem.

[4] In other words, we assume that the decision maker has the capability to derail the peace process. That is what makes it a "significant actor". And once the peace agreement is shelved the armed conflict between the rebel organization and the government resumes.

The **conflict payoff** $c \geq 0$ represents the decision-maker's payoff in a period with conflict. It is meant to be a summary statistic for all possible implications of conflict, including losses incurred due to violence. Following Mesquita's (2005) arguments this parameter can also be thought as a measure of how much access extremists have to resources after moderates shake hands with the government and cease their militant activities. Depending on the case these resources may also include revenues from the trade of natural resources, income from drug and/or human trafficking, transfers from the diaspora, *et cetera*. To make the model nontrivial, we assume that the conflict payoff c is not better than the best possible offer, M, that is, we assume $c \leq M$. (Note that since $c \geq 0$, the conflict payoff is also not worse than the worst possible offer.)

Let y_t be the decision-maker's payoff at period t. Then, $y_t = c$ for every conflict period, and $y_t = \omega$ for every peace period realized after a peace offer ω.

The decision-maker's problem is to devise a strategy to maximize its **expected payoff**

$$E \sum_{t=0}^{\infty} \beta^t y_t$$

where $\beta \in (0, 1)$ is the decision-maker's **patience level**. The parameter β is negatively correlated with the amount of time between two peace offers. In a case where peace offers are frequent (and the time between them, small) the parameter β is close to 1. On the other hand, if a long time passes from one peace offer to the next, it is appropriate that β is small.

Note that the decision-maker modeled above is risk-neutral. The case of a risk-averse decision-maker, though interesting, is out of the scope of this paper and left for future research.

3 The Benchmark Case

In this section, we assume that

A-1 the conflict can only end by a peace settlement, and

A-2 a peace settlement can not be broken in a future date.

In the following sections, we will analyze the implications of dropping each assumption.

Let $v(\omega)$ be the expected value of $\sum_{t=0}^{\infty} \beta^t y_t$ for a decision-maker who has received the offer ω, who is about to decide whether to accept or reject it, and who behaves optimally.

$v(\omega)$ can then be written as

$$v(\omega) = \max\left\{\frac{\omega}{1-\beta}, c + \beta Ev\right\} \tag{1}$$

where $Ev = \int_0^M v(\omega')\, dF(\omega')$ is the excepted value of a future offer, given that the uncertainties in the conflict environment is summarized by F. More specifically, it is the decision maker's expected payoff from rejecting the current offer and acting optimally in response to the next offer. Note that the maximization involves choosing between two options: Since by **A-2**, a peace agreement ω means that the decision-maker will receive ω in every future period, and since future payoffs are discounted by β, accepting the offer ω gives the expected payoff

$$\omega + \beta\omega + ... + \beta^t\omega + ... = \frac{\omega}{1-\beta}.$$

Alternatively, rejecting ω gives the conflict payoff c today and the expected discounted payoff βEv tomorrow. Note that **A-1** is important in shaping this expression. If there was a possibility that the conflict can end with violence, that would have to appear in this expression (as will be the case in Section 4).

Note that $c + \beta Ev$ is independent of ω and $\frac{\omega}{1-\beta}$ is increasing in ω. Therefore, there is a unique **minimum acceptable offer** ω^* such that

$$\frac{\omega^*}{1-\beta} = c + \beta Ev. \tag{2}$$

For $\omega > \omega^*$, the unique optimal action for the decision-maker is to accept the peace offer and end the conflict. However, offers such that $\omega < \omega^*$ are rejected for being too low. In these cases, the decision-maker continues conflict for another period in the hope of receiving a better offer. Finally, at $\omega = \omega^*$, both actions yield the same payoff. That is, the solution to Equation 1 can be written as follows:

$$v(\omega) = \begin{cases} c + \beta Ev & if \quad \omega \leq \omega^*, \\ \frac{\omega}{1-\beta} & if \quad \omega \geq \omega^*. \end{cases} \tag{3}$$

The following proposition summarizes the decision-maker's optimal strategy.

Proposition 1 *The decision-maker's optimal strategy is to set a minimum acceptable offer ω^* and accept a peace offer if and only if it is as high as ω^*. This minimum acceptable offer uniquely satisfies*

$$\omega^* - c = \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\, dF(\omega). \tag{4}$$

The decision-maker's expected payoff from following this strategy is

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega)\right). \tag{5}$$

Finally, under this strategy the average duration of the conflict is

$$\frac{1}{1-F(\omega^*)}. \tag{6}$$

In Equation 4, the left hand side represents the cost of continuing conflict for another period. The right hand side represents the discounted expected gains from rejecting the peace offer. The minimum acceptable offer ω^* is then the offer which equates the cost of continuing conflict to the expected gains from it.

It is useful to note that the last term in the decision maker's expected payoff from conflict (Equation 5), $\int_0^M \omega dF(\omega)$, is the expected peace offer. The expression $\frac{1}{1-\beta}\int_0^M \omega dF(\omega)$ corresponds to receiving this mean payoff every period. Depending on F and ω^* however, the decision-maker receives a premium above this payoff, as can be observed in the first term.

Interestingly, we often hear minimum acceptable offer declarations from armed opposition groups. PKK's "democratic confederalism" condition is one example. Hamas' insistence on a two-state solution with pre-1967 borders and on the right of return for Palestinian refugees is another. Similarly, during 1989-1991 the peace negotiations, Khmer Rouge insisted that Vietnam's complete withdrawal was necessary before any agreement could be reached in the Cambodian conflict.

Equation 6 is the expected number of periods until the decision maker accepts a peace agreement signed by the moderates and the government, and thus it gives the expected duration before the conflict comes to an end. Note that this duration depends on the distribution of possible agreements, and is inversely related to the likelihood of the bargaining sides to agree upon something better than what the decision maker finds minimally acceptable. In other words, the duration of the conflict is determined by the relative bargaining power of the moderates and the government as well as on all the other parameters that determine the minimum acceptable offer for the decision maker.

The following results analyze how the minimum acceptable offer, expected payoff, and duration of conflict are affected by changes in the decision environment.

Proposition 2 *An increase in the decision-maker's payoff from conflict c (or equivalently, a decrease in the cost of conflict) increases the decision-maker's minimum acceptable offer ω^*, increases the average duration of the conflict, and increases the decision-maker's expected* *payoff. Similarly, an increase in the decision-maker's patience level β increases the decision-maker's minimum acceptable offer ω^*, increases the average duration of the conflict, and increases the decision-maker's expected payoff.*

This is an intuitive result. A decrease in the cost of conflict makes it less costly for the decision-maker to reject a peace offer in the hopes of receiving a better one in the future. This increases the decision maker's minimum acceptable offer as well as its expected payoff. Interpreting c as the extent of access to resources, this proposition is in line with Mesquita's (2009) conclusion as well. The more access the extremists have to resources, the more likely we are to witness extremist violence following peace negotiations. Similarly, due to an increase in β the decision-maker will be more likely to wait for a better future offer, thereby increasing its minimum acceptable offer ω^* and expected payoff.

The minimum acceptable offer ω^* also tells us about (i) the probability that the decision maker will accept the peace offer at period t and (ii) the expected value of an accepted offer. An increase in ω^* decreases the probability that the decision maker will accept the offer at period t but at the same time it increases the expected value of an accepted offer for the decision-maker.

We next discuss the implications of two types of change in the offer distribution F. The first, called **first-order stochastic dominance**, is regarding an improvement in the offer distribution. A distribution F stochastically dominates another distribution F' if for every Ω, the probability that F will produce a better offer than Ω is at least as high as the probability that F' will produce a better offer than Ω, that is, $F(\Omega) \leq F'(\Omega)$, with strict inequality holding for at least one Ω. Note that an upward shift in a distribution, increasing its mean, creates a new distribution that first order stochastically dominates the former. But the concept covers a larger class of improvements than a sole increase in mean.

