A Theory of Reference Point Formation*

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Abstract

We introduce a model of reference-dependent choice where the reference point is endogenously determined through maximization of a *conspicuity ranking*. This subjective ranking captures how prominent or eye-catching the alternatives are relative to one another. The most conspicuous alternative in a choice set serves as the reference point and in turn determines the reference-dependent utility function the decision maker maximizes to make a choice. We show that this *conspicuity based endogenous reference model* (CER) is characterized by an intuitive and simple behavioral postulate, called the Single Reversal, and we discuss how choice data can be used to reveal information about CER's parameters. We additionally analyze special cases where a reference-free utility function, combined with psychological constraints, is used to make reference-dependent choices.

Keywords: Conspicuity, Reference-Dependence, Reference Point Formation, Revealed Preference, Psychological Constraints, Choice Reversal

JEL Codes: D01, D11

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1 Introduction

Starting with the seminal works of Markowitz (1952) and Kahneman and Tversky (1979), the idea of reference-dependence has played a very significant role in economics. Numerous empirical and experimental studies have documented that choices are often reference-dependent. With this motivation, researchers have developed a variety of theoretical models in which an exogenously-given reference point affects choice behavior. However, with the exception of a few studies, this literature remains silent on how the reference point is determined. This has been recognized as a major drawback (*e.g.*, see Markowitz, 1952, Tversky and Kahneman, 1991, Levy, 1992, Wakker, 2010, Barberis, 2013). For example, Wakker (2010, p. 245) argues that *"If too much liberty is left concerning the choice of reference points, then the theory becomes too general and is almost impossible to refute empirically. It does not then yield valuable predictions."* In other words, a full-blown theory of reference-dependence necessitates a theory of reference point formation.

Many studies informally relate the determination of a reference point to a notion of conspicuity (or equivalently, salience) and argue that the "most conspicuous alternative" in a choice set becomes the reference point (Brickman et al., 1978, Samuelson and Zeckhauser, 1988, Pratkanis, 2007, DellaVigna, 2009, Larrick and Wu, 2012, Bhatia and Golman, 2019, Bhatia, 2017).[1] To quote Bhatia and Golman (2019), *"reference points are merely options that are especially salient to the decision maker."* For example when purchasing an airline ticket, many consumers sort alternatives according to a criterion most important to them (*e.g.*, price or the duration of the flight), and the top of this list (*e.g.*, the cheapest or the fastest flight) might then act as a reference point when consumers evaluate other options.[2] Similarly, in online platforms like Amazon, the best reviewed or the most purchased alter-

[1] In addition, many empirical findings suggest that conspicuous alternatives are more likely to attract attention and affect decision-making (*e.g.*, see Lohse, 1997, Milosavljevic et al., 2012).

[2] In the marketing literature, price is known to be a particularly important criterion for conspicuity (*e.g.*, see Winer, 1986, Kalyanaram and Winer, 1995, Erdem et al., 2001).

native might serve as a reference point. Note that such reference points are generated by rankings (price, quality, or reviewer rating) that are context independent. In addition, identifying these top ranked alternatives can be effortless in many settings. For example, many online platforms allow consumers to sort available products according to various criteria. Our objective is to formalize this intuition to offer a theory of endogenous reference point formation and analyze its behavioral implications.

In our model, the decision maker is endowed with a subjective *conspicuity ranking* over the alternatives. Subjectivity of the conspicuity ranking is motivated by observations that individuals might differ in their criteria for conspicuity, and typically, analysts do not directly observe an individual's conspicuity ranking.[3] For example, for a price-conscious customer, a cheaper product might be more conspicuous. In general, the conspicuity ranking of a product might depend on multiple attributes, some of which can potentially be irrelevant for its valuation, such as the size or the color of its package (*e.g.*, see Milosavljevic et al., 2012). The conspicuity ranking is the first component of our model.[4] Our key assumption is that the conspicuity ranking is context independent. As stated in the previous paragraph, this assumption is valid in a variety of settings. Furthermore, allowing context-dependent conspicuity rankings results in a theory with very little predictive power.

The second component of our model is a set of reference-dependent utility functions. We do not impose any particular functional form on the utility functions. Generality in the utility component allows our model to encompass a wide range of reference-dependent models proposed in the literature, including those of Tversky and Kahneman (1991), Munro

[3]Subjectivity of the conspicuity ranking is in line with experimental findings which show that different individuals facing the same decision environment might end up with distinct reference points. For example, Terzi et al. (2016) present experimental data that demonstrates heterogeneity among individuals in the reference points they employ.

[4]A related yet different notion is discussed in Bordalo et al. (2013). In their framework, each product has different attributes, and depending on the context and the reference point, one of the attributes becomes salient and receives a higher weight in the final evaluation. Thus, in their model, it is the reference point that determines salience of an *attribute*. Conversely, in our model, the conspicuity ranking determines the reference point. Hence, the two approaches are conceptually different.

and Sugden (2003), Masatlioglu and Ok (2005, 2014), Sagi (2006), Kőszegi and Rabin (2006), and Bordalo et al. (2013).

We are now ready to define the choice procedure of our agent. Given a choice problem S, the alternative which maximizes the conspicuity ranking ($\gg$) serves as the reference point ($r(S)$). Next, given the reference point $r(S)$, the agent maximizes the reference-dependent utility function $U_{r(S)}$ in S to make a choice. This model, summarized in Figure 1, is called the *Conspicuity based Endogenous Reference model* (hereafter, **CER**). In the coming sections we study the basic properties of CER, as well as its economic implications, and discuss to what extent its ingredients can be inferred from choice data.

$$S \longrightarrow r(S) \longrightarrow \max_{x \in S} U_{r(S)}(x) \longrightarrow c(S)$$

Choice Set	Most Conspicuous Alternative	Reference-Dependent Utility Maximization	Choice

Figure 1: Conspicuity based Endogenous Reference model (CER)

The first contribution of our paper is the concept of a *conspicuity ranking* through which reference point formation is endogenized. To highlight the significance of this innovation, consider the constant loss aversion model of Tversky and Kahneman (1991). Due to its tractable form, this highly celebrated reference-dependent model is widely used in applications. Yet it has also been criticized on the basis that it cannot accommodate well-known behavioral patterns such as the attraction and compromise effects due to fact that this model is rational given an exogenous reference point.[5] However, a CER with the constant loss aversion utility function of Tversky and Kahneman (1991) can accommodate both. Figure 2 (left) presents an example with three alternatives $\{A, B, C\}$, where the conspicuity ranking is $A \gg B \gg C$

[5] The compromise effect refers to a phenomenon that consumers tend to choose the compromise alternative of a choice set more frequently than extreme alternatives (see Simonson, 1989, Simonson and Tversky, 1992). The attraction effect refers to findings that when an alternative is presented together with a strictly inferior alternative, its choice probability increases (see Huber et al., 1982, Ratneshwar et al., 1987).

and the constant loss aversion parameter is $\lambda = 2$, as commonly used in the literature. In the figure, B is chosen over C when only these two alternatives are available. However, the choice switches from B to C when (and only when) a third alternative A is added to the colored regions. Note that these regions are predominantly consistent with the underlying motivations for the attraction and compromise effects.

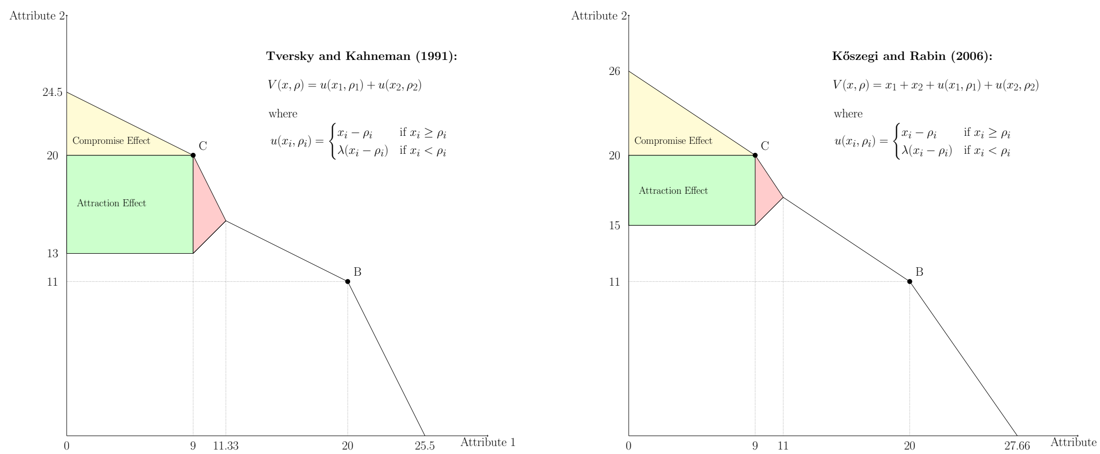

Figure 2: Attraction and compromise effects when CER is based on two well-known reference-dependent utility functions. The conspicuity ranking is $A \gg B \gg C$ and the loss aversion parameter is $\lambda = 2$.

We should point out that this improvement is not just a result of the endogenization of the reference point. To clarify this point, consider Kőszegi and Rabin (2006) who introduced a new endogenous reference-dependent choice model (called Preferred Personal Equilibrium). Their Proposition 3 shows that if the utilities are linear, the implied behavior of their model is identical to the classical model. Hence, similar to Tversky and Kahneman (1991), Kőszegi and Rabin (2006) cannot accommodate attraction and compromise effect for linear utilities. However, with their underlying utility functions, our reference-point formation process can replicate intriguing choice patterns such as the attraction and compromise effects (see Figure 2, right).

In addition to attraction and compromise effects, our model can also accommodate cyclical binary choices. To illustrate, consider a consumer who chooses an alternative A over B, B over C, and C over A. This choice pattern was first experimentally demonstrated by May (1954) and has been replicated in numerous choice environments (e.g., see Tversky, 1969, Loomes et al., 1991, Manzini and Mariotti, 2009, Mandler et al., 2012). Our explanation for this choice pattern uses the idea that the endogenous reference point of the consumer might be different in different choice problems. Overall, the choice of reference point formation process has the potential to improve the performance of existing models significantly by allowing them to accommodate additional (seemingly anomalous) choice patterns.

It is useful to note that we do not view our model as the model to explain these behavioral patterns. For example, going back to Figure 2, if C is chosen from the choice set $\{A, B, C\}$ but not $\{B, C\}$, our model requires that C must be chosen against A in a binary comparison. While this requirement is consistent with the attraction effect, it is not necessarily consistent with a stronger version of the compromise effect where C is chosen only if it is a compromise between A and B. Hence, our model accommodates only certain types of the attraction and compromise effect phenomena. Nevertheless, our main goal in this paper is to provide a parsimonious theory of reference formation, which follows a clear and intuitive procedure. One of the advantages of our model is that we do not assume observability of attributes. Hence, our model can be applied to economically important questions where the relevant dimensions of the problem are not exogenously given, such as job search and/or voting.

Our second contribution is that we allow the conspicuity ranking to be *subjective* (that is, to depend on the decision maker's individual characteristics) and we show how this subjective ranking can be *inferred from choice data*. This inference relies on an important feature of our model concerning choice reversals. A *choice reversal* is said to occur when the elimination of an unchosen alternative affects the choice.[6] In our model such reversals can only be induced

[6]Formally, the statement "x induces a choice reversal in S" can be stated as $x \neq c(S) \neq c(S \setminus x)$ where $c(S)$ is the choice from S.

by the elimination of the most conspicuous alternative in a choice set. This feature allows us to infer the conspicuity ranking from observed choices.

To better understand CER, we explore its behavioral implications. It turns out that one intuitive and simple behavioral postulate, which we call *Single Reversal Axiom*, fully characterizes CER. This axiom is motivated by the aforementioned observation on how CER regulates choice reversals. Single Reversal Axiom requires that if there is a choice problem where an alternative x causes a choice reversal when y is available, there cannot be a choice problem where y causes a choice reversal when x is available.[7] Single Reversal Axiom allows for at most one choice reversal in each choice set. Since the classical Weak Axiom of Revealed Preference (WARP) does not allow any choice reversals, Single Reversal can be thought of as the minimal deviation from it. Overall, CER enjoys an intuitive and simple axiomatic foundation that provides a clear picture on what type of choice behavior CER can address and enables the design of simple experiments to test its validity.

