“Take It or Leave It” Offers: Obstinacy and Endogenous Deadlines in Negotiations*

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Abstract

A bargainer uses “*take it or leave it*” offers to signal that he is not willing to make further concessions (*commitment threat*) and that he will leave the bargaining table unless his demand is accepted (*exit threat*). This paper investigates the impact of these threats on rational negotiators' equilibrium shares and behavior in a bilateral negotiation. The threats are credible because the negotiators are assumed to have the opportunity of mimicking obstinate types—who, for some reason, are constrained to implement their threats—and to build reputation on their obstinacy. The existence of the endogenous deadline option for one of the players has two main effects: (1) it renders the deal unique and efficient, and (2) shifts the bargaining power towards the player who can influence the deadline.

Keywords: Bargaining, Reputation, War of attrition, Continuous-time, Exit, Commitment, Behavioral types.

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1. Introduction

In episode 13 (season 6) of the famous TV series *House M.D.*, the main story revolves around the last eight hours of a critical negotiation between Lisa Cuddy (the dean of Medicine and hospital administrator of the fictional Princeton-Plainsboro Teaching Hospital in New Jersey) and a contract negotiator from Atlantic Net Insurance (the largest insurance company in the area) to renew a contract. Dr. Cuddy and the contract negotiator have been arguing about the contract for eight months, and that day, Dr. Cuddy lays it all on the line. When they meet at 8:30 a.m., Dr. Cuddy makes her final offer that she agrees to capitated care but wants a 12% increase in rates. The contract negotiator refuses Dr. Cuddy's offer immediately. Then she tells him that this is the hospital's final offer and he has until 3:00 p.m. to agree, or she will make a public announcement that they are no longer accepting Atlantic Net. At noon, Dr. Cuddy heads into the board meeting where the board makes it clear that her job is on the line if she cannot renew the contract with Atlantic Net. Dr. Cuddy tracks down the CEO of Atlantic Net at lunch and confronts him about the contract. He blows her off and tells her that her tactic will not work. However, Dr. Cuddy does not back down. The negotiator from Atlantic Net returns and offers an 8% increase as their final offer, and Dr. Cuddy declines and wants the full 12%. After a few stressful hours of waiting, the story ends with good news for Dr. Cuddy. The negotiator revisits her just before 3:00 p.m. (the deadline) and tells her that the company has agreed to her 12% proposal.

Negotiators commonly use "*take it or leave it*" offers as a final strategic maneuver in order to push their rivals towards more acceptable terms (e.g., labor negotiations). It usually begins with a milder threat such as "That is my final offer," and if it does not work, it escalates to a higher level: "Take this one or I am calling off the negotiation." If the bargainers' offers do not get closer to each other after a lengthy delay, followed by offers and counteroffers, then the "take it or leave it" offer could be an optimal strategy to speed things up. This paper is not interested in explaining how negotiators come to this point at the first place, where they threaten each other so vigorously. Instead, the focus is investigating the impacts of these bluffs on rational negotiators' equilibrium strategies and shares in a bilateral negotiation and examining a possible case where the threatened negotiator can mitigate the impacts of these bluffs.

Intimidating the opponent with a threat of not making further concessions (i.e., *commitment threat*) or of leaving the bargaining table (i.e., *exit threat*) would increase the equilibrium share of a bargainer only if they are credible. In this paper, I follow a reputational approach for both commitment and exit threats. For this purpose, I study a stylized four-stage infinite-horizon and continuous-time bargaining game that is adapted from Kambe (1999). The novel twist is that one of the agents announces a deadline for agreement. In particular, there are four defining features of the model. First, two negotiators bargain over the division of a surplus; negotiators begin with announcing their demands. Second, in case the demands are incompatible, negotiator 1 declares a deadline for agreement. Third, they face some small probability of becoming committed to their demands, and for negotiator 1, to walking away at the self-imposed deadline. Fourth, the negotiation phase adopts a war of attrition protocol. During the negotiation phase, each agent knows his type (*obstinate* or *flexible*) but is unsure about the opponent's true type. This uncertainty provides incentives to the flexible negotiators to build reputation on their obstinacy, affecting the flexible negotiators' equilibrium play.

This exercise is important for two reasons. First, commitment and exit threats are studied separately in the bargaining literature, but the current work is the first one that combines these two. Second, allowing a negotiator to leave the bargaining table imposes an endogenous deadline for reputation building that has not been studied in the bargaining and reputation literature before.[1] Exit-threat appears to be an important tactic in reality, but it is usually used together with other threats (e.g., commitment) as a last resort. A weak negotiator, whose opponent has strong bargaining power due to his ability to commit (e.g., reputational advantage), can make his commitments less credible or likely to be effective by using exit-threat. This link has not been explored in the literature, and the current study offers a mechanism to explain this link.

The analysis reveals that the key feature is that negotiator 1 chooses as early a deadline as possible with the constraint that she must have sufficient time to build a reputation for stubbornness $\hat{z}_1(t)$, which is increasing over time, so that negotiator 2 strictly prefers accepting negotiator 1's offer when the deadline K is reached rather than inducing an impasse without a deal with probability $\hat{z}_1(K)$. Agreement is immediate because the negotiators forecast this deadline strategy when choosing their bargaining postures and negotiator 2 adopts the greatest just compatible posture. Thus, the existence of the endogenous deadline option for one of the players has two main effects (vis-a-vis Kambe, 1999 and Abreu and Gul, 2000): (1) it renders the deal unique and efficient, and (2) shifts the bargaining power towards (and thus increases the share of) the player who can influence the deadline.