Proposition 3 *An improvement in the offer distribution F in terms of first-order stochastic dominance increases the decision-maker's minimum acceptable offer ω^* and increases its expected payoff.*

This proposition is also intuitive. An increase in the probability of better future offers will in turn increase the decision-maker's expected payoff from rejecting an existing peace offer. In response, the decision-maker will reject more offers, that is, its minimum acceptable offer will increase. The welfare effect, though also intuitive, is less trivial; but it is obtained after a few iterations of the expected payoff expression, Equation 5. Note that the statements (quoted in the Introduction) of the PKK commander about the collapse of the anti-Kurdish alliance among Syria, Iran and Turkey, and its implications for the PKK is an example of such an improvement in the offer distribution from the PKK's point of view.

The second change in the offer distribution that we consider is called a **mean-preserving spread** (Rothschild and Stiglitz, 1970). As its name suggests, a mean-preserving spread increases the probability of extreme offers in a balanced way so that the mean offer remains constant. Particularly, it increases the distribution's variance. Thus, a mean preserving spread represents an increase in the volatility of the conflict environment. Formally, a distribution F' is a mean-preserving spread of another distribution F if (i) $\int_0^M \omega dF'(\omega) = \int_0^M \omega dF(\omega)$ and (ii) for each $0 \leq \Omega \leq M$, $\int_0^\Omega (F'(\omega) - F(\omega)) d\omega \geq 0$.

Proposition 4 *A mean-preserving spread of the offer distribution F increases the decision-maker's minimum acceptable offer ω^* and increases its expected payoff.*

At first glance, this result might not seem very intuitive. After all, a mean-preserving spread increases the probability of very bad offers as well as very good ones. The intuition lies in the fact that the decision-maker can protect itself against an increase in bad offers by simply keeping on rejecting them. The corresponding increase in the probability of good offers, on the other hand, increases the decision maker's expected payoff from holding on for better offers. This in turn increases its minimum acceptable offer ω^*. The welfare effect trivially follows from Equation 5. As noted above, a testable implication of Proposition 4 is that extremist factions become more demanding in more volatile conflict environments.

While the PKK commanders might interpret the situation in Syria and Iraq as the collapse of an anti-Kurdish alliance, one can also argue that the current authority vacuum in these states also increases the uncertainty about the future of their Kurdish minorities. This is especially so now that a number of Islamist fundamentalist terrorist groups like Isis are gaining power feeding off that vacuum. And how those minorities fare might have a significant impact on the bargaining power of the PKK against the Turkish state. In that sense, the civil war in Syria and Iraq can be considered as an environmental factor which leads to a more volatile offer distribution, or in other words, as a change in the offer distribution in the form of a mean preserving spread. Our model then implies that the higher volatility in the environment is a factor contributing to the reluctance of PKK cadres to abide by the agreement their leader had signed.

Note that propositions 3 and 4 do not state how the average duration of the conflict is affected. It turns out that there are effects in both directions and the net effect depends on the structure of the offer distribution F, which is not specified here.

4 Possibility of Military Victory

In this section, we extend the benchmark model to allow the possibility that the conflict will terminate without a peace settlement, that is, with victory of either side. Formally, we will analyze the implications of dropping assumption **A-1** in the benchmark model.

In a given period of conflict, let $\tau \in [0,1]$ be the probability that the conflict will be terminated with violence, that is, with victory of either side. Let $\rho \in [0,1]$ be the probability that the opposition (i.e. the decision maker) is the victor, conditional on the event that the conflict ends with violence. Thus, with probability $\tau\rho$, the conflict ends with the victory of the opposition and with probability $\tau(1-\rho)$, it ends with the victory of the government. Finally, $(1-\tau)$ represents the probability that the conflict will not be terminated in this way in a given period. This parameter is sometimes called the stalemate probability (e.g. see Fearon, 2004). Assume that if the opposition is the victor, the decision maker receives the highest payoff M and if it looses the conflict, it receives 0 in each of the following periods.

Under these assumptions, the optimal payoff of a decision maker who has received an offer ω can be written as follows:

$$v(\omega) = \max\left\{\frac{\omega}{1-\beta}, c + \beta\tau\rho\frac{M}{1-\beta} + \beta(1-\tau)Ev\right\} \tag{7}$$

where $Ev = \int_0^M v(\omega')\,dF(\omega')$. This expression is quite to similar to Equation 1 in the benchmark model. The first term, $\frac{\omega}{1-\beta}$, is the decision maker's payoff from receiving the agreement payoff ω in all foreseeable future. The second term, which is different now, has three parts. The first item c still shows the payoff from conflict in this period. The other two items are about how this period's conflict can end. The item $\beta\tau\rho\frac{M}{1-\beta}$ says that with probability $\tau\rho$, this period ends with the military victory of the decision maker, guaranteeing him the maximum payoff M in all foreseeable future starting from next period. The last term says that with probability $(1-\tau)$ there is a stalemate and having rejected today's agreement and acting optimally in the next period brings the decision maker the expected payoff βEv. Since the decision-maker's expected payoff from the government's victory is zero, it does not appear in the expression.

The analysis is technically similar to the benchmark case and its outcome is summarized in the following proposition.

Proposition 5 *The decision-maker's optimal strategy is to set a minimum acceptable offer* $\omega^*_{\tau\rho}$ *and accept a peace offer if and only if it is as high as* $\omega^*_{\tau\rho}$*. This minimum acceptable* *offer uniquely satisfies*

$$\omega^*_{\tau\rho} - c = \frac{\beta}{(1-\beta(1-\tau))}\left(\tau(\rho M - c) + (1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega)\right). \tag{8}$$

The decision-maker's expected payoff from following this strategy is

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*_{\tau\rho}} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega)\right). \tag{9}$$

Finally, under this strategy the average duration of the conflict is

$$\frac{1}{1 - F\left(\omega^*_{\tau\rho}\right)(1-\tau)}. \tag{10}$$

The interpretations of the optimal strategy and the expected payoff expressions are similar to the benchmark case. The main difference of Equation 8 from the benchmark case is the term $\tau(\rho M - c)$ in the last parenthesis. This term shows the expected gain of the decision maker from rejecting the current offer in case the conflict ends with military victory at the end of the current period. Similarly, the stalemate probability appears in Expression 10 which determines the expected duration of the conflict.

The following propositions discuss how the problem's parameters affect the decision-maker's optimal strategy and welfare. The first proposition is about the effects of the stalemate (or military termination) probability and the probability that the decision maker will be the victor in case of military termination.

Proposition 6 *Let* $\tau, \rho \in [0,1]$. *Fixing* $\tau > 0$, *an increase in the probability of victory of the decision maker,* ρ, *increases the average duration of the conflict, the decision-maker's minimum acceptable offer and its expected payoff from conflict. On the other hand, the effect of an increase in the termination probability* τ *depends on the problem's parameters. If* $\rho M > c + \int_{\omega^*_{\tau\rho}}^M \frac{\left(\omega - \omega^*_{\tau\rho}\right)}{1-\beta} dF(\omega)$, *an increase in* τ *increases the decision-maker's minimum acceptable offer and expected payoff. If* $\rho M < c + \int_{\omega^*_{\tau\rho}}^M \frac{\left(\omega - \omega^*_{\tau\rho}\right)}{1-\beta} dF(\omega)$, *an increase in* τ *decreases the decision-maker's minimum acceptable offer, its expected payoff, and also, the average duration of the conflict. At the knife-edge case where* $\rho M = c + \int_{\omega^*_{\tau\rho}}^M \frac{\left(\omega - \omega^*_{\tau\rho}\right)}{1-\beta} dF(\omega)$, *an increase in* τ *does not affect the decision-maker's minimum acceptable offer and expected payoff, but it decreases the average duration of the conflict.*

The condition in the above proposition compares the decision-maker's expected payoff from a military termination, ρM, to its expected payoff from the continuation of conflict, $c+$

$\int_{\omega^*_{\tau\rho}}^{M} \frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta} dF(\omega)$. If the former payoff is higher, an increase in the termination probability increases the decision-maker's expected payoff from conflict. This makes the decision-maker more demanding. If, however, the latter payoff is higher, an increase in τ decreases the decision-maker's expected from conflict and thus, makes it less demanding.