One important criticism of reference-dependent choice models is that each reference point induces a new utility function as if there is a new self and this makes welfare comparisons across different reference points problematic. To address this criticism, Masatlioglu and Ok (2014) propose a model where there is a single utility function applied under all reference points, but each reference point induces a "psychological constraint" which eliminates some alternatives from consideration. In Section 4, we analyze a special case of CER where choices are made by the procedure of Masatlioglu and Ok (2014). This special case, called the *Psychologically-Constrained Conspicuity based Endogenous Reference model* (PC-CER), is summarized in Figure 3 (where U is the reference-free utility and $Q(r(S))$ is the set of alternatives which are not eliminated by the reference point $r(S)$).

We analyze the behavioral implications of PC-CER as well. It turns out that our *Consis-*

[7] When stated in terms of "revealed conspicuity," Single Reversal means that if x is revealed to be more conspicuous than y, then y cannot be revealed to be more conspicuous than x.

$$\begin{array}{ccccccc} S & \longrightarrow & r(S) & \longrightarrow & \max\limits_{x \in S \cap Q(r(S))} U(x) & \longrightarrow & c(S) \\ \text{Choice Set} & & \text{Most Conspicuous Alternative} & & \text{Constrained Utility Maximization} & & \text{Choice} \end{array}$$

Figure 3: Psychologically Constrained CER (PC-CER)

tency Axiom, together with Single Reversal, fully characterizes PC-CER. Consistency simply states that the revealed (reference-free) preference of this model has no cycles. This characterization also helps us to compare our study with the previous literature, including Manzini and Mariotti (2007), Masatlioglu et al. (2012), Ok et al. (2015), Masatlioglu et al. (2020), Lleras et al. (2017).

To clarify the contribution of our paper to the literature, we provide a brief discussion of existing reference-dependent models (see Section 5 for a more detailed discussion). The earliest strand of literature on this specification treats the reference point as exogenous (*e.g.*, Tversky and Kahneman, 1991, Munro and Sugden, 2003, Sugden, 2003, Sagi, 2006, Salant and Rubinstein, 2008). Later studies endogenize reference-point formation. In models of Bodner and Prelec (1994), Kivetz et al. (2004), Orhun (2009), Bordalo et al. (2013), and Tserenjigmid (2019), the reference point depends on the structure of the choice set but it is independent of individual characteristics. Thus, these models analyze environments where all decision makers facing the same choice problem have the same reference point. Alternatively, Kőszegi and Rabin (2006) and Freeman (2017) analyze models where the endogenous reference point can differ across individuals. However, in their models, choice always coincides with the reference point. In contrast, our model allows the reference point to differ across individuals and to be distinct from the chosen alternative.

Another strand of literature replaces reference-dependent utilities with a reference-free utility function combined with psychological constraints. This special case of reference-dependent choice is important for welfare comparisons, as mentioned earlier (and further discussed in Section 4). Most studies in this strand of the literature treat the reference point as exogenous (*e.g.*, Masatlioglu and Ok, 2005, 2014, Apesteguia and Ballester, 2009, Masatlioglu and Nakajima, 2013, Dean et al., 2017).[8] One exception is Ok et al. (2015) where the reference point is determined endogenously but it is assumed to be distinct from the chosen alternative. PC-CER is closely related to this strand of literature: it endogenizes reference point formation with a reference-free utility and psychological constraints, and, as opposed to Ok et al. (2015), allows the reference point to coincide with the chosen alternative.

We would like to point out that a model closely related to CER was first studied by Rubinstein and Salant (2006) under the name of Triggered Rationality. While their motivation is distinct from ours, their model can be interpreted as endogenous reference dependent choice. Lim (2020) also studies the same model and generalizes their result to other domains. The main axiom of Rubinstein and Salant (2006) and Lim (2020), Reference Point Property, is significantly different from our Single Reversal Axiom.[9] In addition, while the Single Reversal Axiom can be falsified by using only two observations, Reference Point Property is an existence axiom that requires many observations to falsify, and hence it is harder to test empirically. Lastly, we also consider refinements of the general model where the DM is endowed with a reference free utility function and reference dependent psychological constraints. These refinements help us make welfare evaluations in the presence of reference dependence, and they also impose more structure on observed choices.

Our model is consistent with three well-known behavioral patterns frequently observed in empirical studies, namely, the Compromise Effect, the Attraction Effect, and Cyclical Choice. None of the other studies listed above can accommodate all three. Our study is also

[8] Maltz (2020) presents a hybrid model which combines an exogenous reference point (the endowment) with endogenous reference-point formation. In this model, alternatives are partitioned into categories and the most-preferred feasible alternative in the category that contains the endowment serves as the endogenous reference point. Our study is complementary to this model as we do not assume exogenous reference points.

[9] Reference Point Property states that for any S there exists $x \in S$ such that the choice function satisfies the independence of irrelevant alternatives property for all subsets of S that contain x, i.e. if $x \in S'' \subset S' \subset S$ and $c(S') \in S''$, then $c(S') = c(S'')$.

unique in the sense that it characterizes the distinction between these two types of models. To elaborate, a comparison of Theorem 1 and Theorem 2 shows that (in the confines of our framework) this distinction can be characterized by the Consistency Axiom, which is equivalent to acyclicity of the revealed (reference-free) preference in PC-CER.

The rest of the paper is organized as follows. In Section 2, we introduce our model and present the representation theorem. In Section 3, we show how the primitives of our model are revealed from observed choices. In Section 4, we discuss a special case of our model where psychological constraints introduce additional structure on the reference-dependent utility functions. In Section 5, we discuss the related literature. We conclude in Section 6.

2 Conspicuity and Reference-Dependence

Let X be a finite set of alternatives, and denote by $\mathcal{X}$ the set of all nonempty subsets of X. A *choice problem* is a set of alternatives $S \in \mathcal{X}$ from which the decision maker needs to make a choice. A *choice function* $c : \mathcal{X} \to X$ maps every choice problem $S \in \mathcal{X}$ to an alternative $c(S) \in S$. The choice function c represents data on the choice behavior of the decision maker (hereafter, *DM*).

Our model has two components: (i) a family $\mathcal{U} = \{U_\rho\}_{\rho \in X}$ of *reference-dependent utility functions*, each associated with a potential reference point, and (ii) a *conspicuity ranking* $\gg$, which is assumed to be a *strict linear order* on X.[10] The conspicuity ranking $\gg$ reflects the DM's perception of how prominent or eye-catching the alternatives are relative to one another. We theorize that the reference point in a choice set is the most conspicuous alternative in it. Formally, given $\gg$, the *endogenous reference function* $r : \mathcal{X} \to X$ defined

[10] A binary relation R on X is a strict linear order if it is (i) weakly connected: for every $x, y \in X$, $x \neq y$ implies either xRy or yRx or both, (ii) irreflexive: for every $x \in X$, it is not the case that xRx, and (iii) transitive: for every $x, y, z \in X$, xRy and yRz imply xRz.

by

$$r(S) = \arg\max(\gg, S)$$

maps each choice set S to the reference point $r(S) \in S$.

When the DM faces the choice problem S, she uses the induced reference-dependent utility function $U_{r(S)}$ to evaluate alternatives in S. We assume that each $U_{r(S)}$ is *injective*, that is, that it does not allow indifference between alternatives in X. The maximizer of $U_{r(S)}$ in S is the chosen alternative. This process is formally stated in the next definition.

Definition 1. *A choice function c admits a* ***conspicuity based endogenous reference (CER)*** *representation if there exist a family of reference-dependent injective utility functions $\mathcal{U} = \{U_\rho\}_{\rho \in X}$ and a conspicuity ranking $\gg$ such that for each $S \in \mathcal{X}$,*

$$c(S) = \arg\max_{x \in S} U_{r(S)}(x) \quad \textit{where} \quad r(S) = \arg\max(\gg, S).$$

Without loss of generality, in the above definition, we can impose the widely accepted property from the reference-dependence literature on how two reference-dependent utility functions are related: $U_y(x) > U_y(y)$ implies $U_x(x) > U_x(y)$. This property states that if a person is willing to abandon her reference point y for an alternative x, then she will not abandon x for y when x is the reference point. To see why imposing this property is without loss, note that if x is more conspicuous than y, then y will not be the reference alternative as long as x is available. Hence, this property does not restrict observed behavior in our model.

The standard rational choice model is a special case our model: any CER where all reference-dependent utility functions are identical ($i.e.$, $U_\rho = U$ for all $\rho \in X$) behaves similar to the rational choice model with the utility function U. On the other hand, our model allows for choice patterns that violate the Weak Axiom of Revealed Preference (WARP), which is the cornerstone of rational choice theory. This flexibility allows CER to accommodate intriguing behavioral patterns such as the attraction and compromise effects, which are commonly observed in experiments but cannot be explained by the rational choice model.

While CER is flexible enough to accommodate interesting choice patterns, it has a tight behavioral characterization. Consider the observation $y \neq c(S) \neq c(S \setminus y)$, where y is not the chosen alternative in S but the removal of y from S affects the choice. This choice pattern, which we call a *choice reversal*, is ruled out by the standard rational choice model. In our model, a choice reversal can be observed only if the alternative removed from a choice set is the reference point, and hence the most conspicuous alternative in the choice set. To elaborate, if the reference point was the same in S and $S \setminus y$, then we would have $c(S) = c(S \setminus y)$ as $c(S)$ belongs to $S \setminus y$ and it maximizes $U_{r(S)}$ in S. Hence, the observation $c(S) \neq c(S \setminus y)$ suggests that the reference points in S and $S \setminus y$ are distinct. Since the reference function r maximizes the conspicuity ranking $\gg$, we must have $r(S) = y$. This observation motivates the next axiom, *Single (Choice) Reversal*, which states that in every choice set there is at most one alternative that can induce a choice reversal.

Single Reversal Axiom: For every $S, T \in \mathcal{X}$ and distinct $x, y \in X$ such that $\{x, y\} \subseteq S \cap T$, if $x \neq c(S) \neq c(S \setminus x)$, then either $c(T) = y$ or $c(T \setminus y) = c(T)$.

The Single Reversal Axiom can be thought of as a minimal deviation from the Weak Axiom of Revealed Preference. While WARP rules out any choice reversals, the Single Reversal Axiom allows for at most one choice reversal in each choice set.

Our first main result states that all CER satisfy the Single Reversal Axiom, and, more importantly, any choice data that satisfies this axiom can be represented by a CER.[11]

[11] While we work with choice functions, our results can be extended to choice correspondences. The extended model where the utility functions allow for indifferences can be characterized by the following modified version of the Single Reversal Axiom: for any $S, T \in \mathcal{X}$ and distinct $x, y \in X$ such that $\{x, y\} \subseteq S \cap T$, if $C(S) \neq \{x\}$ and $C(S \setminus x) \neq C(S) \setminus x$, then either $C(T) = \{y\}$ or $C(T \setminus y) = C(T) \setminus y$. The proof follows exactly the same steps as the proof of Theorem 1, and hence it is omitted.

Theorem 1. *A choice function c admits a CER representation if and only if it satisfies the Single Reversal Axiom.*

For the proof of Theorem 1, we refer the interested reader to the Appendix. Here, we provide a sketch of the sufficiency argument. To construct the conspicuity ranking, we first define a binary relation R as follows: for any $x \neq y$, we let xRy if there is a choice problem $S \supseteq \{x, y\}$ such that $x \neq c(S) \neq c(S \setminus x)$. The Single Reversal Axiom is equivalent to asymmetry of this binary relation. Moreover, we show that if the Single Reversal Axiom is satisfied, R must be transitive. Hence, we can find a strict linear order $\gg$ that includes R. We let the conspicuity ranking $\gg$ be an arbitrary strict linear order that contains R, and define the endogenous reference function by $r(S) = \arg\max(\gg, S)$.