Kambe (1999) studies a simpler version of the current model, where none of the negotiators has the option of using exit threat. He shows that there are multiple equilibrium outcomes, some of which are inefficient. However, the set of equilibrium outcomes always contains an efficient allocation, and the negotiators' equilibrium demands get closer to this efficient allocation as the negotiators's commitment probabilities approach zero. For example, if the negotiators' time preferences and commitment probabilities are the same, then Kambe (1999) proves that each negotiator demands the half of the surplus (i.e., 0.5) or something closer to that. This paper, on

[1]Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in an Abreu-Gul setting and explore the role of exogenous outside options. They show that if both agents' outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility, and the unique equilibrium is again the Rubinstein (1982) outcome. Ozyurt (2015 a&b) study a three-player war of attrition game in a competitive environment, where one of the players has the option of leaving his bargaining partner to negotiate with the other. However, Ozyurt (2015 a&b) assumes that the negotiator—who goes back and forth between his opponents—has no commitment for a specific deadline and is free to revisit his bargaining partners as as much as he wants.

the other hand, shows that negotiator 1 can improve his share and payoff approximately 40% when each negotiator's commitment probability is 0.1, and 27.6% when this probability is only 0.01.[2]

In equilibrium, conditional on that negotiators' initial offers are incompatible, negotiator 1 always chooses the exit-time/deadline different from 0. The equilibrium value of the exit-time (i.e., K) is a function of the following: both negotiators' initial demands α_1 and α_2 (where $\alpha_i \in [0, 1]$ denotes negotiator i's share of the surplus), the prior probability that negotiator 1 is the obstinate type (i.e., 1's commitment probability), and negotiator 2's time preferences. If negotiator 2 is more patient, then the deadline is longer (i.e., K is bigger). On the other hand, as negotiator 2 gets more impatient, then negotiator 1 chooses a shorter deadline. Put differently, negotiator 1 applies more time pressure on her rival (by shortening the deadline) if her rival is impatient.

Fixing the negotiators' incompatible initial demands, a shorter deadline (i.e., smaller K) increases negotiator 1's expected payoff if the deadline is not too short. That is, the bigger the time pressure negotiator 1 would impose on her rival, the earlier her rival would concede, and thus, the more surplus (in ex-ante terms) negotiator 1 could extract. However, in equilibrium, negotiator 1 never chooses a deadline so close to 0: This is true because (1) such an immediate exit threat is credible only if negotiator 1's initial demand (i.e., α_1) is very close to $1 - \alpha_2$ (i.e., negotiator 1's demand is almost compatible), and (2) negotiator 1 could increase her expected payoff by choosing an extended deadline and a more aggressive demand. The bottom line is that there is a trade-off between aggressiveness in demand and the length of a deadline in the exit threat.

In equilibrium, negotiators never choose extreme demands. The exit threat always makes agent 1 advantageous in the sense that if both agents' time preferences and commitment probabilities are the same, then negotiator 1's equilibrium share is greater than negotiator 2's. Equilibrium strategies, including the negotiators' initial demands, are unique. This unique equilibrium is efficient in the sense that negotiator 1 offers $\bar{\alpha}_1$, which is a function of both negotiators' commitment probabilities and time preferences, and negotiator 2 offers $1 - \bar{\alpha}_1$ and ends the game with no delay. The intuition behind this uniqueness result is as follows: First, if negotiator 1's initial offer is less than or equal to $\bar{\alpha}_1$, then negotiator 1 can always choose her deadline K in such a way that she is the "*strong*" negotiator and her opponent is the "*weak*" negotiator in the game. Being the strong negotiator in the game is more desirable than being the weak negotiator. In equilibrium where the negotiators' initial demands are α_1 and α_2, expected payoff of the weak negotiator 2 is less than (or equal, if the demands are compatible) to $1 - \alpha_1$. Second, given the negotiators' equilibrium payoffs, negotiator 2's best reply is to make a compatible offer and to finish the game in the first stage. Finally, given the second negotiator's best response,

[2]Please see example 1 in page 15.

the optimal strategy for the first negotiator is to make the highest offer possible, which is $\bar{\alpha}_1$.[3]

Unlike Crawford (1982), Muthoo (1996), and Ellingsen and Miettinen (2008), rationalizing the players' commitment is not a concern for this paper. I take that mechanism as given and assume that prior to their demand (and deadline) selections, the negotiators are uncertain whether they will commit or not. Nevertheless, commitment is risky and costly for the negotiators because it involves a possibility of an impasse and 0 payoff. If the negotiators insure themselves against the cost of commitment (e.g., through a pre-agreement that is signed privately with their principals), then they may take higher risks with more aggressive demands/deadline. Insurance can be a negotiator's informational advantage (vis-a-vis his opponent) at the beginning of the game about his future-self.