Note that, if $\rho M > c + \int_{\omega^*_{\tau\rho}}^{M} \frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta} dF(\omega)$, the overall effect of an increase in the termination probability τ on the average duration of conflict depends on unspecified details about the problem's parameters. This is because in the average duration expression, Equation 10, $F\left(\omega^*_{\tau\rho}\right)$ increases and $(1-\tau)$ decreases, making the change in $F\left(\omega^*_{\tau\rho}\right)(1-\tau)$ unspecified. That is, an increase in τ has a direct effect (as seen as a decrease in the term $(1-\tau)$) and an indirect effect through the minimum acceptable offer (as seen as an increase in the term $F\left(\omega^*_{\tau\rho}\right)$) and these two effects work in opposite directions.

It is also interesting to note that an increase in the probability of victory ρ for the decision-maker also increases the average duration of conflict. This is again due to the indirect effect via the minimum acceptable offer. An increase in ρ makes the decision maker more optimistic as a result of which it rejects more offers. This dynamic constituted a major reason behind the reluctance of PKK officials to support and abide by the agreement their leader Öcalan had signed with the Turkish government. As the military commander of PKK argued, the political situation in the Middle East, the political vacuums that formed in Iraq and Syria has provided the Kurdish movement a more comfortable base, higher maneuverability, and thus, a higher probability of success. The declarations of PKK officials we quoted in the Introduction support the predictions of our model. Now that they see victory more likely, PKK ranks find it hard to accept the deal they have found in their hands.

The following proposition discusses the effect of the conflict payoff c and the decision maker's patience level β.

Proposition 7 *Let $\tau, \rho \in [0,1]$. An increase in the decision-maker's payoff from conflict c (or equivalently, a decrease in the cost of conflict) increases the average duration of the conflict, and increases the decision-maker's minimum acceptable offer ω^* as well as its expected payoff. Similarly, an increase in the decision-maker's patience level β increases the decision-maker's minimum acceptable offer ω^*, increases the average duration of the conflict, and increases the decision-maker's expected payoff.*

The following proposition discusses the effect of changes in the offer distribution F.

Proposition 8 *Let $\tau, \rho \in [0,1]$. An improvement in the offer distribution F in terms of first-order stochastic dominance increases the decision-maker's minimum acceptable offer $\omega^*_{\tau\rho}$ and* *increases its expected payoff. Similarly, a mean-preserving spread of the offer distribution F increases the decision-maker's minimum acceptable offer $\omega^*_{\tau\rho}$ and increases its expected payoff.*

5 The Commitment Problem

In this section, we extend the benchmark model to consider cases where the decision-maker is concerned that, due to outside factors, the conflict might re-ignite after an agreement is reached. Formally, we drop assumption **A-2** in the benchmark model which stated that a peace agreement can not be broken in a future date. This is meant to capture cases where the decision-maker is concerned that the bargaining parties can not fully commit to peace. We analyze the effect of such commitment problems on the decision-maker's optimal strategy and expected payoff, as well as the average conflict duration.[5]

Let α be *the probability that the conflict will re-ignite after peace*. For simplicity, we take this probability to be independent of the duration of the peace period. If the conflict re-ignites, the decision-maker incurs the conflict payoff c for the first conflict period and then starts to receive peace offers again. (Assuming that more periods have to pass before peace negotiations start again does not change our findings in any significant way.) Then, if the conflict re-ignites, the decision-maker incurs the conflict payoff c for the first conflict period and then, its discounted expected payoff βEv for the following periods (which assumes an optimal choice between continuing conflict or accepting a new peace offer; maybe more importantly, it also contains the possibility that future peace settlements might again and again be destroyed due to commitment problems). Therefore, the decision-maker's continuation payoff after the re-ignition of conflict is $c + \beta Ev$ where $Ev = \int v(\omega') \, dF(\omega')$ is the decision-maker's expected continuation payoff. In a peace period, this payoff is received with probability α and discounted by β. With probability $(1-\alpha)$, peace survives for another period. In that case, the decision-maker receives the β discounted value $v(\omega)$.

As explained in the previous paragraph, this extension changes the decision maker's optimal payoff expression as follows:

$$v(\omega) = \max\{\omega + \beta(1-\alpha) v(\omega) + \beta\alpha [c + \beta Ev], c + \beta Ev\}. \tag{11}$$

[5]In our model, the decision maker does not face commitment problems itself. Due to time consistency, if it is optimal to accept an offer today, it will never be optimal for the decision maker to go back on that settlement and re-ignite conflict. A richer model with a stochastic state parameter can be used to enrich our model to analyze the implications of this possibility.

The first expression in the parenthesis is the expected payoff of accepting ω: in this case, the decision maker receives ω this period; and the next period, conflict re-ignites with probability α, leading to $\beta(c+\beta Ev)$; or peace survives with probability $(1-\alpha)$, leading to the expected payoff $\beta v(\omega)$. The second term in the parenthesis, $(c+\beta Ev)$, is the expected payoff from rejecting ω, as in the benchmark model.

Solving Equation 11 in a way similar to the previous section, we obtain the following proposition which summarizes the decision-maker's optimal strategy and expected payoff.

Proposition 9 *The decision-maker's optimal strategy is to set a minimum acceptable offer ω_α^* and accept a peace offer if and only if it is as high as ω_α^*. The minimum acceptable offer uniquely satisfies*

$$\omega_\alpha^* - c = \frac{\beta}{1-\beta(1-\alpha)} \int_{\omega_\alpha^*}^{M} (\omega' - \omega_\alpha^*)\, dF(\omega'). \tag{12}$$

The decision-maker's expected payoff from following this strategy is

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c \right). \tag{13}$$

Finally, under this strategy the average duration of a conflict regime is

$$\frac{1}{1-F(\omega_\alpha^*)}. \tag{14}$$

The interpretations of the optimal strategy and the expected payoff expressions are similar to the benchmark case. In Equation 12, the left hand side represents the cost of continuing conflict for another period. The right hand side represents the discounted expected gains from rejecting the peace offer. The minimum acceptable offer ω^* is then the offer which equates the cost of continuing conflict to the expected gains from it. In Equation 13, the term $\int_0^M \omega dF(\omega)$, is the expected peace offer. The remaining terms capture the premium above this payoff, due to properties of ω_α^*, F, c, and α.

Due to a positive probability of the conflict re-igniting, α, there is a positive probability that there will be multiple peace agreements and conflict regimes in between them. The expression in Equation 14 refers to the average duration of any one of these conflict regimes (which will be identical for all due to the stationarity of F).

The following propositions discuss how the problem's parameters affect the decision-maker's optimal strategy and welfare. The first one is on the effect of commitment parameter α.

Proposition 10 *Let $\alpha \in [0,1]$. An increase in the commitment problem α decreases the minimum acceptable offer ω_α^*. Particularly, for each $\alpha > 0$, ω_α^* is smaller than the reservation peace offer ω^* of the benchmark model. As a result, the average duration of a conflict regime is also decreasing in the commitment problem α.*

Proposition 10 states that an increase in commitment problems creates shorter conflict regimes. This finding, while it might seem surprising at first glance, is quite intuitive. As it becomes less likely that the current agreement stays in power for a long time, the decision-maker becomes less selective in the offers that it accepts. As the decision-maker becomes less demanding, the average duration of a conflict period decreases.