Next, for each reference point $\rho \in X$, we define an associated binary relation P_ρ as follows: for any $x \neq y$, we let $xP_\rho y$ if there is a choice problem $S \supseteq \{x, y\}$ such that $\rho = r(S)$, $x = c(S)$, and $y \in S$. The Single Reversal Axioms guarantees that each such P_ρ is transitive, and hence there is a utility function U_ρ consistent with P_ρ. Lastly, we show that the conspicuity ranking $\gg$ together with the set of reference-dependent utility functions $\{U_\rho\}_{\rho \in X}$ form a CER representation of c.

3 Revealed Information

Our model has two components: the reference-dependent utility functions and the conspicuity ranking. We now discuss how to infer information about them from observed choices.

First, note that our utility functions are ordinal since they represent an ordinal binary relation. That is, any monotonic transformation of a utility function continues to represent the same underlying preference relation. Hence, for simplicity, in this section we will notate a utility function with the preference relation it represents. That is, we will use the representation $(\{\succ_\rho\}_{\rho \in X}, \gg)$ instead of $(\mathcal{U}, \gg)$ where $U_\rho(x) > U_\rho(y)$ if and only if $x \succ_\rho y$.

The following example demonstrates how choice data can be used to infer the components of CER. It also establishes that there can be multiple CER representations of the same choice data.

Example 1. *Consider the following choice data on* $X = \{x, y, z\}$*.*

$$c(x, y, z) = y, \quad c(x, y) = x, \quad c(y, z) = y, \text{ and } c(x, z) = z.$$

As discussed in Section 2, the choice reversal $z \neq c(x, y, z) \neq c(x, y)$ *reveals that* z *is more conspicuous than* x *and* y *(and thus, serves as the reference point of* $\{x, y, z\}$*, as well as* $\{x, z\}$ *and* $\{y, z\}$*). Thus, any CER* $(\{\succ_\rho\}_{\rho \in X}, \gg)$ *consistent with this choice data needs to exhibit* $z \gg x$ *and* $z \gg y$*. Furthermore,* $c(x, y, z) = y$ *and* $c(x, z) = z$ *imply that any CER* $(\{\succ_\rho\}_{\rho \in X}, \gg)$ *that represents* c *also needs to satisfy* $y \succ_z z \succ_z x$*. However, the conspicuity ranking between* x *and* y*, as well as the preferences* $\succ_x$ *and* $\succ_y$ *are not identified.*

Since there might be multiple CER representations of the same choice data, we need to give formal definitions for revealed preference and revealed conspicuity.

Definition 2. *Assume that* c *admits* k *different CER representations,* $(\{\succ_\rho\}^i_{\rho \in X}, \gg^i)_{i \in \{1, \ldots, k\}}$*. Then*

1. x *is revealed to be preferred to* y *under the reference point* ρ *if* $x \succ^i_\rho y$ *for all* $i \in \{1, \ldots, k\}$*,*

2. x *is revealed to be more conspicuous than* y *if* $x \gg^i y$ *for all* $i \in \{1, \ldots, k\}$*.*

This definition is very conservative. For example, we say that x is revealed to be preferred to y under the reference point ρ only when *all* possible representations agree on this preference. This conservative approach, proposed by Masatlioglu et al. (2012), guarantees that we do not make any claims that are not fully implied by the data.

The above definition raises a potential difficulty. For example, if one wants to know whether x is more conspicuous than y, it appears necessary to check the consistency of every pair $(\{\succ_\rho\}^i_{\rho\in X}, \gg^i)$ with this claim. However, this is not practical, especially when there are too many alternatives. Instead we now provide a method to obtain revealed conspicuity and revealed preference.

We first discuss revealed conspicuity. As argued earlier, if x causes choice reversal when y is in the choice set, then we can conclude that x is more conspicuous than y in every CER representation of c. More importantly, the converse is also true, that is, such choice reversals fully characterize revealed conspicuity. In other words, if x does not induce a choice reversal when y is available, then there exists a CER representation of the choice behavior where y is ranked as more conspicuous than x. Formally, let R be a binary relation such that for any $x \neq y$,

$$xRy \text{ if there is } S \supseteq \{x, y\} \text{ such that } x \neq c(S) \neq c(S \setminus x).$$

The following remark establishes that R fully characterizes revealed conspicuity.

Remark 1. *(Revealed Conspicuity) Suppose c admits a CER representation. Then x is revealed to be more conspicuous than y if and only if xRy.*

We next discuss revealed preference. To this end, for any x, y, z such that $x \neq y$, we define

$$\begin{aligned} xP_zy \text{ if there is } S \supseteq T \supseteq \{x, y, z\} \text{ such that } (i) &\quad z \neq c(S) \neq c(S \setminus z), \textit{ and} \\ (ii) &\quad x = c(T). \end{aligned}$$

By Remark 1, the first part guarantees that z is the most conspicuous alternative in S, and hence also in $T \subseteq S$. Therefore, z is the reference alternative in T. The second part of the definition then reveals that x must be preferred to y under the reference point z in all possible representations. This is formally stated in the next remark.

Remark 2. *(Revealed Preference) Suppose c admits a CER representation. Then x is revealed to be preferred to y under the reference point z if and only if xP_zy.*

4 Reference-Dependent Choice with Psychological Constraints

The model we considered so far assumes that choices are represented by maximization of a reference-dependent utility function U_ρ which is potentially distinct for different reference points. That is,

$$c(S|\rho) := \arg\max_{x \in S} \; U_\rho(x).$$

One disadvantage of this model is that, since the utility function changes with the reference point, there is no overall welfare criterion. To overcome this criticism, Masatlioglu and Ok (2014) propose an alternative model of reference-dependent choice. Their model views choice as arising from a "psychologically constrained utility maximization" where the constraints are induced by one's initial endowment.[12] This model allows construction of a (reference-independent) ranking of alternatives that can be used to carry out meaningful welfare analyses.

Reference-dependent choice in Masatlioglu and Ok (2014) involves a *psychological constraint function*, formally defined as follows.

Definition 3. *A* ***psychological constraint function*** *$Q : X \to \mathcal{X}$ maps each alternative $\rho \in X$ to its psychological constraint set $Q(\rho) \in \mathcal{X}$ such that $\rho \in Q(\rho)$.*

Namely, $Q(\rho)$ denotes the set of alternatives "acceptable" to the DM when her point of reference is ρ. The only restriction imposed on Q is $\rho \in Q(\rho)$, that is, a reference point does

[12]This idea was first introduced by Samuelson and Zeckhauser (1988), who state that "Assuming that he or she understands his or her current plan, a reasonable strategy would be to undertake a comparative analysis including only some subset of competing plans (ignoring the others altogether)."

not exclude itself from consideration.

If the choice set is S and the reference point is $\rho \in S$, the DM only considers alternatives in $Q(\rho) \cap S$, and she makes her choice by maximizing a reference-free utility function U on this set. Formally,

$$c_{MO}(S|\rho) = \underset{x \in S \cap Q(\rho)}{\arg\max}\, U(x).$$

This reference-dependent choice procedure is a special case of our general model. To see this, start with reference-free utility and psychological constraint functions, and note that if an alternative does not belong to the psychological constraint set of the reference point, it should not be chosen in the presence of this reference point. Hence, when constructing the associated reference-dependent utility function, such alternatives can be assigned any utility level lower than the (reference-free) utility level of the reference point. Otherwise, we can assign the original (reference-free) utility level. Formally, given U and Q, and letting $m < \min_{x \in X} U(x)$, define the reference-dependent utility functions U_ρ as follows:

$$U_\rho(x) := \begin{cases} U(x) & \text{if } x \in Q(\rho), \\ m & \text{if } x \notin Q(\rho). \end{cases}$$

We now define a special case of CER where the underlying reference-dependent choice is based on Masatlioglu and Ok (2014). The rest of the choice procedure is the same.

Definition 4. *A choice function c admits a* ***Psychologically-Constrained CER (PC-CER)*** *representation if there exist a (reference-independent) injective utility function U, a conspicuity ranking $\gg$, and a psychological constraint function Q such that for each $S \in \mathcal{X}$,*

$$c(S) = \underset{x \in S \cap Q(r(S))}{\arg\max}\, U(x) \quad \textit{where} \quad r(S) = \arg\max(\gg, S).$$

The standard rational choice model is still a special case. When $Q(\rho) = X$ for all $\rho \in X$, the DM behaves in an identical manner to a rational-choice agent whose utility function is

U. Similarly, when $Q(\rho) = \rho$ for each $\rho \in X$, the model behaves in an identical manner to a rational choice agent whose utility function has the same ranking as $\gg$. For choice data that satisfies WARP, it is impossible to distinguish these two special cases.

PC-CER exhibits both endogenous reference-point formation (as in CER) and a reference-free utility function (as in the rational choice model). Being a special case of CER, PC-CER satisfies the Single Reversal Axiom and exhibits choice reversals only due to a change in the most conspicuous alternative. Due to the existence of a reference-free utility function, PC-CER also satisfies an additional consistency property which is closely related to the following property of the rational choice model.

For every $S \in \mathcal{X}$, there is $x \in S$ such that if $x \in T$ then either $c(T) = x$ or $c(T) \notin S$.

The above statement says that there is the best element x in S, and if x and another alternative $y \in S$ are considered together in some other set T, it cannot be that y is chosen from T as the best alternative: either x or an alternative in $T \setminus S$ must be chosen from T. This simple consistency requirement, however, does not take into account the fact that, in case of reference effects, x might be considered as an alternative in S but might be ruled out in some other set T by the psychological constraint of T's reference point. To exclude this possibility, we revise the above statement to additionally require that the reference point in T does not rule out x from consideration. In sum, to capture PC-CER (which is more general than the standard rational choice model), we relax the above statement as follows.

Consistency: For every $S \in \mathcal{X}$, there is $x \in S$ such that if $\{x, z\} \subseteq T \subseteq T'$, $z \neq c(T') \neq c(T' \setminus z)$, and $x = c(x, z)$, then either $c(T) = x$ or $c(T) \notin S$.

In the above statement, the choice reversal $z \neq c(T') \neq c(T' \setminus z)$ tells us that z is the reference point in T', and hence in its subsets T and $\{x, z\}$ (see the discussion preceding Remark 1). The statement $x = c(x, z)$ additionally informs us that the reference point z does not rule out x from consideration. Since x is the best alternative in S and it is considered in T, it is not possible that another alternative $y \in S$ is chosen instead of x.

Our original model CER may violate Consistency.[13] Hence, Consistency is a formulation of the behavioral difference between CER and PC-CER. This behavioral difference arises from the replacement of reference-dependent utilities in CER with a combination of the reference-free utility and psychological constraints in PC-CER. Consistency is thus expected to be related to revealed (reference-free) preference ranking of PC-CER. Indeed, it guarantees that this preference ranking has no cycles. For choice data that is consistent with CER but not with PC-CER, such reference-free preferences cannot even be constructed.

The following theorem states that any PC-CER satisfies both Single Reversal and Consistency. Furthermore, any choice data that satisfies these two axioms can be rationalized by a PC-CER.

Theorem 2. *A choice function c admits a PC-CER representation if and only if it satisfies Single Reversal and Consistency.*

We provide a sketch of the sufficiency argument. To construct the conspicuity ranking, we first define a binary relation R as in the proof of Theorem 1. As before, Single Reversal guarantees that R is transitive. However, unlike in Theorem 1, we cannot take any arbitrary completion of R to be our conspicuity ranking. Instead, the conspicuity ranking $\gg$ is constructed as follows: for any $x \neq y$, $x \gg y$ if either xRy or x and y cannot be compared by R and $x = c(x, y)$. We show that binary relation $\gg$ is weakly connected and transitive.