The second negotiator, who is not able to choose a deadline, can diminish the effects of his rival's exit threat by insuring himself against some of the utility loss he will suffer when he commits to his demand. I will prove the results for a "*partial-insurance*" case where the compensation amounts are minimal. However, various other insurance schemes would yield the same results. Negotiator 2's share and expected payoff are higher in the partial-insurance case. The reason for this result is simple. When negotiator 2 is partially insured, he will be protected against the risk of an impasse (and 0 payoff) in case he commits to his demand. As a result of this, negotiator 2 will be able to make more aggressive initial demands, which partially neutralizes the effects of negotiator 1's exit threat. There are multiple equilibrium demands when negotiators are partially insured. The upper bound for negotiator 1's initial offers is $\bar{\alpha}_1$. That is, the efficient outcome of the no-insurance case is just one of the equilibrium outcomes in the partial-insurance case; all the others are inefficient because the negotiators' demands are incompatible and there is some (probabilistic) delay in reaching agreement. In any equilibrium under the partial-insurance case, where negotiators' initial offers are α_1 and α_2, negotiator 2 is always weak in the game, but his expected payoff is $1 - \alpha_1$. Although negotiator 2's expected payoff in the game is exactly equal to what he can achieve by conceding to negotiator 1 right away, the temptation of reaching a better deal (where negotiator 1 accepts α_2 with some positive probability) causes a delay in equilibrium. However, this temptation fades away when there is no insurance.

Section 2 explains the details of the infinite-horizon bargaining game. Section 3 presents the main results of the study. Section 4 considers the bargaining phase (i.e., the concession game) and characterizes the concession strategies. Section 5 discusses some extensions, and Section 6 concludes and discusses the related literature.

[3] In any equilibrium, the first negotiator's highest demand should be lower than $\bar{\alpha}_1$ because if negotiator 1 ever offers α_1 that is higher than $\bar{\alpha}_1$, then negotiator 2 can deviate to a demand that is sufficiently close to but higher than $1-\alpha_1$ and make himself strong (with a payoff more than $1-\alpha_1$) regardless of negotiator 1's deadline choice.

2. Preliminaries

Two negotiators, 1 (she) and 2 (he), bargain over the division of a surplus. For $i \in \{1, 2\}$, negotiator i's rate of time preference is r_i. The *bargaining game* between the negotiators is a four-stage infinite-horizon, continuous-time game. All stages start at time 0, and the first three stages also end at time 0. In stage 1, negotiators simultaneously demand a share of the surplus. Let $\alpha_i \in [0, 1]$ denote negotiator i's first-stage demand. If the negotiators' demands are compatible (i.e., $\alpha_1 + \alpha_2 \leq 1$), then the game ends, and one of the two divisions of surplus $(\alpha_1, 1 - \alpha_1)$ or $(1 - \alpha_2, \alpha_2)$ is implemented with a probability of $1/2$ each. The bargaining game continues to the second stage if the negotiators' initial demands are incompatible. In stage 2, after observing his opponent's demand, negotiator 1 announces a particular time $K \in [0, \infty)$ as the deadline for the game.

In stage 3, nature *independently* and *privately* sends one of two messages, c or d, to each negotiator. Negotiators commit to their demands and negotiator 1 leaves the bargaining game at time K if they receive the message c. Each negotiator i receives the message c with probability $z_i \in (0, 1)$. However, if a negotiator receives the message d, "don't commit," he will continue to play the game with no commitment to his initial demand and deadline. Once the third stage is finalized, each negotiator knows his own type (either flexible or obstinate), but not the opponent's true type. The initial priors (i.e., z_1 and z_2) are common knowledge. Therefore, similar to that in Section 4 of Crawford (1982), Kambe (1999), Wolitzky (2012), and Ellingsen and Miettinen (2014), the probability of obstinacy is independent of the chosen demands, and for player 1, of the chosen deadline. No discounting applies before time $t = 0$.

Upon the beginning of stage 4 (still at time 0), negotiators immediately begin to play the following continuous-time *concession game*: At any given time $t \geq 0$, a negotiator either accepts his opponent's initial demand or waits for his concession. Concession of a negotiator marks the completion of the game. Negotiator 2 can never leave the bargaining table. However, negotiator 1—whether she is flexible or not—can leave the game at any time $t \geq 0$. The deadline K is not binding for the flexible players. That is, the game may continue forever if the flexible negotiator 1 never leaves the bargaining game. Since the game is in continuous time, there occurs some measure-theoretic pathologies associated with the behaviors of negotiator 1's flexible and obstinate (committed) types, and I resolve this in the manner introduced by Abreu and Pearce (2007) and later used by Abreu, Pearce and Stacchetti (2012). In particular, for any $t \geq 0$, corresponding to the "conventional time" t, I suppose two logically consecutive stages t_1 and t_2 of time t. No discounting applies between these "two stages." A negotiator can concede to his opponent at both stages t_1 and t_2. However, negotiator 1 can leave the bargaining table only at time t_1.

If a flexible negotiator leaves the bargaining table, then he receives the outside option of 0. If agreement is never reached, the negotiators also receive 0. Both negotiators are risk neutral.

If the game finishes at time $t \geq 0$ with negotiator i's concession, then the payoffs to negotiators i and j are $(1-\alpha_j)e^{-r_i t}$ and $\alpha_j e^{-r_j t}$, respectively.[4] Flexible negotiators maximize the expected discounted values of their shares. The entire structure of the game is common knowledge among the negotiators. *I denote this infinite-horizon, continuous-time bargaining game by* G.

There are examples of negotiations between fiduciaries or agents (e.g., lawyers, union leaders, sports agents, political leaders) and not between the principals themselves, where the preferences of the principals and their delegates may or may not diverge in the course of a settlement. Therefore, one can naturally construe the bargaining game G and the third stage from this perspective. When a negotiator (agent) receives the message d, he may follow a strategy in which he waits until some time $t \leq K$ and concedes at this time if the game has not ended yet. However, when a negotiator receives the message c, he will be constrained to wait until time K and receive the payoff of 0 if his rival has also received the message c. Therefore, in equilibrium, the expected payoff of a negotiator when he receives the message c is strictly less than his expected payoff when he receives the message d.