The earlier literature (e.g. Walter, 2009) argues that an increase in commitment problems will also increase the expected duration of the conflict. Note that this is not in contradiction with Proposition 10 since the latter concerns with the length of a conflict regime in between two peace agreements. While this period gets shorter as shown in the proposition, the increase in α also means that peace agreements will be broken more frequently and thus, such conflict regimes will be repeated more frequently. Thus, it might very well be that the aggregate duration of the conflict increases while the duration of each individual conflict regime decreases.

Another claim in the literature is that the existence of commitment problems makes a conflict more likely. If we hold that an increase in the decision-maker's conflict payoff makes it more likely to initiate conflict in the first place, then we see that our model does not necessarily support the earlier view. Depending on the specifics of the conflict environment (as summarized in the distribution F) an increase in the commitment problem can decrease (as well as increase) the decision-maker's payoff from conflict and thus, make it less likely to initiate conflict in the first place.

We next discuss the effects of the conflict payoff c and the patience parameter β.

Proposition 11 *Let $\alpha \in [0,1]$. An increase in the decision-maker's payoff from conflict c (or equivalently, a decrease in the cost of conflict) increases the decision-maker's minimum acceptable offer ω_α^* and increases its expected payoff. It also increases the average duration of a conflict regime. Similarly, an increase in the decision-maker's patience level β increases the decision-maker's minimum acceptable offer ω_α^* and increases its expected payoff. It also increases the average duration of a conflict regime.*

The following proposition discusses the effect of changes in the uncertainties shaping the conflict environment, i.e. the offer distribution F.

Proposition 12 *Let $\alpha \in [0,1]$. An improvement in the offer distribution F in terms of first-order stochastic dominance increases the decision-maker's minimum acceptable offer ω_α^* and increases its expected payoff. A mean-preserving spread of the offer distribution F increases the decision-maker's minimum acceptable offer ω_α^* and increases its expected payoff.*

Once again, the specification of F is not precise enough to deduce the net effect of the changes analyzed in Proposition 12 on the average duration of a conflict regime.

6 Conclusion

Armed oppositions are ideologically heterogenous groups. It is the moderates in an armed opposition the governments negotiate with, but the fate of their agreement depends on the approval of all other significant factions within the opposition. In this paper, we constructed a dynamic model of conflict in which such a faction (the decision maker) is to decide whether to accept a peace agreement signed by the moderates or not.

Our model, described in Section 2, is a stochastic dynamic decision problem where at each period, the decision maker receives a peace agreement to end a conflict situation and faces a trade off between ending the costly conflict by accepting the offer and rejecting the offer and holding on for the possibility of receiving a better offer in the future.

In Section 3, we analyze this benchmark model. We first calculate the optimal decision rule of a rational decision maker and show that it is in the form of an ex-ante fixed minimum acceptable offer, all offers above which are accepted and all below rejected. Using this finding, we are able to calculate the expected duration of conflict as well as the decision maker's expected payoff from conflict. Then, we conduct sensitivity analyses to see how changes in the conflict environment impact upon the decision maker and hence the likelihood of peaceful resolution. We show that an increase in the conflict payoff or the decision maker's patience level makes the decision maker more demanding, and at the same time, increases the decision maker's expected payoff from conflict as well as the expected duration of the conflict. Similarly, an improvement in the uncertainty regarding the conflict environment makes the decision maker more demanding and increases its expected payoff from conflict.

In Section 4, we introduce the possibility that the conflict will end with military victory of either side. It turns out that this does not upset any of the findings for the benchmark model. We then show that an increase in the decision maker's likelihood of military victory makes it more demanding, increases its expected payoff from conflict, and increases the expected conflict duration. On the other hand, the effect of an increase in the military termination probability depends on how the decision maker's payoff from a military victory compares to her payoff from the continuation of conflict. If the former (latter) is higher, the decision maker becomes more (less) demanding and its expected conflict payoff increases (decreases).

In Section 5, we introduce the decision maker's concern for commitment problems, modeled as the probability that a party other than the decision maker will violate the terms of the peace agreement and the conflict will reignite. We show that this extension does not upset any of the findings for the benchmark model. Interestingly, an increase in the commitment problem makes the decision maker less demanding and decreases the expected duration of a conflict regime (even though it introduces the possibility that a conflict regime will be re-ignited in future periods).

Our analysis sheds light on one of the many facets of civil conflicts, namely the complicated decision problem faced by a decision maker who is party to it. As our literature review shows, this problem has not received attention previously. Yet, as we argue in the introduction, it is an important component of many real life conflicts. Using our model, we are able to quantify and compare how the optimal decision depends on the many important parameters characterizing a conflict environment. And our analysis produces several empirically testable hypotheses as summarized in the above paragraphs. In this sense, we hope that our work contributes to the extensive literature on civil conflicts and complements previous studies that have focused on other facets.

References

Acemoğlu, Daron, Davide Ticchi, and Andrea Vindigni. 2010. "Persistence of Civil Wars." *Journal of the European Economic Association* 8: 664–676.

Bapat, Navin. 2005. "Insurgency and the Opening of Peace Processes." *Journal of Peace Research* 42: 699–717.

Blattman, Christopher. and Edward Miguel. 2010. "Civil War." *Journal of Economic Literature* 48(1): 3-57.

Bueno de Mesquita, Bruce, and Randolph M. Siverson.1995. "War and the Survival of Political Leaders: A Comparative Study of Regime Types and Political Accountability." *American Political Science Review* 89(4): 841–855.

Bueno de Mesquita, Bruce, James D. Morrow, Randolph M. Siverson, and Alastair Smith. 1999. "An Institutional Explanation of the Democratic Peace." *American Political Science*

Review 93(4): 791–807.

Bueno de Mesquita, Ethan, and Eric S. Dickson. 2007. "The Propaganda of the Deed: Terrorism, Counterterrorism, and Mobilization." *American Journal of Political Science* 51(2): 364–381.

Burdett, Kenneth. 1979. "Unemployment Insurance Payments as a Search Subsidy: A Theoretical Analysis," *Economic Inquiry* 17(3): 333-343.

Burdett, Kenneth, and Tara Vishwanath. 1988. "Declining Reservation Wages and Learning." *Review of Economic Studies* 55(4): 655-665.

Cunningham, David E.. 2006. "Veto Players and Civil War Duration." *American Journal of Political Science* 50(4): 875-892.

Fearon, James. 1994. "Domestic Political Audiences and the Escalation of International Disputes." *American Political Science Review* 88(3): 577–592.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* Summer: 379-414.

Fearon, James D. 2004. "Why do some civil wars last so much longer than others?." *Journal of Peace Research* 3:275–301.

Filson, Darren, and Suzanne Werner. 2002. "A Bargaining Model of War and Peace: Anticipating the Onset, Duration, and Outcome of War." *American Journal of Political Science* 46(4): 819-837.

Garfinkel, Michelle R., and Stergios Skaperdas. 2007. "Economics of Conflict: An Overview." in *Handbook of Defense Economics*, Volume 2, *Defense in a Globalized World*, edited by Todd Sandler, and Keith Hartley, 649–710. Amsterdam and Oxford: Elsevier, North-Holland.

Haavelmo, Trygve. 1954. *A Study in the Theory of Economic Evolution.* Amsterdam: North-Holland.

Hirshleifer, Jack. 1988. "The Analytics of Continuing Conflict." *Synthese* 76(2): 201–33.

Hirshleifer, Jack. 1989. "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success." *Public Choice* 63(2): 101–12.

Jackson, Matthew O., and Massimo Morelli. 2009. "The Reasons for War: An Updated Survey." in *The Handbook on the Political Economy of War,* edited by Chris Coyne. Elgar Publishing.