Our next step is to construct the psychological constraint function Q. We define that $y \in Q(x)$ if and only if $x \gg y$ and $y = c(x, y)$. This psychological constraint function is

[13] To see this, consider the following example consistent with CER: the conspicuity ranking is $t \gg z \gg y \gg x$, and x is the most preferred alternative and y is the second most preferred alternative under references x and z, while y is the most preferred alternative and x is the second most preferred alternative under references y and t. Then we observe the following choice data: $c(x, y, z, t) = c(y, z) = c(x, y) = y$ and $c(x, y, z) = c(x, t) = x$. It can easily be shown that this data violates Consistency for $S = \{x, y\}$.

minimal in the following sense: if there is another representation of the same choice behavior given by $(U', \gg', Q')$, then $Q(x) \subseteq Q'(x)$ for all $x \in X$.

The third step is to define preferences. To this end, we define a binary relation P as follows: for any $x \neq y$, xPy if there is $z \in X$ such that $\{x, y\} \subseteq Q(z)$ and $c(x, y, z) = x$. Consistency guarantees that P defined as such is acyclic. Using this, we then let U be a utility function consistent with P. Lastly, we show that the triple $(U, \gg, Q)$ is a PC-CER representation of c.

4.1 Revealed Preference

In this section, we illustrate how to infer the DM's utility function/preference ranking from her observed choices, given choice data that is consistent with PC-CER.[14] For revealed conspicuity and revealed psychological constraint, see the Appendix.

The following example demonstrates how choice data can be used to infer the components of PC-CER.

Example 2. *Consider the following choice data on $X = \{x, y, z\}$:*

$$c(x, y, z) = x, \ \ c(x, y) = y, \quad c(y, z) = y, \quad \textit{and} \ \ c(x, z) = x.$$

The choice reversal $z \neq c(x, y, z) \neq c(x, y)$ reveals that z is more conspicuous than x and y. We also observe that $c(x, z) = x$ and $c(y, z) = y$. Since x and y are chosen over z despite the fact that they are less conspicuous than z, we infer that x and y must both be preferred to z. We also learn that the psychological constraint under z does not rule out either x or y. In particular, y is considered in the choice set $\{x, y, z\}$. This, together with the observation that $c(x, y, z) = x$, lead us to conclude that x is preferred to y. Finally, since x is preferred to y but $y = c(x, y)$, it must be that y is more conspicuous than x.

[14] As in Section 3, we focus on the preference relations rather than their utility representations. Hence, we write $(\succ, \gg, Q)$ instead of $(U, \gg, Q)$.

Overall, we uniquely identify preference and conspicuity as $x \succ y \succ z$ and $z \gg y \gg x$. The psychological constraint is identified as $Q(z) = \{x, y, z\}$, $\{y\} \subseteq Q(y) \subseteq \{y, z\}$, and $\{x\} \subseteq Q(x)$.

As demonstrated in Example 2, under a fixed reference point choices from bigger sets are revealed to be preferred to choices from smaller sets. Using this idea, we construct the following binary relation. For any x, y, z such that $x \neq y$, we define

$$\begin{aligned} xPy \text{ if } \exists S, T \text{ with } \{x,y,z\} \subseteq T \subseteq S \text{ such that } (i) &\quad z \neq c(S) \neq c(S \setminus z), \\ (ii) &\quad c(y,z) = y, \textit{ and} \\ (iii) &\quad x = c(T). \end{aligned} \tag{1}$$

Condition (i) implies that z is the most conspicuous alternative and the reference point in S as well as in T and $\{y, z\}$. Since y is chosen from $\{y, z\}$, we can infer that y is in the psychological constraint set of z. Then observing $x = c(T)$ reveals that x must be preferred over y.

Since the binary relation P is not necessarily transitive (but all PC-CER that represent c have transitive preferences), we let P^T be the transitive closure of P. The next remark establishes that P^T fully characterizes the revealed preference ranking.

Remark 3. *(Revealed Preference) Suppose c admits a PC-CER representation. Then x is revealed to be preferred to y if and only if xP^Ty.*

The same construction that is used in the proof of Theorem 2 can be used to prove Remark 3, as well as the remarks on revealed conspicuity and revealed psychological constraint, presented in the Appendix.

4.2 Ordered Psychological Constraints

In this section, we consider a special case of PC-CER where the psychological constraint is *ordered with respect to the conspicuity relation.* Intuitively, this means that a more conspicuous alternative induces a harsher psychological constraint than a less conspicuous one.

Definition 5. *A psychological constraint function Q is ordered with respect to $\gg$ if*

$$x \gg y \textit{ implies } Q(x) \subseteq Q(y).$$

The ordering assumption on Q is natural under a wide range of circumstances. For example, consider a consumer whose reference point is the cheapest alternative in the menu and her psychological constraint rules out alternatives that are too expensive relative to the reference point. More specifically, if the consumer is willing to spend at most $p+m$ dollars on a purchase when her reference point costs p dollars, her constraint function can be written as $Q(x) = \{y \in X | \, p_y \leq p_x + m\}$. In this example, cheaper alternatives are more conspicuous, that is, $x \gg y$ is equivalent to $p_x < p_y$. This in turn implies $Q(x) \subseteq Q(y)$. Hence, Q is ordered with respect to $\gg$.

As a second example, consider the willpower model of Masatlioglu et al. (2020). The conspicuity ranking captures the amount of temptation each alternative creates. Under this interpretation, the most tempting alternative in a choice set is the reference point. Let $v(x)$ denote the temptation value of an alternative x and assume that the DM has a willpower stock w which she can use to resist temptation. That is, a DM with a reference point x is able to consider an alternative y only if its temptation value is not less than $v(x) - w$, i.e., $v(y) \geq v(x) - w$. Then it is easy to see that the psychological constraint function Q defined as $Q(x) = \{y \in X | \, v(y) \geq v(x) - w\}$ is ordered with respect to $\gg$, where $x \gg y$ whenever $v(x) > v(y)$.

For such examples, the following restriction of the PC-CER is appropriate.

Definition 6. *A choice function c admits an* ***ordered PC-CER*** *representation if there exists a PC-CER representation $(U, \gg, Q)$ of c such that Q is ordered with respect to $\gg$.*

The additional ordering assumption on the psychological constraint allows us to learn more about the DM's preferences. To illustrate this, we revisit Example 1:

$$c(x, y, z) = y, \quad c(x, y) = x, \quad c(y, z) = y, \text{ and } c(x, z) = z.$$

As discussed before, $z \neq c(x, y, z) = y \neq c(x, y)$ tells us that z is more conspicuous than both x and y, y is in the constraint set of z, and y is preferred to z. Without an ordering assumption on Q, this is all we can learn. With an ordered Q, however, y being in the constraint set of z implies that y is also in the constraint set of (the less conspicuous) x. We now show that x is preferred to y. To see this, first suppose y is more conspicuous than x. Then $c(x, y) = x$ implies the desired conclusion. Alternatively, suppose x is more conspicuous than y. Then, since y is in the constraint set of x and $c(x, y) = x$, we again conclude that x is preferred to y. Hence, with the ordering assumption on Q, the revealed preference ranking becomes $x \succ y \succ z$. The ordering assumption on Q, however, does not help us to identify the conspicuity ranking between x and y. We can have $x \gg y$ and $Q(x) = \{x, y, z\}$ or $y \gg x$ and $Q(y) = \{x, y, z\}$.

As illustrated above, we now have two types of preference revelations. The first one is the binary relation P defined in Equation (1). The second type of preference revelation is new. It is related to "Less is More" concept in the literature, which states that excess of options can be welfare reducing for the DM. In our model, the ordering assumption on psychological constraints, combined with the assumption that the most conspicuous alternative is the reference point, lead the DM to consider fewer alternatives as the number of alternatives increases.[15] Therefore, the alternative chosen from a smaller choice set is revealed to be

[15] In the limited attention literature, Lleras et al. (2017) formalize this idea that decision makers might

preferred to the one chosen in a larger choice set. To be more precise, if x is chosen over y in some choice set T and y is chosen over x in some superset S of T, then x is revealed to be preferred to y. To see why, notice that since y is chosen over x in S it must belong to the psychological constraint set induced by the most conspicuous alternative in that set. Due to the ordered nature of the psychological constraint function, y must also belong to the psychological constraint set induced by the most conspicuous alternative in T. Since x is chosen over y in T, x must be more preferred than y. Formally, for any $x \neq y$,

$$xP'y \text{ if } \exists S \supset T \supseteq \{x, y\} \text{ such that } c(T) = x \text{ and } c(S) = y.$$

It turns out that a necessary condition for a choice function c to have an ordered PC-CER representation is acyclicity of the union of these two revealed preferences: $P_o = P \cup P'$. The next axiom imposes this requirement. In that sense, it is very similar to the Strong Axiom of Revealed Preference (SARP), commonly used in the literature.

Acyclicity: P_o is acyclic.

The main result of this section states that ordered PC-CER satisfies both Acyclicity and Single Reversal axioms. Furthermore, any choice function that satisfies these two axioms admits an ordered PC-CER representation.

Theorem 3. *A choice function c admits an ordered PC-CER representation if and only if it satisfies Single Reversal and Acyclicity.*

The proof of the Theorem also provides the characterization of the revealed preference. Let P_o^T denote the transitive closure of P_o. The next remark says that P_o^T is indeed the revealed preference ranking.

consider fewer alternatives in larger choice sets due to increased competition for attention. The second type of revealed preference in ordered PC-CER coincides with the revealed preference in their model. Please see Section 5 for a more detailed discussion about the relationship between two models.

Remark 4. *(Revealed Preference) Suppose c admits an ordered PC-CER representation. Then x is revealed to be preferred to y if and only if $xP_o^T y$.*

5 Related Literature

In this section, we discuss two strands of literature that is related to our work, namely, (i) models of reference dependence and (ii) other choice procedures involving choice reversals.

Reference-Dependence: Kőszegi and Rabin (2006), Bordalo et al. (2013), Ok et al. (2015), and Tserenjigmid (2019) are most closely related to ours. We have already discussed these papers, and in particular the relationship with Kőszegi and Rabin (2006), in depth in the introduction. Below we provide a few additional details omitted from the introduction.

As discussed in the introduction, in Kőszegi and Rabin (2006), due to the assumption that expectations are rational, the reference point coincides with the actual choice. Hence, this model cannot accommodate attraction and compromise effects. Alternatively, the reference point in Ok et al. (2015) can never coincide with actual choice. In addition in Ok et al. (2015), the DM does not exhibit reference-dependence in binary choice problems, which rules out cyclical choice behavior. This assumption allows choices from binary problems to fully reveal the DM's (reference-independent) preferences. In contrast, CER, PC-CER and ordered PC-CER allow reference-dependence in binary choice problems and do not assume reference-independent preferences to be observable from binary choice problems. In line with its main motivation, their model can accommodate the attraction effect phenomenon. However, it does not accommodate the compromise effect or cyclical choice behavior.[16]

[16] Ok et al. (2015) rule out cyclical binary choice by assumption. We show that their model does not accommodate the compromise effect as well. Using their terminology, we say that z "helps" x against y and z "hurts" y against x if $y = c(x, y)$ but $x = c(x, y, z)$. One of the key assumptions in Ok et al. (2015) is that if z "helps" x against one alternative, it can never "hurt" x against any other alternative. Now consider four alternatives x, y, z, and t such that x is the compromise alternative in $\{x, y, z\}$ and t is the compromise alternative in $\{x, z, t\}$. Then, we might observe that z "helps" x against y, but z "hurts" x against t, violating the key assumption in Ok et al. (2015).

Bordalo et al. (2013) model a commodity as a vector of K attributes and identify the reference point as the commodity with *average attributes* (see also Bodner and Prelec, 1994, Kivetz et al., 2004). Alternatively, Tserenjigmid (2019) identify the reference point as the commodity with the *worst attributes*. In these models, the reference point need not be in the choice set or in the commodity space. Under certain parameters, both models can explain versions of attraction and compromise effects. The model in Bordalo et al. (2013) is also capable of accommodating cyclical binary choices by utilizing violations of monotonicity (where an alternative which is dominated in all attributes can be chosen). For a behavioral foundation, see Ellis and Masatlioglu (2021).