If the negotiators are somehow insured against the payoff loss of commitment, then the negotiators may be inclined to take higher risks by making more aggressive demands in the first stage. Insurance can be thought of a valuable outside option or a side payment directly paid by a third party; for example, the principals. Alternatively, insurance can be the negotiators' informational advantage at the beginning of the game about their future-selves. For example, the nature's move in the third stage would be interpreted as the third party's (i.e., principals') move that determine whether their agents will commit for the rest of the bargaining game or not. Thus, a privately signed pre-agreement between a negotiator and his principal, ensuring that the principal will not force the agent to commit, could work as an insurance. An important question naturally arises when an effective insurance exists: which negotiator will benefit from such an insurance? This is investigated in the second part of Section 3.

The Information Structure and Obstinate Types: The only source of uncertainty in the game is the negotiators' actual types, and it matters only in the fourth stage of the game. In the first two stages, all negotiators choose their strategies, given their beliefs, to maximize their expected payoffs, and this is common knowledge. However, following the third stage, a negotiator is either flexible or obstinate. As is standard in the literature, the obstinate types never back down from their demands. In addition to this, the obstinate type of negotiator 1 leaves the bargaining game at time K (in particular, at stage K_1) that negotiator 1 announced in stage 1.

[4] In case of simultaneous concession, one of the two divisions of surplus, $(\alpha_1, 1-\alpha_1)$ or $(1-\alpha_2, \alpha_2)$, is implemented with the probability of $1/2$ each.

Strategies of the Flexible (Rational) Negotiators: In stage 1, a strategy for negotiator i is a pure action $\alpha_i \in [0,1]$. Since the subsequent analysis is quite involved, I restrict the negotiators to play pure strategies in stage 1. Given the negotiators' demand selections, a strategy for negotiator 1 in stage 2 is also a pure action $K \in [0,\infty)$. Although K depends on α_1 and α_2, this connection is omitted for notational simplicity.

In the war of attrition phase (i.e., in stage 4), a strategy for negotiator i is a right-continuous distribution function $F_i : [0,\infty) \to [0,1]$, representing the probability of flexible negotiator i conceding to negotiator j by time t (inclusive). Once again, F_i depends on α_1, α_2, and K, but I will omit this relation for notational simplicity. Let $B_i : [0,\infty) \to [0, 1-z_i]$ denote the probability of negotiator i conceding to negotiator j by time t (inclusive). That is, B_i indicates negotiator j's belief about i accepting α_j and finishing the concession game prior to time t. Therefore, we have $B_i(t) = F_i(t)(1-z_i)$.

Note that flexible negotiator 1 receives the payoff of 0 if she leaves the bargaining table. Hence, it is never optimal for her to leave the bargaining game. For this reason, I ignore flexible negotiator 1's exit strategies. Suppose that negotiators' strategies are α_1, α_2, and K in the first two stages and F_1 and F_2 in the fourth stage. Therefore, given that negotiator i receives message d in stage 3, flexible negotiator i's expected payoff of conceding to negotiator j at time t is

$$\begin{aligned} U_i^d(t, F_j) \equiv \alpha_i \int_{y<t} e^{-r_i y} dB_j(y) + \frac{1}{2}(1+\alpha_i-\alpha_j)[B_j(t) - B_j(t^-)]e^{-r_i t} \\ + (1-\alpha_j)[1 - B_j(t)]e^{-r_i t}, \end{aligned} \tag{1}$$

with $B_j(t^-) = (1-z_j)\lim_{y\uparrow t} F_j(y)$. Moreover, given that negotiator i receives the message c in the third stage, obstinate negotiator i's expected payoff in the fourth stage is[5]

$$U_i^c(F_j) = \alpha_i \int_{y\in[0,\infty)} e^{-r_i y} dB_j(y). \tag{2}$$

Therefore, negotiator i's expected payoff in the game is[6]

$$U_i(F_i, F_j) = (1-z_i) \int_{y\in[0,\infty)} U_i^d(y, F_j) dF_i(y) + z_i\, U_i^c(F_j). \tag{3}$$

[5] Note that $\int_{y\in[0,\infty)} e^{-r_i y} dB_j(y) = B_j(0) + \int_0^\infty e^{-r_i y} dB_j(y)$.

[6] The payoffs presented in (1) and (2) are calculated at the beginning of stage 4, that is after the nature's move. However, the payoff in (3) is calculated before the nature's move.

A Benchmark Result

Kambe (1999) considers a simpler version of the current model, where none of the negotiators can exit the game and announce an exit time. Therefore, the negotiators' concession game strategies are as given by Abreu and Gul (2000). Kambe shows that the immediate settlement (x_1^*, x_2^*) is an equilibrium outcome, where[7]

$$x_i^* = \frac{r_j \log z_j}{r_i \log z_i + r_j \log z_j}.$$

Therefore, if the negotiators are identical, then $x_i^* = 0.5$ for each i is an equilibrium outcome. There are other (inefficient) equilibrium outcomes where each negotiator demands a share that is not too far from 0.5.