Kıbrıs, Arzu. 2012. "Uncertainty and Ratification Failure." *Public Choice* 3:439-467.

Ljungqvust, Lars, and Thomas Sargent. 2000. *Recursive Macroeconomic Theory.* MIT Press.

McBride, Michael, and Stergios Skaperdas. 2007. "Explaining Conflict in Low-Income Countries: Incomplete Contracting in the Shadow of the Future." in *Institutions and Norms in Economic Development*, edited by Mark Gradstein and Kai A. Konrad, 141–62. Cambridge and London: MIT Press.

McCall, John J. 1970. "Economics of Information and Job Search." *Quarterly Journal of Economics* 81(4):113-126.

Mortensen, Dale T. 1970. "A Theory of Wage and Employment Dynamics." in *Microeconomic Foundations of Employment and Inflation Theory*, edited by Phelps et al. New York: Norton.

Page Fortna, Virginia. 2004. *Peace Time: Cease-fire Agreements and the Durability of Peace.* Princeton: Princeton University Press.

Powell, Robert. 1999. *In the Shadow of Power.* Princeton: Princeton University Press.

Powell, Robert. 2004. "Bargaining and Learning While Fighting." *American Journal of Political Science* 48(2): 344–361.

Powell, Robert. 2006. "War as a Commitment Problem." *International Organization* 60(1): 169–203.

Rogerson, Richard, Robert Shimer, and Randall Wright. 2005. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature* 43(4): 959-988.

Rothschild, Michael, and Joseph Stiglitz. 1970. "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2(3): 225-243.

Schultz, Kenneth. 1999. "Do Democratic Institutions Constrain or Inform." *International Organization* 53 (Spring): 23-66.

Slantchev, Branislav L. 2003. "The power to hurt: Costly conflict with completely informed states." *American Political Science Review* 97 (1), 123-133.

Smith, Alastair. 1998. "International Crises and Domestic Politics." *American Political Science Review* 92 (September): 623-38.

Smith, Alastair, and Allan C. Stam. 2004. "Bargaining and the Nature of War." *Journal of*

Conflict Resolution 48(6): 783-813.

Stedman, Stephen John. 1997. “Spoiler Problems in Peace Processes.” *International Security* 22(2): 5-53.

Stigler, George. 1961. “The Economics of Information.” *Journal of Political Economy* 69(3): 213-25.

Walter, Barbara. 1997. “The Critical Barrier to Civil War Settlement.” *International Organization* 51(3): 335–64.

Walter, Barbara. 2009. “Bargaining Failures and Civil War.” *American Review of Political Science* 12: 243-261.

7 Appendix (Online only)

This section contains the proofs.

Proof. (Proposition 1) The existence of the minimum acceptable offer and the resulting value function (Equation 3) are already presented in the text. Here, we will derive Equation 4. For this, use Equation 3 to rewrite Equation 2 as

$$\frac{\omega^*}{1-\beta} = c + \beta\left(\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^{M} \frac{\omega}{1-\beta} dF(\omega)\right).$$

Extending the left hand side,

$$\begin{aligned}
\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^{M} \frac{\omega^*}{1-\beta} dF(\omega) &= c + \beta\left(\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^{M} \frac{\omega}{1-\beta} dF(\omega)\right) \\
\omega^* \int_0^{\omega^*} dF(\omega) - c &= \frac{1}{1-\beta} \int_{\omega^*}^{M} (\beta\omega - \omega^*)\, dF(\omega).
\end{aligned}$$

Adding $\omega^* \int_{\omega^*}^{M} dF(\omega)$ to both sides gives

$$\omega^* - c = \frac{\beta}{1-\beta} \int_{\omega^*}^{M} (\omega - \omega^*)\, dF(\omega).$$

To obtain the expected payoff expression, Equation 5, note that by definition

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF(\omega) + \int_{\omega^*}^{M} \omega dF(\omega)\right).$$

It can be rewritten as

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} (\omega^* - \omega)\, dF(\omega) + \int_0^{M} \omega dF(\omega)\right).$$

Integrating by parts,

$$\int_0^{\omega^*} (\omega^* - \omega)\, dF(\omega) = (\omega^* - \omega) F(\omega) \mid_0^{\omega^*} - \int_0^{\omega^*} F(\omega)\, d(\omega^* - \omega) = \int_0^{\omega^*} F(\omega)\, d(\omega).$$

Using this equality, we obtain

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} F(\omega)\, d(\omega) + \int_0^{M} \omega dF(\omega)\right),$$

the desired expression.

To obtain the expression for the average duration of the conflict, let $\xi = F(\omega^*)$ be the probability that a peace offer is rejected. Then the conflict lasts for t periods with probability $prob\,(L = t) = (1-\xi)\,\xi^{t-1}$ (since the first $t-1$ offers are rejected with probability ξ^{t-1} and the last one is accepted with probability ξ). That means, the duration of the conflict is geometrically distributed. Then, the average duration of the conflict is

$$\sum_{t=1}^{\infty} t\,(1-\xi)\,\xi^{t-1} = \frac{1}{1-\xi},$$

the desired expression. ■

Proof. **(Proposition 2)** An increase in c decreases the left hand side in Equation 4. Alternatively, an increase in β increases the right hand side. In either case, however, the left hand side is smaller than the right hand side.

Now, the right hand side of Equation 4 is continuously decreasing in ω^* and the left hand side is increasing in ω^*. Thus, to equate the two sides again, ω^* must increase, thereby decreasing the right hand side and increasing the left hand side continuously.

The increase in ω^* also increases $F(\omega^*)$, thus increasing the average duration of the conflict due to expression 6.

It follows from Equation 5 that an increase in c, by increasing ω^*, leads to an increase in Ev. ■

Proof. **(Proposition 3)** Suppose F' first-order stochastically dominates F. Suppose ω^* is the minimum acceptable offer under F. Then ω^* satisfies Equation 4:

$$\omega^* - c = \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\,dF(\omega)\,.$$

By definition of first-order stochastic dominance, this equation implies

$$\omega^* - c < \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\,dF'(\omega)\,.$$

To compensate, ω^* must increase, decreasing the right hand side and increasing the left hand side, until the two are equal again. Thus, the reservation wage under F', $\omega^{*\prime}$ must satisfy $\omega^{*\prime} > \omega^*$.

To show the welfare effect, note that Equation 5 can be rewritten as

$$\begin{aligned}
Ev &= \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF(\omega) + \int_{\omega^*}^{M} \omega dF(\omega)\right) \\
Ev &= \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF(\omega) + \int_{\omega^*}^{M} \omega dF(\omega) + \int_{\omega^*}^{M} \omega^* dF(\omega) - \int_{\omega^*}^{M} \omega^* dF(\omega)\right) \\
Ev &= \frac{1}{1-\beta}\left(\omega^* + \int_{\omega^*}^{M} (\omega - \omega^*)\,dF(\omega)\right)
\end{aligned}$$

Integrating by parts,

$$\begin{aligned}
\int_{\omega^*}^{M} (\omega - \omega^*)\, dF(\omega) &= (\omega - \omega^*)\, F(\omega) \mid_{\omega^*}^{M} - \int_{\omega^*}^{M} F(\omega)\, d(\omega - \omega^*) \\
&= M - \omega^* - \int_{\omega^*}^{M} F(\omega)\, d(\omega).
\end{aligned}$$

Using this equality, we obtain

$$Ev = \frac{1}{1-\beta}\left(M - \int_{\omega^*}^{M} F(\omega)\, d(\omega)\right).$$