Other Choice Procedures: Our model is also related to the following papers, though to a lesser extent. While these models have different psychological motivations from ours (and from each other), similar to our study they all generate choice reversals.

The limited attention model of Masatlioglu et al. (2012) follows a two-stage choice process: in the first stage, a decision maker focuses on a small set of alternatives (limited attention), and in the second stage she maximizes her preference among the alternatives which survive the first stage (preference maximization). This model is conceptually different from CER. First, in our model, the DM has full attention, hence no first-stage. Second, while their model assumes a single preference, the DM in our model has multiple reference-dependent preferences. Surprisingly, despite these structural differences, we next show that our model is a subset of theirs in terms of observed choice behavior .

First, note that in their model consideration sets satisfy the following simple property: if an alternative is not considered in some choice set, then its removal from the choice set cannot change the DM's consideration set. Hence, if x is chosen from S but it is not chosen from $S \setminus z$, this reveals that the consideration sets in S and $S \setminus z$ are distinct. Therefore, z must have been considered in S, and hence x must be preferred to z. Their axiomatization then requires that this revealed preference relation has no cycles. Now note that the same choice pattern in our model reveals that z is more conspicuous than x and every other alternative in S (Remark 1). Since the conspicuity ranking in our model is a linear order, the revealed conspicuity ranking cannot have any cycles. Hence, our model necessarily satisfies their axiom. On the other hand, the "difficult choice" pattern $c(x, y, z) = x$, $c(x, y) = y$, and $c(x, z) = z$, can be captured by their model but not by ours. This shows that their model is strictly more general than ours in terms of observed choice behavior.

Lleras et al. (2017) also study choice under limited attention by imposing an alternative restriction on the first stage. They assume that the DM pays attention to fewer alternatives in larger choice sets due to increasing competition for attention. The differences highlighted in the previous paragraph between CER and the model of Masatlioglu et al. (2012) also apply to this model. Hence, CER is also conceptually different from the model of Lleras et al. (2017). In addition, the two models are distinct in terms of observed choice behavior.

In the rational shortlisting model of Manzini and Mariotti (2007), the DM iteratively applies two asymmetric binary relations to make a choice. CER and PC-CER are not logically related to this model.[17] Surprisingly, the ordered PC-CER turns out to be a rational shortlist method in which the first binary relation is an interval order and the second binary relation is a linear order.[18] To see this, let $\rhd$ be a binary relation given by $x \rhd y$ if and only if $y \notin Q(x)$ where Q is ordered with respect to a linear order $\gg$. It is easy to show that $\rhd$ is an interval order.[19] In addition, for any interval order $\rhd$, the constraint function Q that satisfies the above definition must be ordered.[20] Hence, our characterization result also

[17] The models of Manzini and Mariotti (2012a), Cherepanov et al. (2013), Lleras et al. (2017) are generalizations of Manzini and Mariotti (2007). All of them satisfy Weak-WARP (for any $\{x, y\} \subset T \subset S$, $c(x, y) = x$ and $c(S) = x$ imply $c(T) = x$), which is logically independent of our Single Reversal axiom.

[18] Some of the related papers contributing to this literature are Houy (2008), Au and Kawai (2011), Manzini and Mariotti (2012a,b), Apesteguia and Ballester (2013), Tyson (2013), Dutta and Horan (2015), Horan (2016), Matsuki and Tadenuma (2018).

[19] Since $x \in Q(x)$, $\rhd$ is irreflexive. Moreover, suppose $x \rhd y$ and $z \rhd t$. If $t \in Q(x)$ and $y \in Q(z)$, then, by the ordering assumption on Q, either $t \in Q(x) \subseteq Q(z)$ or $y \in Q(z) \subseteq Q(x)$, which contradicts the original hypothesis. Hence, either $t \notin Q(x)$ ($x \rhd t$) or $y \notin Q(z)$ ($z \rhd y$), as desired.

[20] Suppose not. Then there exist x, y, z, t such that $y \in Q(z) \setminus Q(x)$ and $t \in Q(x) \setminus Q(z)$. By definition, $x \rhd y$ and $z \rhd t$, but neither $x \rhd t$ nor $z \rhd y$, a contradiction to the hypothesis that $\rhd$ is an interval order.

provides behavioral implications of this special case of the rational shortlist method.

Masatlioglu et al. (2020) propose a model of temptation where individuals have imperfect control over their immediate urges and they are able to overcome temptation by exerting mental effort. In their model this ability, called willpower, is a limited resource.[21] We uncover a surprising relationship between our work and two nested models considered in Masatlioglu et al. (2020). It turns out that both of their models are special cases of PC-CER. In fact, since their general limited willpower model is equivalent to a rational shortlist method where the first binary relation is an interval order and the second binary relation is a linear order (see Masatlioglu et al. (2020)), a choice function c admits an ordered PC-CER representation if and only if it admits a generalized limited willpower representation.

Noor and Takeoka (2010) extends the costly self-control model of Gul and Pesendorfer (2001). In their model, deviation from the most tempting alternative in the menu imposes a cost. The decision maker maximizes the welfare utility minus the cost of deviation from the most tempting alternative in the menu. For the self-control cost, they consider a general form which depends on both the choice and the maximum temptation value in the menu. Even though the domain of this paper is ex-ante menu preferences, one can focus on the implied ex-post choices to make a comparison. Indeed, the ex-post choices in this model exhibit choice-reversals as in ours. Furthermore, these ex-post choices satisfy our Single Reversal axiom.

Recently, Ravid and Steverson (2018) focus on the ex-post choices and show that the cost function of Noor and Takeoka (2010) can be written as a function of the difference between the realized utility from choice and the minimum utility possible in the menu. The authors call this the "bad temptation" model since the temptation value is the opposite of welfare utility. Given this equivalence result, the bad temptation model also satisfies Single

[21] As opposed to our study, Masatlioglu et al. (2020) take a preference-choice function pair $(\succeq, c)$ as their data and analyze conditions under which the limited willpower model can represent the pair $(\succeq, c)$.

Reversal. As opposed to Noor and Takeoka (2010), Ravid and Steverson (2018) provide a characterization of the bad temptation model in ex-post choice. They prove that their model is characterized by the Axiom of Revealed Temptation which is identical to Reference Point Property of Rubinstein and Salant (2006) (see footnote 9). Hence, these models are indistinguishable from each other on the basis of choice data alone even though they capture very different positive models of behavior.

6 Conclusion

We provide a simple model of reference-dependent choice (CER) in which the reference point is determined endogenously. The main component of CER is a conspicuity ranking, whose maximization in a choice set determines the set's reference point. The reference point in turn determines the reference-dependent preferences. We show that CER can be characterized by a simple and easily testable Single Reversal axiom on observed choices. We also demonstrate how one can reveal the conspicuity ranking and reference-dependent preferences of a decision maker. Imposing an additional Consistency axiom on choice behavior results in a more specialized model (PC-CER) where the reference point affects choices through a psychological constraint and the DM has reference-free preferences that can be used to make welfare evaluations. We also analyze the implications of an order structure on the psychological constraints (ordered PC-CER).

We would like to point out that this paper studies just one particular theory of reference point formation. Alternative and equally valuable theories of reference points such as expectations, aspirations, minimal and average alternatives as references have already been explored in the literature. More research is needed to determine which theory of reference point formation is the most appropriate in different applications. We hope that this paper will contribute to this line of research.

References

Apesteguia, J. and M. A. Ballester (2009): "A Theory of Reference-Dependent Behavior," *Economic Theory*, 40(3), 427–455.

——— (2013): "Choice by Sequential Procedures," *Games and Economic Behavior*, 77(1), 90–99.

Au, P. H. and K. Kawai (2011): "Sequentially Rationalizable Choice with Transitive Rationales," *Games and Economic Behavior*, 73(2), 608–614.

Barberis, N. C. (2013): "Thirty Years of Prospect Theory in Economics: A Review and Assessment," *Journal of Economic Perspectives*, 27(1), 173–195.

Bhatia, S. (2017): "Comparing Theories of Reference-Dependent Choice," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 43(9), 1490–1517.

Bhatia, S. and R. Golman (2019): "Attention and Reference Dependence," *Decision*, 6(2), 145–170.

Bodner, R. and D. Prelec (1994): "The Centroid Model of Context Dependent Choice," *Unpublished Manuscript, MIT*.

Bordalo, P., N. Gennaioli, and A. Shleifer (2013): "Salience and Consumer Choice," *Journal of Political Economy*, 121(5), 803–843.

Brickman, P., D. Coates, and R. Janoff-Bulman (1978): "Lottery Winners and Accident Victims: Is Happiness Relative?" *Journal of Personality and Social Psychology*, 36(8), 917 – 927.

Cherepanov, V., T. Feddersen, and A. Sandroni (2013): "Rationalization," *Theoretical Economics*, 8(3), 775–800.

Dean, M., Ö. Kıbrıs, and Y. Masatlioglu (2017): "Limited Attention and Status Quo Bias," *Journal of Economic Theory*, 169, 93–127.

DellaVigna, S. (2009): "Psychology and Economics: Evidence from the Field," *Journal of Economic Literature*, 47(2), 315–72.

Dutta, R. and S. Horan (2015): "Inferring Rationales from Choice: Identification for Rational Shortlist Methods," *American Economic Journal: Microeconomics*, 7(4), 179–201.

Ellis, A. and Y. Masatlioglu (2021): "Choice with Endogenous Categorization," *The Review of Economic Studies*.

Erdem, T., G. Mayhew, and B. Sun (2001): "Understanding Reference Price Shoppers: A Within and Cross-Category Analysis," *Journal of Marketing Research*, 38(4), 445–457.

Freeman, D. J. (2017): "Preferred Personal Equilibrium and Simple Choices," *Journal of Economic Behavior & Organization*, 143, 165–172.

Gul, F. and W. Pesendorfer (2001): "Temptation and Self-Control," *Econometrica*, 69(6), 1403–1435.

Horan, S. (2016): "A Simple Model of Two-Stage Choice," *Journal of Economic Theory*, 162, 372–406.

Houy, N. (2008): "Progressive Knowledge Revealed Preferences and Sequential Rationalizability," *Unpublished Manuscript.*

Huber, J., J. W. Payne, and C. Puto (1982): "Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis," *Journal of Consumer Research*, 9(1), 90–98.

Kahneman, D. and A. Tversky (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47(2), 263–292.

Kalyanaram, G. and R. S. Winer (1995): "Empirical Generalizations from Reference Price Research," *Marketing Science*, 14(3), G161–G169.

Kivetz, R., O. Netzer, and V. Srinivasan (2004): "Alternative Models for Capturing the Compromise Effect," *Journal of Marketing Research*, 41(3), 237–257.

Kőszegi, B. and M. Rabin (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121(4), 1133–1165.

Larrick, R. P. and G. Wu (2012): "Risk in Negotiation: Judgments of Likelihood and Value," *Oxford University Press.*

Levy, J. S. (1992): "An Introduction to Prospect Theory," *Political Psychology*, 13(2), 171–186.

Lim, X. Z. (2020): "Ordered Reference Dependent Choice," *Working Paper.*

Lleras, J. S., Y. Masatlioglu, D. Nakajima, and E. Y. Ozbay (2017): "When More Is Less: Limited Consideration," *Journal of Economic Theory*, 170, 70–85.

Lohse, G. L. (1997): "Consumer Eye Movement Patterns on Yellow Pages Advertising," *Journal of Advertising*, 26(1), 61–73.

Loomes, G., C. Starmer, and R. Sugden (1991): "Observing Violations of Transitivity by Experimental Methods," *Econometrica*, 59(2), 425–439.

Maltz, A. (2020): "Exogenous Endowment - Endogenous Reference Point," *The Economic Journal*, 130(625), 160–182.

Mandler, M., P. Manzini, and M. Mariotti (2012): “A Million Answers to Twenty Questions: Choosing by Checklist,” *Journal of Economic Theory*, 147(1), 71–92.

Manzini, P. and M. Mariotti (2007): “Sequentially Rationalizable Choice,” *American Economic Review*, 97(5), 1824–1839.