3. Main Results

I describe flexible negotiator i's behavior in the concession game by a probability distribution over concession times, $F_i(t) = Pr(\text{flexible } i \text{ will quit prior to } t)$, where we allow $F_i(0) > 0$, so i may concede immediately with positive probability. Let $\lambda_i(t)$ be flexible negotiator i's instantaneous concession (or hazard) rate at time t conditional on that no negotiator has conceded before this time. That is, $\lambda_i(t) = \frac{dB_i(t)/dt}{1-B_i(t)} = \frac{(1-z_i)dF_i(t)/dt}{1-(1-z_i)F_i(t)}$. Here we look for an equilibrium where flexible negotiator j mixes between accepting α_i and waiting.[8] Therefore, flexible j is indifferent between conceding at time t and waiting for an infinitesimal period Δ and then conceding at time $t + \Delta < K$, where K is negotiator 1's deadline announcement in stage 2, if and only if

$$(1-\alpha_i)e^{-r_j t} = \alpha_j e^{-r_j t}\lambda_i(t)\Delta + [1-\lambda_i(t)\Delta](1-\alpha_i)e^{-r_j(t+\Delta)},$$

where $\lambda_i(t)\Delta$ is the probability that i concedes during the interval Δ. Solving this equation for $\lambda_i(t)$ and taking its limit as Δ approaches 0 yields

$$\lambda_i(t) = \frac{r_j(1-\alpha_i)}{\alpha_1 + \alpha_2 - 1}.$$

Integrating up the hazard rate gives

$$F_i(t) = \frac{1}{1-z_i} - \frac{c_i}{1-z_i}e^{-\lambda_i t},$$

[7]Kambe claims that there are multiple equilibria and negotiator i's equilibrium demands are in an ϵ neighborhood of x_i^*, where ϵ is a factor of the probability that both negotiators are obstinate (i.e., $z_1 z_2$). However, he does not provide a formal statement or a proof.

[8]In the next section, I formally prove that there is no pure strategy equilibrium of the concession game (see Propositions 2 and 3).

where $c_i = 1 - F_i(0)$.

Therefore, if negotiator 1 chooses $K > 0$, negotiators' equilibrium strategies are somewhat constrained by the fixed hazard rates λ_1 and λ_2. Different values of $K > 0$ will change the horizon of the game G, beyond which no negotiator will concede. Thus, the equilibrium values of $F_1(0)$ and $F_2(0)$ will change with K.

A weak negotiator, whose opponent has strong bargaining power due to his ability to commit, can make his commitments less credible or likely to be effective by using exit-threat. This link is explored in this study. To understand the mechanism that explains this link, suppose for now that the negotiators have identical demands and time preferences (i.e., $\alpha_i = \alpha > 1/2$ and $r_i = r$ for $i = 1, 2$), but negotiator 2 has a reputational advantage (i.e., $z_2 > z_1$). In equilibrium, the negotiators' concession game strategies, $F_1(t)$ and $F_2(t)$, must be continuous and increasing functions of t with a uniquely determined hazard (growth) rate $\lambda = \frac{r(1-\alpha)}{2\alpha-1}$ as briefly discussed above. Concession game strategies may have jumps only at $t = 0$, but we must have $F_1(0) \times F_2(0) = 0$. That is, in equilibrium, both negotiators cannot concede with positive probabilities at $t = 0$ because j would always want to wait for some small $\epsilon > 0$ amount of time in order to enjoy the discrete chance of i quitting. However, the identity of the negotiator who concedes with a positive probability at $t = 0$ is critical for players' equilibrium payoffs. If negotiator j concedes with a positive probability at time 0 (i.e., $F_j(0) > 0$), then negotiator i's payoff is strictly positive (i.e., negotiator i is strong), and his opponent's (i.e., negotiator j's) game payoff is strictly smaller than i's payoff. For this reason, we call negotiator j as the weak negotiator. Finally, given the flexible negotiator i's strategy, his reputation at time t (conditional on that the game has not yet finished) is given by the Bayes' rule as

$$\hat{z}_i(t) = \frac{z_i}{z_i + (1 - z_i)(1 - F_i(t))}.$$

These claims are standard results in the bargaining and reputation literature.

Figure: $1a$

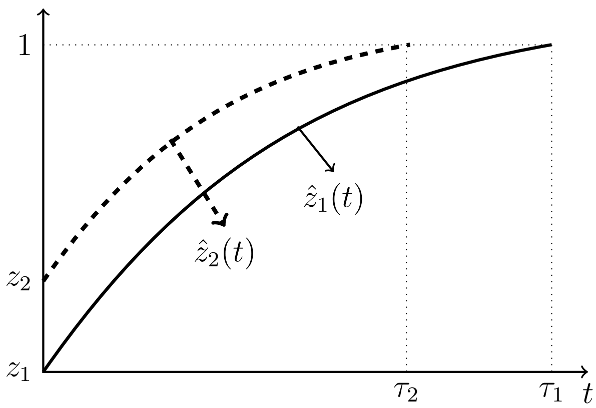
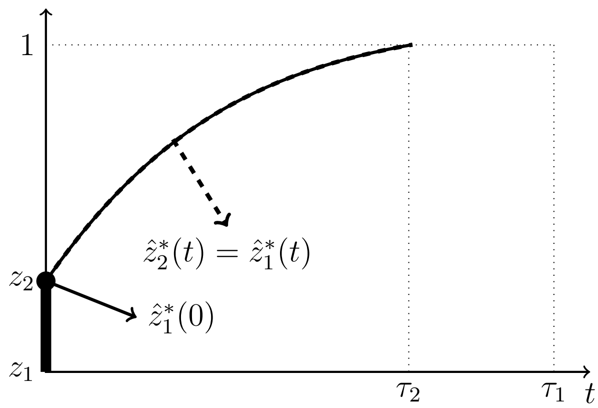


Figure: $1b$



Now suppose that negotiator 1 cannot pick a deadline, and so we are in the world of Kambe

(1999) or Abreu and Gul (2000). The negotiators' reputation functions $\hat{z}_1^*(t)$ and $\hat{z}_2^*(t)$ in equilibrium must look like those in Figure 1b. That is, negotiator 1 must concede at time 0 with a positive probability, and so the value of her reputation function at time 0 (i.e., $\hat{z}_1^*(0)$) is strictly higher than z_1, and she is the weak negotiator in the game.