By first-order stochastic dominance and the fact that $\omega^{*\prime} > \omega^*$

$$\int_{\omega^{*\prime}}^{M} F'(\omega)\, d(\omega) < \int_{\omega^*}^{M} F(\omega)\, d(\omega)$$

establishing

$$\frac{1}{1-\beta}\left(M - \int_{\omega^{*\prime}}^{M} F'(\omega)\, d(\omega)\right) > \frac{1}{1-\beta}\left(M - \int_{\omega^*}^{M} F(\omega)\, d(\omega)\right),$$

that is, Ev increases from F to F'. ■

Proof. (Proposition 4) Suppose F' is a mean-preserving spread of F. Suppose ω^* is the minimum acceptable offer under F. We will first rewrite Equation 4 as follows:

$$\begin{aligned}
\omega^* - c &= \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\, dF(\omega) \\
\omega^* - c &= \frac{\beta}{1-\beta}\left(\int_{\omega^*}^{M} (\omega - \omega^*)\, dF(\omega) + \int_{0}^{\omega^*} (\omega - \omega^*)\, dF(\omega) - \int_{0}^{\omega^*} (\omega - \omega^*)\, dF(\omega)\right) \\
\omega^* - c &= \frac{\beta}{1-\beta}\left(E\omega - \omega^* - \int_{0}^{\omega^*} (\omega - \omega^*)\, dF(\omega)\right) \\
\omega^* - (1-\beta)\, c &= \beta E\omega - \beta\int_{0}^{\omega^*} (\omega - \omega^*)\, dF(\omega)
\end{aligned}$$

and integrating by parts gives

$$\omega^* - c = \beta\,(E\omega - c) + \beta\int_{0}^{\omega^*} F(\omega)\, d\omega.$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega^*} (F'(\omega) - F(\omega))\, d\omega \geq 0$. Thus,

$$\omega^* - c \leq \beta\,(E\omega - c) + \beta\int_{0}^{\omega^*} F'(\omega)\, d\omega.$$

Both sides are increasing in ω^*, but the right hand at a smaller rate. Thus, increasing ω^* increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega^{*\prime} - c = \beta\left(E\omega - c\right) + \beta \int_0^{\omega^{*\prime}} F'\left(\omega\right) d\omega$$

implies $\omega^{*\prime} \geq \omega^*$, with strict inequality if $\int_0^{\omega^*} \left(F'\left(\omega\right) - F\left(\omega\right)\right) d\omega > 0$ (that is, if the mean-preserving spread is strict at ω^*).

The welfare effect follows from Equation 5:

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} F\left(\omega\right) d\left(\omega\right) + \int_0^M \omega dF\left(\omega\right)\right).$$

By definition, a mean-preserving spread does not effect $\int_0^M \omega dF\left(\omega\right)$ and increases $\int_0^{\omega^*} F\left(\omega\right) d\left(\omega\right).$ Combining this with the fact that $\omega^{*\prime} \geq \omega^*$ we obtain

$$\frac{1}{1-\beta}\left(\int_0^{\omega^*} F\left(\omega\right) d\left(\omega\right) + \int_0^M \omega dF\left(\omega\right)\right) < \frac{1}{1-\beta}\left(\int_0^{\omega^{*\prime}} F'\left(\omega\right) d\left(\omega\right) + \int_0^M \omega dF'\left(\omega\right)\right),$$

that is, Ev increases from F to F'. ■

Proof. (Proposition 5) In Equation 7, the second expression is independent of ω and the first expression is increasing in ω. Therefore, there is a unique minimum acceptable offer $\omega^*_{\tau\rho}$ such that$v\left(\omega\right) = \max\left\{\frac{\omega}{1-\beta}, c + \beta\tau\rho\frac{M}{1-\beta} + \beta\left(1-\tau\right)Ev\right\}$

$$\frac{\omega^*_{\tau\rho}}{1-\beta} = c + \frac{\beta\tau\rho M}{1-\beta} + \beta\left(1-\tau\right)Ev.$$

Note that

$$Ev = \int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF\left(\omega\right).$$

Thus we have

$$\frac{\omega^*_{\tau\rho}}{1-\beta} = c + \frac{\beta\tau\rho M}{1-\beta} + \beta\left(1-\tau\right)\left(\int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF\left(\omega\right)\right).$$

Extending the left hand side,

$$\begin{aligned}
\int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) &= c + \frac{\beta\tau\rho M}{1-\beta} + \beta\left(1-\tau\right)\begin{pmatrix} \int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) \\ + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF\left(\omega\right) \end{pmatrix} \\
\left(1-\beta\left(1-\tau\right)\right)\int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) &= c + \frac{\beta\tau\rho M}{1-\beta} + \int_{\omega^*_{\tau\rho}}^M \frac{\beta\left(1-\tau\right)\omega - \omega^*_{\tau\rho}}{1-\beta} dF\left(\omega\right) \\
\left(1-\beta\left(1-\tau\right)\right)\int_0^{\omega^*_{\tau\rho}} \omega^*_{\tau\rho} dF\left(\omega\right) &= \left(1-\beta\right)c + \beta\tau\rho M + \int_{\omega^*_{\tau\rho}}^M \left(\beta\left(1-\tau\right)\omega - \omega^*_{\tau\rho}\right) dF\left(\omega\right)
\end{aligned}$$

Adding $(1-\beta(1-\tau))\int_{\omega^*_{\tau\rho}}^{M}\omega^*_{\tau\rho}dF(\omega)$ to both sides gives

$$\omega^*_{\tau\rho}-c=\frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c+\tau\rho M+(1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF(\omega)\right)$$

The derivation of the expected payoff expression, Equation 9, is identical to that in the proof of Proposition 1.

To obtain the expression for the average duration of the conflict, let $\xi=F\left(\omega^*_{\tau\rho}\right)$ be the probability that a peace offer is rejected. Let L be the duration of time until the conflict ends either with a successful peace settlement or with victory of one side.

The conflict lasts for one period with probability $prob(L=1)=\tau+(1-\tau)(1-\xi)$, that is, either the conflict terminates with violence (with probability τ) at the end of the first period, or the first peace offer is accepted (with probability $(1-\tau)(1-\xi)$). Similarly, the conflict lasts for two periods with probability $prob(L=2)=(1-\tau)\xi(\tau+(1-\tau)(1-\xi))$ where $(1-\tau)\xi$ covers the probability that in the first period the conflict does not terminate with violence and the first offer is rejected. The second part of the expression is as explained in the previous case. Generalizing, the conflict lasts for t periods with probability $prob(L=t)=(1-\tau)^{t-1}\xi^{t-1}(\tau+(1-\tau)(1-\xi))$. Then, the average duration of the conflict is

$$\begin{aligned}
\sum_{t=1}^{\infty}t(1-\tau)^{t-1}\xi^{t-1}(\tau+(1-\tau)(1-\xi)) &= \\
(\tau+(1-\tau)(1-\xi))\sum_{t=1}^{\infty}t(1-\tau)^{t-1}\xi^{t-1} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{t=1}^{\infty}\sum_{k=1}^{t}(1-\tau)^{t-1}\xi^{t-1} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{k=0}^{\infty}\sum_{t=1}^{\infty}(1-\tau)^{t-1+k}\xi^{t-1+k} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{k=0}^{\infty}\frac{(1-\tau)^{k}\xi^{k}}{1-(1-\tau)\xi} &= \\
\frac{(\tau+(1-\tau)(1-\xi))}{1-(1-\tau)\xi}\sum_{k=0}^{\infty}(1-\tau)^{k}\xi^{k} &= \frac{1}{1-\xi(1-\tau)}
\end{aligned}$$

which is the desired expression. ■

Proof. (Proposition 6)

Effect of ρ: An increase in ρ increases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must increase, increasing the left hand side and decreasing the right hand side of Equation

8. By Equation 9, this increases the decision-maker's expected payoff. By Equation 10, it also increases the average duration of the conflict.