——— (2009): “Consumer Choice and Revealed Bounded Rationality,” *Economic Theory*, 41(3), 379–392.

——— (2012a): “Categorize Then Choose: Boundedly Rational Choice and Welfare,” *Journal of the European Economic Association*, 10(5), 1141–1165.

——— (2012b): “Choice by Lexicographic Semiorders,” *Theoretical Economics*, 7, 1–23.

Markowitz, H. (1952): “The Utility of Wealth,” *Journal of Political Economy*, 60(2), 151–158.

Masatlioglu, Y. and D. Nakajima (2013): “Choice by Iterative Search,” *Theoretical Economics*, 8(3), 701–728.

Masatlioglu, Y., D. Nakajima, and E. Y. Ozbay (2012): “Revealed Attention,” *American Economic Review*, 102(5), 2183–2205.

Masatlioglu, Y., D. Nakajima, and E. Ozdenoren (2020): “Revealed Willpower,” *Theoretical Economics*, 15(1), 279–317.

Masatlioglu, Y. and E. A. Ok (2005): “Rational Choice with Status Quo Bias,” *Journal of Economic Theory*, 121(1), 1–29.

——— (2014): “A Canonical Model of Choice with Initial Endowments,” *Review of Economic Studies*, 81(2), 851–883.

Matsuki, J. and K. Tadenuma (2018): “Choice via Grouping Procedures,” *International Journal of Economic Theory*, 14(1), 71–84.

May, K. O. (1954): “Intransitivity, Utility, and the Aggregation of Preference Patterns,” *Econometrica*, 22(1), 1–13.

Milosavljevic, M., V. Navalpakkam, C. Koch, and A. Rangel (2012): “Relative Visual Saliency Differences Induce Sizable Bias in Consumer Choice,” *Journal of Consumer Psychology*, 22(1), 67–74.

Munro, A. and R. Sugden (2003): “On the Theory of Reference-Dependent Preferences,” *Journal of Economic Behavior & Organization*, 50(4), 407–428.

Noor, J. and N. Takeoka (2010): “Uphill Self-Control,” *Theoretical Economics*, 5, 127–158.

Ok, E. A., P. Ortoleva, and G. Riella (2015): "Revealed (P)reference Theory," *American Economic Review*, 105(1), 299–321.

Orhun, A. Y. (2009): "Optimal Product Line Design When Consumers Exhibit Choice Set-Dependent Preferences," *Marketing Science*, 28(5), 868–886.

Pratkanis, A. R. (2007): "Social Influence Analysis: An Index of Tactics," *The Science of Social Influence: Advances and Future Progress*, 17–82.

Ratneshwar, S., A. D. Shocker, and D. W. Stewart (1987): "Toward Understanding the Attraction Effect: The Implications of Product Stimulus Meaningfulness and Familiarity," *Journal of Consumer Research*, 13(4), 520–533.

Ravid, D. and K. Steverson (2018): "Bad Temptation," *SSRN Working Paper 3127575*.

Rubinstein, A. and Y. Salant (2006): "Two comments on the principle of revealed preference," *Unpublished paper*.

Sagi, J. S. (2006): "Anchored Preference Relations," *Journal of Economic Theory*, 130(1), 283–295.

Salant, Y. and A. Rubinstein (2008): "(A, f): Choice with Frames," *Review of Economic Studies*, 75(4), 1287–1296.

Samuelson, W. and R. Zeckhauser (1988): "Status Quo Bias in Decision Making," *Journal of Risk and Uncertainty*, 1(1), 7–59.

Simonson, I. (1989): "Choice Based on Reasons: The Case of Attraction and Compromise Effects," *Journal of Consumer Research*, 16(2), 158–174.

Simonson, I. and A. Tversky (1992): "Choice in Context: Tradeoff Contrast and Extremeness Aversion," *Journal of Marketing Research*, 29(3), 281.

Sugden, R. (2003): "Reference-Dependent Subjective Expected Utility," *Journal of Economic Theory*, 111(2), 172–191.

Terzi, A., K. Koedijk, C. N. Noussair, and R. Pownall (2016): "Reference Point Heterogeneity," *Frontiers in Psychology*, 7:1347.

Tserenjigmid, G. (2019): "Choosing with the Worst in Mind: A Reference-Dependent Model," *Journal of Economic Behavior & Organization*, 157, 631–652.

Tversky, A. (1969): "Intransitivity of Preferences," *Psychological Review*, 76(1), 31–48.

Tversky, A. and D. Kahneman (1991): "Loss Aversion in Riskless Choice: A Reference-Dependent Model," *Quarterly Journal of Economics*, 106(4), 1039–1061.

Tyson, C. J. (2013): "Behavioral Implications of Shortlisting Procedures," *Social Choice and Welfare*, 41(4), 941–963.

Wakker, P. P. (2010): *Prospect Theory: For Risk and Ambiguity*, Cambridge University Press.

Winer, R. S. (1986): "A Reference Price Model of Brand Choice for Frequently Purchased Products," *Journal of Consumer Research*, 13(2), 250–256.

Appendix

Proof of Theorem 1

Here we show that Single Reversal is sufficient for a CER representation. To construct the conspicuity ranking, we first define the following binary relation. For each $x, y \in X$ such that $x \neq y$, let xRy if there exists $S \in \mathcal{X}$ such that $\{x, y\} \subset S$ and $x \neq c(S) \neq c(S \setminus x)$.

Claim 1. *R is asymmetric.*

Proof. Directly follows from the statement of Single Reversal Axiom. $\square$

Claim 2. *If $c(\bigcup_{i=1}^n S_i) \in \bigcap_{i=1}^n S_i$, then $c(\bigcup_{i=1}^n S_i) = c(S_i)$ for some $i \in \{1, ..., n\}$.*

Proof. Let $S = S_1$ and $T = \bigcup_{i=2}^n S_i$.

Suppose $c(S \cup T) \in S \cap T$, but $c(S \cup T) \notin \{c(S), c(T)\}$. Then $T \setminus S$ and $S \setminus T$ are both nonempty. Enumerate $T \setminus S = \{t_1, \dots, t_k\}$ and $S \setminus T = \{s_1, \dots s_l\}$. By our supposition, there are $k' \in \{1, ..., k\}$ and $l' \in \{1, ..., l\}$ such that $c(S \cup T) = c(T \cup \{s_1, ..., s_{l'}\}) \neq c(T \cup \{s_1, ..., s_{l'-1}\})$ and $c(S \cup T) = c(S \cup \{t_1, ..., t_{k'}\}) \neq c(S \cup \{t_1, ..., t_{k'-1}\})$. Also, since $c(S \cup T) \in S \cap T$, we have $c(S \cup T) \notin \{s_{l'}, t_{k'}\}$. But then $t_{k'} \neq c(S \cup \{t_1, ..., t_{k'}\}) \neq c(S \cup \{t_1, ..., t_{k'-1}\})$ implies $t_{k'} R\ s_{l'}$. Similarly, $s_{l'} \neq c(T \cup \{s_1, ..., s_{l'}\}) \neq c(T \cup \{s_1, ..., s_{l'-1}\})$ implies $s_{l'} R\ t_{k'}$. This contradicts asymmetry of R, which was established in Claim 1.

The previous paragraph guarantees that the claim holds when $n = 2$. Next, assume that the claim holds for all $n' < n$. We will prove it for n. By the previous paragraph, either $c(S) = c(\bigcup_{i=1}^n S_i)$ or $c(T) = c(\bigcup_{i=1}^n S_i)$. If the former is true, we are done. Otherwise, since $T = \bigcup_{i=2}^n S_i$, by the induction hypothesis, for some $i \in \{2, ..., n\}$ we have $c(S_i) = c(T) = c(\bigcup_{i=1}^n S_i)$. $\square$

Claim 3. *Assume $x \neq c(S) \neq c(S \setminus x)$ and $y \neq c(T) \neq c(T \setminus y)$. Then for any $S' \in \mathcal{X}$ such that $\{x, y\} \subseteq S' \subseteq S \cup T$ and $c(S \cup T) \in S'$, we have $c(S') = c(S \cup T)$.*

Proof. Suppose not. Let $\{t_1, \dots t_n\} = (S \cup T) \setminus S'$. For each $i \in \{1, ..., n\}$, define $T'_i = S' \cup \{t_i\}$. Since $\bigcup_{i=1}^n T'_i = S \cup T$ and $\bigcap_{i=1}^n T'_i = S'$, we have $c(\bigcup_{i=1}^n T'_i) \in \bigcap_{i=1}^n T'_i$. This, by Claim 2, implies $c(T'_i) = c(S \cup T)$ for some $i \in \{1, ..., n\}$. Since $c(S \cup T) \in S'$, we have $t_i \neq c(T'_i)$. Since $c(S') \neq c(S \cup T)$, we have $c(T'_i) \neq c(S')$. Since $\{x, y\} \subseteq S'$, we then have $t_i R x$ and $t_i R y$. But if $t_i \in S$, by our assumption xRt_i, contradicting asymmetry of R established in Claim 1. Alternatively if $t_i \in T$, we have a similar contradiction due to yRt_i. $\square$

Claim 4. *R is transitive.*

Proof. Assume $xRyRz$. Then there are $S \supset \{x, y\}$ and $T \supset \{y, z\}$ such that $x \neq c(S) \neq c(S \setminus x)$ and $y \neq c(T) \neq c(T \setminus y)$. Note that, by asymmetry of R we have $x \notin T$. If $z \in S$, then xRz and we are done. Alternatively assume $z \notin S$. Then there are two cases to consider.

Case 1: $c(S \cup T) \in S$.

Let $\bar{S} = S \cup \{z\}$. Since $\{x, y\} \subset S \subset \bar{S} \subseteq S \cup T$, Claim 3 implies $c(S) = c(\bar{S}) = c(S \cup T)$.

Thus, $x \neq c(\bar{S})$. First, assume $c(\bar{S}) = c(\bar{S} \setminus x)$. But, then $c(S) = c(\bar{S}) = c(\bar{S} \setminus x) = c(S \setminus x)$, in contradiction to the original hypothesis. Hence, $c(\bar{S}) \neq c(\bar{S} \setminus x)$ and xRz follows.

Case 2: $c(S \cup T) \in T \setminus S$.
Let $\bar{T} = T \cup \{x\}$. Since $\{x, y\} \subset \bar{T} \subseteq S \cup T$, Claim 3 implies $c(\bar{T}) = c(S \cup T)$. Since $c(S \cup T) \in T \setminus S$, $c(\bar{T}) \notin \{x, y\}$. Hence, if $c(\bar{T}) \neq c(T)$, we have xRz, the desired result. Alternatively assume $c(\bar{T}) = c(T)$. Since $y \neq c(\bar{T})$, if $c(\bar{T}) \neq c(\bar{T} \setminus y)$, we have yRx, contradicting asymmetry of R. Thus, $c(T) = c(\bar{T}) = c(\bar{T} \setminus y)$. If $c(\bar{T} \setminus y) = c(T \setminus y)$, the previous equality implies $c(T) = c(T \setminus y)$, contradicting the original hypothesis. Thus, $c(\bar{T} \setminus y) \neq c(T \setminus y)$. Since $x \neq c(\bar{T}) = c(\bar{T} \setminus y)$, this implies xRz, the desired conclusion. □

Let $\gg$ be any completion of R. Using $\gg$, we next define the reference function. For each $S \in \mathcal{X}$, let

$$r(S) = \arg\max(\gg, S).$$

Claim 5. *If $x \in S$ is such that $x \notin \{r(S), c(S)\}$, then $c(S) = c(S \setminus x)$.*

Proof. Suppose we have $x \neq r(S)$ and $x \neq c(S) \neq c(S \setminus x)$. By definition of R, this implies $xRr(S)$, contradicting the definition of r. □

Now for each $\rho \in X$, we define the following binary relation. Let $xP_\rho y$ if there exists a choice set such that $r(S) = \rho$, $c(S) = x$ and $y \in S$.