The intuition for this result is simple: Suppose for a contradiction that none of the negotiators concede at time 0. In that case, the negotiators' reputation functions must look like those in Figure 1a. Because $z_2 > z_1$ and both players' reputations grow at the same rate, negotiator 2's reputation must reach one earlier. Let τ_i denote the time at which negotiator i's reputation reach one. Negotiator 1 should not continue to concede (or delay her concession) beyond time τ_2 as in $\hat{z}_1(t)$ because she learns at this time that her opponent is obstinate, contradicting with the optimality of equilibrium. The same logic applies to the case where negotiator 2 concedes at time 0. Thus, since both negotiators cannot concede at the same time, equilibrium strategies should yield reputation functions as those presented in Figure 1b.

Now, I will explain how negotiator 1 can turn this disadvantaged situation into her advantage by declaring a deadline. Negotiator 1 first calculates p_H, which is negotiator 1's reputation level at the time of deadline that makes negotiator 2 indifferent between conceding to negotiator 1 and waiting for her concession. Therefore, it is a function of α_1 and α_2 only. Suppose that p_H is sufficiently low as given in Figure 1c. Then, the optimality of equilibrium implies that negotiator 1 should announce her deadline as K^* and negotiator 2 should concede to negotiator 1 with a positive probability at time 0 (see Figure 1d), implying that negotiator 1 will be the strong negotiator with a higher payoff.

Figure: 1c

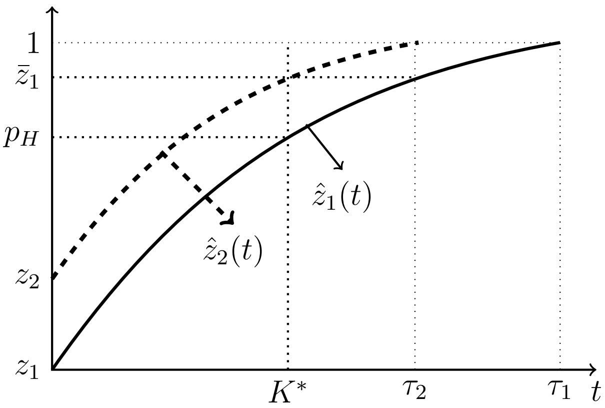
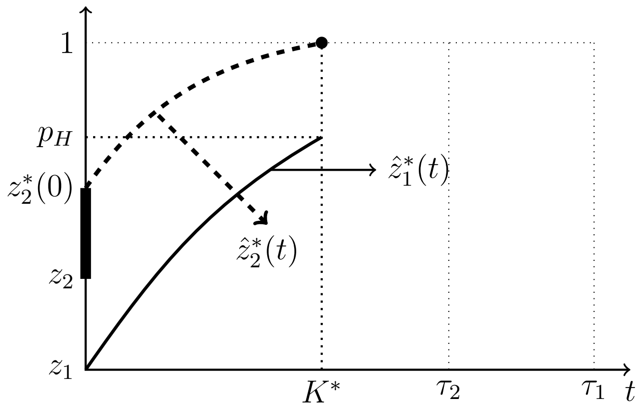


Figure: 1d



If negotiator 1's strategy is such that her reputation function $\hat{z}_1^*(t)$ is given in Figure 1d, then negotiator 2's best reply is to concede to negotiator 1 with certainty before the deadline K^*. Postponing concession beyond time K^* is simply not optimal for negotiator 2 because at this time, negotiator 1's reputation will reach to p_H and flexible negotiator 2 will prefer conceding to 1 rather than inducing an impasse without a deal with probability p_H. Because the growth rate of the concession strategies are fixed and negotiator 2's concession game strategy must be continuous over the interval $[0, K^*]$, which I formally prove in Section 4, negotiator 2 has no choice other than playing a strategy in which it yields the reputation function $\hat{z}_2^*(t)$ in Figure 1*d*. That is, negotiator 2 must concede with a positive probability at $t = 0$. Similarly, not conceding to negotiator 2 at time 0 is a best response for negotiator 1 as well. Note that negotiator 1's reputation does not have to hit one at time K^*. All she needs is to build her reputation up to the level p_H before the deadline, and she can achieve this—without making an initial concession at time 0—if she picks her deadline no earlier than K^*.

To sum, a negotiator who is weak because of her reputational disadvantage can reverse the situation by exit threat. When p_H is very high (in particular, if $p_H \geq \bar{z}_1$ where $\bar{z}_1 = \hat{z}_1(\tau_2)$ as presented in Figure 1*c*), no deadline would make negotiator 1 strong.[9] However, each negotiator has the power to affect the value of p_H through their demand choices. When p_H is higher than $\bar{z}_1$, for example, negotiator 1's optimal action is to reduce her share α_1 and demand something more acceptable for negotiator 2. The parameter $\bar{\alpha}_1$ (please see Theorem 2)—a function of the negotiators' initial reputations and time preferences—denotes the highest price negotiator 1 can pick so that p_h is lower than $\bar{z}_1$ for all (incompatible) demands negotiator 2 would pick. If negotiator 1's price is above $\bar{\alpha}_1$, then negotiator 2's best response is to pick an incompatible demand to increase the value of p_H above $\hat{z}_1$. The negotiators' payoff functions determine their optimal demand selections and these results are presented in Theorems 2 and 3.