Effect of τ : The derivative of the right hand side of Equation 8 with respect to τ is

$$\frac{\partial\left(\frac{\beta\left(-\tau c+\tau\rho M+(1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF(\omega)\right)}{(1-\beta(1-\tau))}\right)}{\partial\tau}=-\frac{\beta\left(c\left(1-\beta\right)-M\rho\left(1-\beta\right)+\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF\left(\omega\right)\right)}{\left(\beta\tau-\beta+1\right)^2}$$

If $\rho M > c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the sign of this derivative is positive, that is, an increase in τ increases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must increase, increasing the left hand side and decreasing the right hand side of Equation 8. By Equation 9, this increases the decision-maker's expected payoff.

Alternatively if $\rho M < c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the sign of this derivative is negative, that is, an increase in τ decreases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must decrease, decreasing the left hand side and increasing the right hand side of Equation 8. By Equation 9, this decreases the decision-maker's expected payoff. By Equation 10, it also decreases the average duration of the conflict.

Finally, if $\rho M = c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the above derivative is zero. Thus, a change in τ has no effect on equations 8 and 9. In Equation 10, it does not affect $F\left(\omega^*_{\tau\rho}\right)$ but decreases the overall expression due to its direct effect. ■

Proof. (Proposition 7) An increase in c or β makes the left hand side smaller than the right hand side in Equation 8. To equate the two sides, one has to increase $\omega^*_{\tau\rho}$, which increases the left hand side and decreases the right hand side continuously. The new value of $\omega^*_{\tau\rho}$ is thus greater as a result of an increase in c or β.

The increase in $\omega^*_{\tau\rho}$ also increases $F\left(\omega^*_{\tau\rho}\right)$, thus increasing the average duration of the conflict due to expression 10.

It follows from Equation 9 that an increase in c or β, by increasing $\omega^*_{\tau\rho}$, leads to an increase in Ev. ■

Proof. (Proposition 8) Suppose F' first-order stochastically dominates F. Suppose $\omega^*_{\tau\rho}$ is the minimum acceptable offer under F. Then $\omega^*_{\tau\rho}$ satisfies Equation 8:

$$\omega^*_{\tau\rho}-c=\frac{\beta}{\left(1-\beta\left(1-\tau\right)\right)}\left(-\tau c+\tau\rho M+\left(1-\tau\right)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF\left(\omega\right)\right).$$

By definition of first-order stochastic dominance, this equation implies

$$\omega^*_{\tau\rho}-c<\frac{\beta}{\left(1-\beta\left(1-\tau\right)\right)}\left(-\tau c+\tau\rho M+\left(1-\tau\right)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF'\left(\omega\right)\right).$$

To compensate, $\omega^*_{\tau\rho}$ must increase, decreasing the right hand side and increasing the left hand side, until the two are equal again. Thus, the reservation wage under F', $\omega^{*\prime}_{\tau\rho}$ must satisfy $\omega^{*\prime}_{\tau\rho} > \omega^*_{\tau\rho}$.

For the second claim, suppose F' is a mean-preserving spread of F. Suppose $\omega^*_{\tau\rho}$ is the minimum acceptable offer under F. We will first rewrite Equation 5 as follows:

$$\begin{aligned}
\omega^*_{\tau\rho} - c &= \frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c + \tau\rho M + (1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega)\right) \\
\omega^*_{\tau\rho} - c &= \frac{\beta}{(1-\beta(1-\tau))}\begin{pmatrix} -\tau c + \tau\rho M + (1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega) \\ +(1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega) - (1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega)\end{pmatrix} \\
\omega^*_{\tau\rho} - c &= \frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c + \tau\rho M + (1-\tau)\left(E\omega - \omega^*_{\tau\rho}\right) - (1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right) dF(\omega)\right)
\end{aligned}$$

and integrating by parts gives

$$\omega^*_{\tau\rho} - c = \beta\left(\tau\rho M + (1-\tau) E\omega - c + (1-\tau)\int_0^{\omega^*_{\tau\rho}} F(\omega)\, d(\omega)\right).$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega^*_{\tau\rho}}\left(F'(\omega) - F(\omega)\right) d\omega \geq 0$. Thus,

$$\omega^*_{\tau\rho} - c \leq \beta\left(\tau\rho M + (1-\tau) E\omega - c + (1-\tau)\int_0^{\omega^*_{\tau\rho}} F'(\omega)\, d(\omega)\right)$$

Both sides are increasing in $\omega^*_{\tau\rho}$, but the right hand at a smaller rate. Thus, increasing $\omega^*_{\tau\rho}$ increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega^{*\prime}_{\tau\rho} - c = \beta\left(\tau\rho M + (1-\tau) E\omega - c + (1-\tau)\int_0^{\omega^{*\prime}_{\tau\rho}} F'(\omega)\, d(\omega)\right)$$

implies $\omega^{*\prime}_{\tau\rho} \geq \omega^*_{\tau\rho}$, with strict inequality if $\int_0^{\omega^*_{\tau\rho}}\left(F'(\omega) - F(\omega)\right) d\omega > 0$ (that is, if the mean-preserving spread is strict at $\omega^*_{\tau\rho}$).

The proof of the welfare effect is identical to that in the proofs of propositions 3 and 4.

■

Proof. (Proposition 9) In Equation 11, the second term is constant in ω. Postulating that $v(\omega)$ is increasing in ω, we obtain the following optimal action in the minimum acceptable offer form. Let ω^* be the minimum acceptable offer. For $\omega \leq \omega^*$ then, $v(\omega) = c + \beta E v$. For $\omega \geq \omega^*$ on the other hand, we have

$$v(\omega) = \omega + \beta(1-\alpha) v(\omega) + \beta\alpha\left[c + \beta E v\right]$$

which simplifies to

$$v(\omega) = \frac{\omega + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1-\alpha)}.$$

Note that this expression is increasing in ω, which is consistent with our postulate. Thus, the resulting value function is:

$$v(\omega) = \begin{cases} c + \beta Ev & if \quad \omega \leq \omega_\alpha^* \\ \frac{\omega + \beta\alpha[c+\beta Ev]}{1-\beta(1-\alpha)} & if \quad \omega \geq \omega_\alpha^* \end{cases} \tag{15}$$

where ω_α^* equates the two expressions:

$$\frac{\omega_\alpha^* + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1-\alpha)} = c + \beta Ev.$$

Solving this equality, we obtain

$$\omega_\alpha^* = (1-\beta)(c + \beta Ev).$$

By definition,

$$\begin{aligned} Ev &= \int_0^{\omega_\alpha^*} v(\omega)\, dF(\omega) + \int_{\omega_\alpha^*}^M v(\omega)\, dF(\omega) \\ Ev &= \int_0^{\omega_\alpha^*} \frac{\omega_\alpha^* + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1-\alpha)} dF(\omega) + \int_{\omega_\alpha^*}^M \frac{\omega + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1-\alpha)} dF(\omega). \end{aligned}$$

Solving it for Ev, we obtain

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \int_{\omega_\alpha^*}^M \omega dF(\omega) + \beta\alpha c \right).$$

Inserting Ev into $\omega_\alpha^* = (1-\beta)(c + \beta Ev)$, we then obtain

$$(1 + \alpha\beta)\,\omega_\alpha^* = (1 + \alpha\beta - \beta)\, c + \beta \int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \beta \int_{\omega_\alpha^*}^M \omega dF(\omega).$$

Adding and subtracting $\beta \int_{\omega_\alpha^*}^M \omega_\alpha^* dF(\omega)$ from the right hand side, the expression simplifies into

$$\omega_\alpha^* - c = \frac{\beta}{(1 + \alpha\beta - \beta)} \int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega),$$

the desired expression for ω_α^*.