Claim 6. *For each $\rho \in X$, P_ρ is a transitive binary relation.*

Proof. Suppose $xP_\rho y$ and $yP_\rho z$. Then there exists S and T with $y \in S$ and $z \in T$ such that $r(S) = r(T) = \rho$, $c(S) = x$ and $c(T) = y$. Now consider $S \cup T$. Since $r(S) = r(T) = \rho$, we must have $r(S \cup T) = \rho$. We claim that $c(S \cup T) = x$. By the way of contradiction, suppose $c(S \cup T) = t \neq x$. Since $t \in S \cup T$, either $t \in S$ or $t \in T$. If $t \in S$, then by repeated application of the previous claim we get that $c(S) = t$, which is a contradiction as $t \neq x$. Suppose $t \in T \setminus S$. By repeated application of the previous claim, we get that $c(T) = t$ so that $t = y$. But this is a contradiction as $y \in S$. Hence, we conclude that $c(S \cup T) = x$. Since $r(S \cup T) = \rho$ and $z \in S \cup T$, by definition, $xP_\rho z$. □

For each $r \in X$, let $\succ_\rho$ be any completion of P_ρ and let U_ρ be the utility representation of $\succ_\rho$, i.e., $x \succ_\rho y$ if and only if $U_\rho(x) > U_\rho(y)$. Now by definition, $x = c(S)$ if and only if $U_{r(S)}(x) > U_{r(S)}(y)$ for all $y \in S \setminus x$. This completes the proof of the theorem. ■

Proof of Theorem 2

Let R be as in the proof of Theorem 1. Note that then, R is asymmetric and transitive. We next define a second binary relation, which we will later use together with R to construct the conspicuity ranking. For each $x, y \in X$ such that $x \neq y$, let $xR'y$ if $\neg(xRy)$, $\neg(yRx)$ and $c(x, y) = x$.[22]

[22]The notation $\neg(xRy)$ denotes "not xRy".

Claim 7. *If $xR'yR'z$, then $\neg(zRx)$.*

Proof. First note that, x, y, z are distinct alternatives. Also, by definition $c(x, y) = x$ and $c(y, z) = y$. Now, if $c(x, y, z) = y$, we have $z \neq c(x, y, z) \neq c(x, y)$ implying zRy. This contradicts $yR'z$. Similarly, if $c(x, y, z) = z$, then $x \neq c(x, y, z) \neq c(y, z)$ implying xRy. This contradicts $xR'y$. Therefore, $c(x, y, z) = x$.

Suppose zRx. Then there exists $S \supset \{x, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$. Note that since $yR'z$, $y \notin S$. Let $\bar{S} = S \cup \{y\}$. There are two cases to consider.

Case 1: $c(\bar{S}) \neq y$.

Since $\neg(yRx)$, we have $c(S) = c(\bar{S})$ and thus, $z \neq c(\bar{S})$. Since $\neg(zRy)$, we then have $c(\bar{S}) = c(\bar{S} \setminus z)$. But then, $y \neq c(\bar{S} \setminus z)$. This, and $\neg(yRx)$ imply $c(\bar{S} \setminus z) = c(S \setminus z)$. But combining these equalities, we get $c(S) = c(S \setminus z)$, a contradiction.

Case 2: $c(\bar{S}) = y$.

Since $c(\{x, y, z\}) = x$, the set $\bar{S} \setminus \{x, y, z\} = \{t_1, ..., t_n\}$ is nonempty. For each $i \in \{1, ..., n\}$, let $T_i = \{x, y, z, t_i\}$. Since $c(\bigcup_{i=1}^{n} T_i) = c(\bar{S}) \in \bigcap_{i=1}^{n} T_i$, by Claim 2 $c(\bar{S}) = c(T_i)$ for some $i \in \{1, ..., n\}$. But then $t_i \neq y = c(x, y, z, t_i) \neq c(x, y, z) = x$, implies t_iRz. This contradicts zRt_i due to Claim 1.

Since both cases lead to a contradiction we cannot have zRx. □

We define the conspicuity ranking as $\gg = R \cup R'$. The next claim establishes that $\gg$ is a strict linear order.

Claim 8. *The conspicuity ranking $\gg$ is a strict linear order.*

Proof. For any $x \neq y$, either x and y are compared by R or by R'. Hence, $\gg$ is weakly connected. By definition, it is also irreflexive. To establish that $\gg$ is a strict linear order, we show that $\gg$ is transitive. Assume $x \gg y \gg z$. We have a few cases to consider.

Case 1: $xRyRz$.

We then have xRz by Claim 4.

Case 2: $xR'yR'z$.

By Claim 7, $\neg(zRx)$. If xRz, we are done. Alternatively assume $\neg(xRz)$. Note that by definition, $c(x, y) = x$ and $c(y, z) = y$. Now, if $c(x, y, z) = y$, we have $z \neq c(x, y, z) \neq c(x, y)$ implying zRy. This contradicts $yR'z$. Similarly, if $c(x, y, z) = z$, then $x \neq c(x, y, z) \neq c(y, z)$ implying xRy. This contradicts $\neg(xRy)$. Therefore, $c(x, y, z) = x$. If $c(x, z) = z$, then $y \neq c(x, y, z) \neq c(x, z)$, contradicting $yR'z$. Therefore, $c(x, z) = x$. This implies $xR'z$.

Case 3: $xRyR'z$.

Note that if zRx, by Claim 4 we have zRy. This contradicts $yR'z$. Alternatively if $zR'x$, by Claim 7 we have $\neg(xRy)$, again a contradiction. Therefore, either xRz or $xR'z$.

The case where $xR'yRz$ is similar to Case 3. □

Using $\gg$, we next define the reference function. For each $S \in \mathcal{X}$, let

$$r(S) = \arg\max(\gg, S).$$

Claim 5 implies that if $x \in S$ is such that $x \notin \{r(S), c(S)\}$, then $c(S) = c(S \setminus x)$.

Given two alternatives x, y which are dominated by z according to binary relation R, it is useful to understand under which conditions we can find a choice set which includes $\{x, y, z\}$ and z causes choice reversal in that set. The next claim provides a sufficient condition which will be useful in the proof of the final claim.

Claim 9. *Assume zRx, zRy and $c(x, y) = y$. Then either xRy or there is $S \supseteq \{x, y, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$.*

Proof. Since zRx and zRy, there are $T_1, T_2 \in \mathcal{X}$ such that $x \in T_1$, $y \in T_2$, $z \neq c(T_1) \neq c(T_1 \setminus z)$ and $z \neq c(T_2) \neq c(T_2 \setminus z)$.

Let $\bar{T}_2 = T_2 \cup \{x\}$. If $T_2 = \bar{T}_2$, letting $S = T_2$ concludes the proof. Alternatively, assume $T_2 \neq \bar{T}_2$. If $c(\bar{T}_2) = z$, then $x \neq c(\bar{T}_2) \neq c(T_2)$, implying xRz. This contradicts asymmetry of R. So $c(\bar{T}_2) \neq z$. We have two cases.

Case 1: $c(\bar{T}_2) = x$.
Since $z \neq c(T_2) \neq c(T_2 \setminus z)$, Single Reversal then implies $c(x, y, z) = x$. But then $z \neq c(x, y, z) \neq c(x, y)$ and letting $S = \{x, y, z\}$ concludes the proof.

Case 2: $c(\bar{T}_2) \neq x$.
Then, by asymmetry of R, $c(T_2) = c(\bar{T}_2)$. Also, if $c(\bar{T}_2) \neq c(\bar{T}_2 \setminus z)$, letting $S = \bar{T}_2$ concludes the proof. So assume $c(\bar{T}_2) = c(\bar{T}_2 \setminus z)$. Then $x \neq c(\bar{T}_2 \setminus z)$. If $c(\bar{T}_2 \setminus z) = c(T_2 \setminus z)$, then $c(T_2) = c(\bar{T}_2) = c(\bar{T}_2 \setminus z) = c(T_2 \setminus z)$, which is a contradiction to our original hypothesis. So $c(\bar{T}_2 \setminus z) \neq c(T_2 \setminus z)$. But then xRy, concluding the proof. $\square$

We next define the psychological constraint function $Q : X \to \mathcal{X}$ as

$$Q(x) = \{y \in X \mid r(x, y) = x \text{ and } c(x, y) = y\}.$$

Our next objective is to define the preference ranking $\succ$. For each $x, y \in X$ such that $x \neq y$, let xPy if there are $z \in X \setminus x$ and $T, S \in \mathcal{X}$ such that $\{x, y, z\} \subseteq T \subseteq S$, $z \neq c(S) \neq c(S \setminus z)$, $c(T) = x$, and $c(y, z) = y$.

Claim 10. *P is acyclic.*

Proof. Suppose $x_1 P x_2 P \cdots P x_n P x_{n+1}$ where $x_1 = x_{n+1}$. Then there are $\{(z_i, T_i, S_i)\}_{i=1}^n$ such that for each $i \in \{1, ..., n\}$, we have $\{x_i, x_{i+1}, z_i\} \subseteq T_i \subseteq S_i$, $z_i \neq c(S_i) \neq c(S_i \setminus z_i)$, $c(x_{i+1}, z_i) = x_{i+1}$, and $c(T_i) = x_i$. Now consider the set $S^* = \{x_2, \ldots, x_{n+1}\}$. For any $x_i \in S^*$, there is z_{i-1} such that $z_{i-1} \neq c(S_{i-1}) \neq c(S_{i-1} \setminus z_{i-1})$, $c(x_i, z_{i-1}) = x_i$, $\{x_i, z_{i-1}\} \subseteq T_{i-1} \subseteq S_{i-1}$ and $c(T_{i-1}) \in S^*$, but $c(T_{i-1}) \neq x_i$, contradicting Consistency. $\square$

Finally, we define the preference ranking $\succ$ to be any completion of P. The next claim shows that $c(S)$ is $\succ$-maximal in $Q(r(S)) \cap S$, thus concluding the proof.

Claim 11. *$c(S)$ maximizes $\succ$ in $Q(r(S)) \cap S$.*

Proof. First, if $c(S) = r(S)$, then by Claim 5, $c(r(S), x) = r(S)$ for all $x \in S$. Hence, $Q(r(S)) \cap S = \{r(S)\}$ and the claim trivially holds.

Alternatively assume $c(S) \neq r(S)$. By Claim 5, $c(r(S), c(S)) = c(S)$ which implies $\neg(r(S)R'c(S))$. But then by Claim 8, $r(S)Rc(S)$. Hence there is $T \supset \{r(S), c(S)\}$ such that $r(S) \neq c(T) \neq c(T \setminus r(S))$. Then by definition of P, $c(S)Pr(S)$. Thus, $c(S) \succ r(S)$.

Notice that $c(r(S), c(S)) = c(S)$ implies $c(S) \in Q(r(S))$. Now let $x \in Q(r(S)) \cap S$ be such that $x \notin \{c(S), r(S)\}$. By definition, $c(x, r(S)) = x$. We need to show that $c(S)Px$. There are two possible cases.

Case 1: $c(x, c(S)) = x$.
By Claim 5, $c(x, r(S), c(S)) = c(S)$. Hence, $r(S) \neq c(x, r(S), c(S)) \neq c(x, c(S))$. Since $x = c(x, r(S))$, by definition of P, $c(S)Px$.

Case 2: $c(x, c(S)) = c(S)$.
Since $c(r(S), c(S)) = c(S)$ and $c(x, r(S)) = x$, we have $\neg(r(S)R'x)$ and $\neg(r(S)R'c(S))$. Therefore, by Claim 7, it must be the case that $r(S)Rx$ and $r(S)Rc(S)$. Then by Claim 9, either there is $T \supseteq \{x, r(S), c(S)\}$ such that $r(S) \neq c(T) \neq c(T \setminus r(S))$ or $xRc(S)$. In both cases $c(S)Px$ follows. ■

Independence of Axioms

See footnote 12 for an example of choice data that satisfies Single Reversal but not Consistency. To see that Consistency does not imply Single Reversal, let $c(x, y, z) = x$, $c(x, y) = c(y, z) = y$ and $c(x, z) = z$. Then removing either y or z from $\{x, y, z\}$ causes choice reversal. However these choices satisfy Consistency.