Theorem 1. *In any sequential equilibrium with $\alpha_1+\alpha_2 > 1$, let $p_H = \frac{\alpha_1+\alpha_2-1}{\alpha_2}$ and $p_L = z_2^{\lambda_1/\lambda_2} p_H$, where $\lambda_i = \frac{r_j(1-\alpha_i)}{\alpha_1+\alpha_2-1}$ for $i = 1, 2$.*

1. *If $z_1 \geq p_H$, then negotiator 1 chooses $K = 0$, negotiator 2 accepts α_1 at time 0_1 with certainty, and negotiator 1 accepts α_2 only at time 0_2 if the game has not ended yet.*
2. *If $p_L \leq z_1 < p_H$, then negotiator 1 chooses $K = \frac{-\ln(z_1/p_H)}{\lambda_1}$, and the concession game strategies are*
   i. $F_1(t) = \frac{1}{1-z_1}\left(1 - e^{-\lambda_1 t}\right)$ *for all* $t \in [0, K_1]$ *and* $F_1(t) = 1$ *for all* $t \geq K_2$, *and*
   ii. $F_2(t) = \frac{1}{1-z_2}\left(1 - z_2 \left(p_H/z_1\right)^{\lambda_2/\lambda_1} e^{-\lambda_2 t}\right)$ *for all* $t \in [0, K_1]$ *and* $F_2(t) = F_2(K_1)$ *for all* $t \geq K_2$.
3. *If $z_1 < p_L$, then negotiator 1 chooses any $K \in \left[-\frac{\ln z_2}{\lambda_2}, \infty\right)$, and the concession game strategies are $F_i(t) = \frac{1}{1-z_i}\left(1 - z_i e^{-\lambda_i\left(\frac{\ln z_2}{\lambda_2}+t\right)}\right)$ for all $t \leq -\frac{\ln z_2}{\lambda_2}$ and $F_i(t) = 1$ for all $t \geq -\frac{\ln z_2}{\lambda_2}$.*

All the proofs in this section are deferred to the appendix. Theorem 1 characterizes the equilibrium strategies of the flexible negotiators given their first-stage offers. Flexible negotiator

[9] The condition $p_H > \bar{z}_1$ implies that $p_H > \hat{z}_1(\tau_2)$, which yields the third condition in Theorem 1 (i.e., $z_1 < p_L$).

1's type will be revealed at time K_2 (i.e., the second stage of the conventional time K) because the obstinate type will leave the bargaining at time K_1 for sure. Therefore, in equilibrium, flexible negotiator 1 will accept α_2 and finish the game at time K_2 with certainty if the game ever reaches this point. Given this behavior of flexible negotiator 1 and given that the game ever reaches time K, flexible negotiator 2's instantaneous payoff at time K is $[1-\hat{z}_1(K)]\alpha_2$ if he does not concede to negotiator 1. Here, $z_1 \leq \hat{z}_1(K) \leq 1$ indicates the posterior probability that negotiator 1 is the obstinate type at time K_1. However, if flexible negotiator 2 accepts α_1 and finishes the game at time K_1, then his instantaneous payoff will be $1-\alpha_1$. Therefore, in equilibrium, flexible negotiator 2 will certainly accept α_2 prior to K if and only if $\hat{z}_1(K) \geq \frac{\alpha_1+\alpha_2-1}{\alpha_2}$ $(=p_H)$.

The above arguments suggest that if z_1 is sufficiently high (i.e., $z_1 \geq p_H$), then negotiator 1 should declare $K=0$ in the second stage and force flexible negotiator 2 to concede immediately at time 0. However, if z_1 is less than this threshold, then negotiator 1 should build her reputation prior to the deadline. Therefore, negotiator 1 should choose the deadline no earlier than the time K^* satisfying $\hat{z}_1(K^*) \geq p_H$. Otherwise, with some positive probability, negotiator 2 never concedes to negotiator 1. Since Bayes' rule implies that $\hat{z}_1(K^*) = \frac{z_1}{1-B_1(K^*)}$, we have $K^* = -\ln(z_1/p_H)/\lambda_1$. It is crucial to note that choosing the deadline K bigger than K^* is not optimal for negotiator 1 because the earlier the deadline is, the sooner negotiator 2 accepts α_1. Since negotiator 1 discounts time, an earlier deadline implies a greater expected payoff for negotiator 1. As a result, negotiator 1 chooses her deadline as K^* when z_1 is strictly less than p_H and is higher than p_L (the requirement $p_L \leq z_1$ is explained in the next paragraph).

On the other hand, if negotiator 1's initial reputation is very low (i.e., $z_1 < p_L$), then the time required for negotiator 1 to build her reputation to p_H (i.e., K^*) will exceed time $\tau_2 = -\frac{\ln z_2}{\lambda_2}$, indicating the time that negotiator 2's reputation reaches 1 if he concedes with a constant hazard rate λ_2 with no positive concession at time 0. Therefore, the deadline threat will have no use for negotiator 1 for small values of z_1, satisfying $z_1 < p_L$.