To obtain the expected payoff expression, note that

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \int_{\omega_\alpha^*}^M \omega dF(\omega) + \beta\alpha c \right)$$

can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} (\omega_\alpha^* - \omega)\, dF(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c \right).$$

Integrating by parts,

$$\int_0^{\omega_\alpha^*} (\omega_\alpha^* - \omega)\, dF(\omega) = (\omega_\alpha^* - \omega) F(\omega) \mid_0^{\omega_\alpha^*} - \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega_\alpha^* - \omega) = \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega).$$

Using this equality, we obtain

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c \right),$$

the desired expected payoff expression.

The proof for the average duration of the conflict is identical to the proof of Proposition 1. ■

Proof. (Proposition 10) Let $\alpha_1 < \alpha_2$ and let $\omega_{\alpha_1}^*$ and $\omega_{\alpha_2}^*$ be the resulting minimum acceptable offers. By Equation 12,

$$\omega_{\alpha_1}^* - c = \frac{\beta}{(1+\alpha_1\beta-\beta)} \int_{\omega_{\alpha_1}^*}^M \left(\omega - \omega_{\alpha_1}^*\right) dF(\omega)$$

and by $\alpha_1 < \alpha_2$,

$$\omega_{\alpha_1}^* - c > \frac{\beta}{(1+\alpha_2\beta-\beta)} \int_{\omega_{\alpha_1}^*}^M \left(\omega - \omega_{\alpha_1}^*\right) dF(\omega).$$

The left hand side is increasing and the right hand side is decreasing in ω_α^*. By definition, $\omega_{\alpha_2}^*$ solves

$$\omega_{\alpha_2}^* - c = \frac{\beta}{(1+\alpha_2\beta-\beta)} \int_{\omega_{\alpha_2}^*}^M \left(\omega - \omega_{\alpha_2}^*\right) dF(\omega).$$

Therefore, we have $\omega_{\alpha_1}^* > \omega_{\alpha_2}^*$.

Now $\frac{1}{1-F(\omega_\alpha^*)}$ is increasing in ω_α^*. Thus, an increase in α, by decreasing ω_α^*, also decreases the average duration of a conflict regime.

Regarding the decision-maker's expected payoff, Equation 13 can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\omega_\alpha^* + \int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega) + \beta\alpha c \right).$$

Integrating by parts

$$\int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega) = M - \omega_\alpha^* - \int_{\omega_\alpha^*}^M F(\omega)\, d(\omega).$$

Using this equality,

$$\begin{aligned} Ev &= \frac{1}{(1-\beta)(\alpha\beta+1)}\left(M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega) + \beta\alpha c\right) \\ Ev &= \frac{M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega)}{(1-\beta)(\alpha\beta+1)} + \frac{\beta\alpha c}{(1-\beta)(\alpha\beta+1)} \end{aligned}$$

The first part of this expression is decreasing in α. However, the second part is increasing in α. The overall effect of α on Ev thus depends on the relative sizes of these opposite effects. ■

Proof. (Proposition 11) The proof that β and c both increase ω_α^* is identical to the proof of Proposition 2. Since an increase in ω_α^* also increases, $\frac{1}{1-F(\omega_\alpha^*)}$, we also have the effect on the average duration.

It is straightforward to see from Equation 13 that an increase in c increases the decision-maker's expected payoff. The effect of β is less obvious. To see it, note that

$$\frac{\partial\left(\frac{\beta\alpha c}{(1-\beta)(\alpha\beta+1)}\right)}{\partial\beta} = \frac{\alpha c\left(\alpha\beta^2+1\right)}{(\beta-1)^2(\alpha\beta+1)^2} > 0.$$

Thus, an increase in β increases the third term in Equation 13. It also increases the first term since, ω_α^* is increasing in β. Finally,

$$\frac{\partial\left(\frac{1}{(1-\beta)(\alpha\beta+1)}\right)}{\partial\beta} = \frac{(2\alpha\beta-\alpha+1)}{(\beta-1)^2(\alpha\beta+1)^2} > 0.$$

Therefore, an increase in β increases Ev. ■

Proof. (Proposition 12) The proof that first-order stochastic dominance increases ω_α^* is similar to that of Proposition 3. To see that it increases expected payoff, note that Equation 13 can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\omega_\alpha^* + \int_{\omega_\alpha^*}^{M} (\omega - \omega_\alpha^*)\, dF(\omega) + \beta\alpha c\right).$$

Integrating by parts,

$$\int_{\omega_\alpha^*}^{M} (\omega - \omega_\alpha^*)\, dF(\omega) = M - \omega_\alpha^* - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega)$$

and inserting in the previous equality

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)}\left(M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega) + \beta\alpha c\right).$$

By first-order stochastic dominance and the fact that ω_α^* is increasing, the expression $\int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega)$ is decreasing. As a result, the whole expression for Ev is increasing.

Suppose F' is a mean-preserving spread of F. Suppose ω_α^* and $\omega_\alpha^{*\prime}$ are the minimum acceptable offers under F and F', respectively. We will first rewrite Equation 12 as follows:

$$\begin{aligned}
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta} \int_{\omega_\alpha^*}^{M} (\omega - \omega_\alpha^*)\, dF(\omega) \\
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta} \begin{pmatrix} \int_{\omega_\alpha^*}^{M} (\omega - \omega_\alpha^*)\, dF(\omega) \\ + \int_0^{\omega_\alpha^*} (\omega - \omega_\alpha^*)\, dF(\omega) - \int_0^{\omega_\alpha^*} (\omega - \omega_\alpha^*)\, dF(\omega) \end{pmatrix} \\
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta} \left(E\omega - \omega_\alpha^* - \int_0^{\omega_\alpha^*} (\omega - \omega_\alpha^*)\, dF(\omega) \right) \\
(1+\alpha\beta)\,\omega_\alpha^* - (1+\alpha\beta-\beta)\, c &= \beta E\omega - \beta \int_0^{\omega_\alpha^*} (\omega - \omega_\alpha^*)\, dF(\omega) \\
(1+\alpha\beta)\,\omega_\alpha^* - (1+\alpha\beta)\, c &= \beta\,(E\omega - c) + \beta \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) \\
\omega_\alpha^* - c &= \frac{\beta}{(1+\alpha\beta)} \left((E\omega - c) + \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) \right)
\end{aligned}$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega_\alpha^*} (F'(\omega) - F(\omega))\, d\omega \geq 0$. Thus,

$$\omega_\alpha^* - c \leq \frac{\beta}{(1+\alpha\beta)} \left((E\omega - c) + \int_0^{\omega_\alpha^*} F'(\omega)\, d(\omega) \right).$$

Both sides are increasing in ω_α^*, but the right hand at a smaller rate. Thus, increasing ω_α^* increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega_\alpha^{*\prime} - c = \frac{\beta}{(1+\alpha\beta)} \left((E\omega - c) + \int_0^{\omega_\alpha^{*\prime}} F'(\omega)\, d(\omega) \right)$$

implies $\omega_\alpha^{*\prime} \geq \omega_\alpha^*$, with strict inequality if $\int_0^{\omega_\alpha^*} (F'(\omega) - F(\omega))\, d\omega > 0$ (that is, if the mean-preserving spread is strict at ω^*).

The welfare effect of a mean-preserving spread follows from Equation 13:

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^{M} \omega dF(\omega) + \beta\alpha c \right).$$

By definition, a mean-preserving spread does not effect $\int_0^{M} \omega dF(\omega)$ and increases $\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega)$.

Combining this with the fact that $\omega_\alpha^{*\prime} \geq \omega_\alpha^*$ we obtain

$$\begin{aligned} & \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c\right) \\ < \; & \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\int_0^{\omega_\alpha^{*\prime}} F'(\omega)\, d(\omega) + \int_0^M \omega dF'(\omega) + \beta\alpha c\right), \end{aligned}$$

that is, Ev increases from F to F'. ■