Proof of Theorem 3

Let $\gg$ and $Q : X \to \mathcal{X}$ be as in the proof of Theorem 2. We define $Q' : X \to \mathcal{X}$ by

$$Q'(x) = \{y \in X | \, y \in Q(z) \text{ where } z \gg x \text{ or } z = x\}$$

Notice that Q' is ordered with respect to $\gg$. For any $x \neq y$, we define xP_oy if

- there exist z and $S \supseteq \{x, y, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$, $c(y, z) = y$, and $c(x, y, z) = x$; or,
- there exist $T \subset S$ containing $\{x, y\}$ such that $c(T) = x$ and $c(S) = y$.

Acyclicity axiom states that P_o is acyclic. Let $\succ$ be any completion of P_o.

Claim 12. *$c(S)$ maximizes $\succ$ in $Q'(r(S)) \cap S$.*

Proof. Firstly, suppose $c(S) = r(S)$. By Single Reversal, $c(x, r(S)) = r(S)$ for all $x \in S$. Hence, $\{r(S)\} = Q(r(S)) \cap S$. This implies that if $x \in Q'(r(S)) \cap (S \setminus r(S))$, then there exists $y \gg r(S)$ such that $c(x, y) = x$. Then x, y, and $r(S)$ are distinct as $y \gg r(S) \gg x$. There are three cases to consider.

Case 1: $c(x, y, r(S)) = y$.
Notice that $r(S) \neq c(x, y, r(S)) \neq c(x, y)$. This implies $r(S) \gg y$, a contradiction.

Case 2: $c(x, y, r(S)) = x$.
Since $c(x, r(S)) = r(S)$, by definition, $r(S)P_o x$. Thus $r(S) \succ x$.

Case 3: $c(x, y, r(S)) = r(S)$.
Since $y \gg x$, we should have $c(y, r(S)) = r(S)$. Since we also have $c(x, y) = x$, it cannot be the case that $yR'r(S)$ or $yR'x$ where R' is defined as in the proof of Theorem 2. By definition of $\gg$, yRx and $yRr(S)$. Also notice that $c(x, r(S)) = r(S)$. By Claim 9, either there exists $T \supseteq \{x, y, r(S)\}$ such that $y \neq c(T) \neq c(T \setminus y)$ or $xRr(S)$. Since $xRr(S)$ is not possible, the former must be true. Then, by definition, $c(x, y, r(S)) = r(S)$ and $c(x, y) = x$ imply $r(S)P_o x$, and hence $r(S) \succ x$.

Now suppose $c(S) \neq r(S)$. Let $x \in Q'(r(S)) \cap (S \setminus c(S))$ be given. In the proof of Theorem 2, we already showed that $c(S) \succ x$ for all $x \in Q(r(S)) \cap (S \setminus c(S))$. Hence, we can assume $x \notin Q(r(S))$. Then there exists $y \gg r(S)$ such that $c(x, y) = x$. Since $r(x, c(S), r(S)) = r(S)$, we have $c(x, c(S), r(S)) = c(c(S), r(S)) = c(S)$. There are four cases to consider.

Case 1: $c(x, y, c(S), r(S)) = y$.
Since $r(x, y, c(S), r(S)) = y$, we have $c(x, y) = y$, a contradiction.

Case 2: $c(x, y, c(S), r(S)) = x$.
Since $c(x, c(S), r(S)) = c(S)$, by definition, $c(S)P_o x$ and we are done.

Case 3: $c(x, y, c(S), r(S)) = r(S)$.
Since $c(x, c(S), r(S)) = c(S)$, we have $y \neq c(x, y, c(S), r(S)) \neq c(x, c(S), r(S))$. Single Reversal implies $c(x, y, r(S)) = r(S)$. This together with $c(x, y) = x$ imply $r(S)P_o x$. Furthermore, $c(r(S), c(S)) = c(S)$ and $c(x, y, c(S), r(S)) = r(S)$ imply $c(S)P_o r(S)$. Hence, $c(S) \succ x$.

Case 4: $c(x, y, c(S), r(S)) = c(S)$.
Since $y \gg r(S)$, we have $c(x, y, c(S)) = c(S)$. If $c(x, c(S)) = x$, then $c(x, y, c(S)) = c(S) \neq c(x, c(S))$ and $c(x, y) = x$ imply $c(S)P_o x$ and we are done. Suppose $c(x, c(S)) = c(S)$. Since $y \gg x$, we have $c(y, c(S)) = c(S)$. Then it cannot be the case that $yR'c(S)$ or $yR'x$. Hence, by definition, $yRc(S)$ and yRx. By Claim 9, either there exists $T \supseteq \{x, y, c(S)\}$ such that $y \neq c(T) \neq c(T \setminus y)$ or $xRc(S)$. In both cases, $c(S)P_o x$ follows. ∎

Revealed Information Under PC-CER

Continuing the revealed preference discussion on Section 4, we illustrate here how to infer the DM's conspicuity ranking and psychological constraint from her observed choices, given that the choice data is consistent with PC-CER.

First, we extend Definition 2 to state what revealed psychological constraint means under PC-CER. Assume that c admits k different PC-CER representations, $(\succ_i, \gg_i, Q_i)_{i \in \{1,\dots,k\}}$. Then x is revealed to be in the psychological constraint of ρ if $x \in Q_i(\rho)$ for all i. Similarly, x is revealed to be outside the psychological constraint of ρ if $x \notin Q_i(\rho)$ for all i.

Recall that the construction in Theorem 2 uses the minimal possible Q. This guarantees that if the preference between x and y is not identified by P^T, then we can pick either $x \succ y$ or $y \succ x$ without affecting choice behavior.

We next use the same idea to provide a characterization of revealed psychological constraint. If we know that x is the reference point of a set T, the fact that y is chosen from T informs us that y belongs to the psychological constraint set of x. Given that the reference point is endogenously determined, we can learn that x is the reference point of T if it induces a choice reversal in T or a superset of it. Formally,

$$Q_M(x) = \{y \in X \mid \text{there is } S \supseteq T \supseteq \{x, y\} \text{ such that } x \neq c(S) \neq c(S \setminus x) \text{ and } y = c(T)\}.$$

As will be discussed later, observing choice reversals is not necessary for revealed conspicuity in PC-CER. In other words, x may be revealed to be more conspicuous than y even when x never causes a choice reversal in the existence of y. Since the definition above only uses choice reversals, one may wonder if it does fully capture revealed consideration. It turns out that the answer is yes. Indeed, if x and y cannot be compared by R, which is the binary relation capturing choice reversals, then whenever $c(x, y) = y$ we can construct a PC-CER representation of c with $y \gg x$. This is the construction used in the proof of Theorem 2. Hence, if x is revealed to be more conspicuous than y in the absence of a choice reversal, then $c(x, y) = x$ and $c(T) \neq y$ for any T with $r(T) = x$ must be true.

Choice data can also inform us whether an alternative lies outside the psychological constraint of another. Consider the observations that x is revealed to be preferred to y and $y = c(x, y)$. Then it must be the case that y is more conspicuous than x and x is outside the psychological constraint set of y. The opposite is also true. If x is not revealed to be preferred to y or $c(x, y) = x$, then we cannot reveal that x is outside the psychological constraint set of y. To see the first point, notice that if x is not revealed to be preferred to y, then there exists a representation $(\succ, \gg, Q)$ with $y \succ x$ and $x \in Q(y)$. To illustrate the second point, if $c(x, y) = x$, then in any representation $(\succ, \gg, Q)$, either x is more conspicuous than y in which case it is without loss to assume $x \in Q(y)$ or y is more conspicuous than x, which in turn implies $x \in Q(y)$. The following remark summarizes these points.

Remark 5. *(Psychological Constraint) Suppose c admits a PC-CER representation. Then (i) x is revealed to be in the psychological constraint set of y if and only if $x \in Q_M(y)$, (ii) x is revealed to be outside the psychological constraint set of y if and only if xP^Ty and $c(x, y) = y$.*

Finally, we discuss revealed conspicuity. As noted earlier, if x causes choice reversal when y is in the choice set, then we can conclude that x is more conspicuous than y. However, Example 2 suggests that more information about conspicuity can be revealed: the fact that x is revealed to be preferred to y and $y = c(x, y)$ informed us that y is more conspicuous than x. Hence, we need to modify the revealed conspicuity relation to accommodate this additional revelation: For any $x \neq y$

$$x\bar{R}y \text{ if } (i) \quad \exists S \supseteq \{x,y\} \text{ such that } x \neq c(S) \neq c(S \setminus x), \text{ or}$$
$$(ii) \quad yP^T x \text{ and } x = c(x,y).$$

Let $\bar{R}^T$ stand for the transitive closure of $\bar{R}$.

Remark 6. *(Revealed Conspicuity) Suppose c admits a PC-CER representation. If $x\bar{R}^T y$ then x is revealed to be more conspicuous than y.*

A natural question to ask is whether $\bar{R}^T$ characterizes revealed conspicuity. The following example shows that the answer is no.

Example 3. *Consider the following choice data on $X = \{x,y,z,t\}$.*

S	$xyzt$	xyz	xyt	xzt	yzt	xy	xz	xt	yz	yt	zt
$c(S)$	x	x	y	x	y	y	x	x	y	y	z

Since $z \neq c(x,y,z,t) \neq c(x,y,t)$ we have $z\bar{R}x$, $z\bar{R}y$, and $z\bar{R}t$. Furthermore, $c(x,y,z) = x$ and $c(y,z) = y$ imply $xPyPz$. Now xPy and $c(x,y) = y$ imply $y\bar{R}x$. Hence, we have $z\bar{R}y\bar{R}x$ and $z\bar{R}t$. Now suppose we take the following completion of $\bar{R}$: $z \gg t \gg y \gg x$. Even though $\gg$ includes $\bar{R}$ there is no PC-CER representation of c with this $\gg$. First notice that given $\gg$, any psychological constraint function rationalizing this data must satisfy $Q(z) \supseteq \{x,y,z\}$ and $Q(t) \supseteq \{x,y,t\}$. Furthermore, since the preference ranking must include P, we have $x \succ y$. But then $r(x,y,t) = t$, $Q(t) \cap \{x,y,t\} = \{x,y,t\}$ and $c(x,y,t) \neq x$. Hence, no PC-CER representation of this data with this $\gg$ exists.

The following table summarizes admissible preference-conspicuity combinations in Example 3.

	$t \succ x \succ y \succ z$	$x \succ t \succ y \succ z$	$x \succ y \succ t \succ z$	$x \succ y \succ z \succ t$
$z \gg y \gg x \gg t$	✓	✓	✓	✓
$z \gg y \gg t \gg x$	✗	✓	✓	✓
$z \gg t \gg y \gg x$	✗	✗	✗	✗

Notice that even though $y\bar{R}t$ does not hold, in all possible PC-CER representations we have $y \gg t$. To put it differently, there is no preference ranking that contains the revealed preference and is also compatible with the conspicuity ranking $z \gg t \gg y \gg x$. In this case, we say that $z \gg t \gg y \gg x$ is not consistent with revealed preference. Our next definition generalizes the intuition from Example 3.

Definition 7. *A conspicuity ranking $\gg$ is consistent with revealed preference if there is a completion of revealed preference P^T denoted by $\succ$ and a psychological constraint function Q defined by $Q(x) = \{y \in X | \, x \gg y \text{ and } c(x,y) = y\}$ such that*

$$c(S) = \arg\max(\succ, Q(\arg\max(\gg, S)) \cap S))$$

for all $S \in \mathcal{X}$.

Notice that if $(\succ, \gg, Q)$ is a CER representation of a choice function c, then $\succ$ must include revealed preference P^T, and, given $\gg$, Q must include the minimal possible psychological constraint. But then, by definition, $\gg$ is consistent with revealed preference. It then follows from definitions that for x to be revealed to be more conspicuous than y, it must be ranked higher in all conspicuity rankings consistent with revealed preference and vice versa.