In equilibrium with $\alpha_1+\alpha_2>1$ and $K>0$, the flexible negotiators are indifferent between quitting at time 0 and waiting for some time $t \leq K$ and then conceding at this time. Therefore, flexible negotiator i's equilibrium payoff of conceding to negotiator j at any time t is equivalent to what he can achieve at the beginning of this stage (i.e., time 0). Therefore, by equation (1)

$$U_i^d\big(t, F_j\big) = \alpha_i(1-z_j)F_j(0) + (1-\alpha_j)\Big[1-(1-z_j)F_j(0)\Big] \tag{4}$$

for all $0 \leq t \leq K$. Thus, negotiator i's equilibrium game payoff is given by

$$\begin{aligned} U_i\big(F_1, F_2\big) \equiv (1-z_i)\Big(&\alpha_i(1-z_j)F_j(0) + (1-\alpha_j)\Big[1-(1-z_j)F_j(0)\Big]\Big) \\ &+ z_i\alpha_i \int_{y\in[0,K)} e^{-r_i y} dB_j(y) \end{aligned} \tag{5}$$

Definition 1. *In any sequential equilibrium of the game G, negotiator i is called **strong** if $F_j(0) > 0$ and **weak** otherwise.*

Remark 1 According to Theorem 1, in any sequential equilibrium where the negotiators' initial offers are incompatible, only one negotiator can be strong. If negotiator i is weak, then $U_i^d(t, F_j) = 1 - \alpha_j$ for all $t \leq K$, and thus, his expected payoff in the game is strictly less than $1 - \alpha_j$.

For the fixed values of the primitives $z_1, z_2 \in (0,1)$ and r_1, r_2, let

$$p_L(\alpha_1, \alpha_2) = \frac{\alpha_1 + \alpha_2 - 1}{\alpha_2} z_2^{\frac{r_2(1-\alpha_1)}{r_1(1-\alpha_2)}} \quad \text{and} \quad p_H(\alpha_1, \alpha_2) = \frac{\alpha_1 + \alpha_2 - 1}{\alpha_2}.$$

Then define the set

$$D \equiv \left\{ \alpha_1 \in [0,1] \middle| \sup_{\alpha_2 \in [1-\alpha_1, 1]} p_L(\alpha_1, \alpha_2) \leq z_1 \right\}$$

and

$$\bar{\alpha}_1 \equiv \sup_{\alpha_1 \in [0,1]} D.$$

Note that $\bar{\alpha}_1$ is well defined because set D is nonempty. This is true because for any values of z_1 and z_2, we can find α_1 sufficiently close to 0 so that $sup_{\alpha_2 \in [1-\alpha_1, 1]} p_L(\alpha_1, \alpha_2)$ is also close to 0, and thus less than z_1.

According to Theorem 1, negotiator 1's equilibrium choice of K makes her strong except when z_1 is very low (i.e., $z_1 < p_L$). In that case, negotiator 1 can never make herself strong by her choice of K. Therefore, given negotiator 2's offer α_2, negotiator 1's best response is to announce $\alpha_1 \in [0,1]$ such that the inequality $p_L(\alpha_1, \alpha_2) \leq z_1$ holds. For this reason, $\bar{\alpha}_1$ represents the highest possible demand that negotiator 1 could announce in equilibrium—given the fixed parameters z_1, z_2, r_1, and r_2. If negotiator 1 offers $\bar{\alpha}_1$ or anything less than $\bar{\alpha}_1$, then there is no $\alpha_2 > 1 - \bar{\alpha}_1$ such that negotiator 2 can make himself strong and negotiator 1 weak.

Hence, in any equilibrium, negotiator 1 will choose some α_1^* satisfying $\alpha_1^* \leq \bar{\alpha}_1$, and so negotiator 2 will be weak. If negotiator 2 makes an incompatible demand in stage 1, then by equation (5), his expected payoff in the game will be strictly less than $1 - \alpha_1^*$. Therefore, negotiator 2's best response is to make a compatible demand $1 - \alpha_1^*$ and finish the game in stage 1. Anticipating this in equilibrium, negotiator 1 will announce the highest possible demand she could pick, which is $\bar{\alpha}_1$, as the previous arguments imply. Hence, the equilibrium is unique, and the negotiators' first-stage offers are compatible. The game never reaches the second stage, where negotiator 1 threatens negotiator 2 with leaving the bargaining table.

Theorem 2. *In the unique sequential equilibrium of the game G, negotiator 1 demands $\bar{\alpha}_1$, negotiator 2 demands $1 - \bar{\alpha}_1$, and the game ends at the first stage with no delay.*

Example 1 (the value of $\bar{\alpha}_1$): Suppose that $r_i = r$ and $z_i = z$ for $i = 1, 2$. Although it is not possible to give a closed form solution for $\bar{\alpha}_1$, we can find its value numerically. First, the term $p_L(\alpha_1, \alpha_2)$ is a concave function of α_2 with a unique maximizer (see figure 2). In fact,

$$\hat{\alpha}_2(\alpha_1, z) = -\frac{1}{\ln z + 1}\left(\frac{\sqrt{(\ln z)(\ln z - 4\alpha_1 + \alpha_1^2 \ln z - 2\alpha_1 \ln z)} - (1-\alpha_1)\ln z}{2} - 1\right)$$

maximizes $p_L(\alpha_1, \alpha_2)$ for given values of z and α_1.

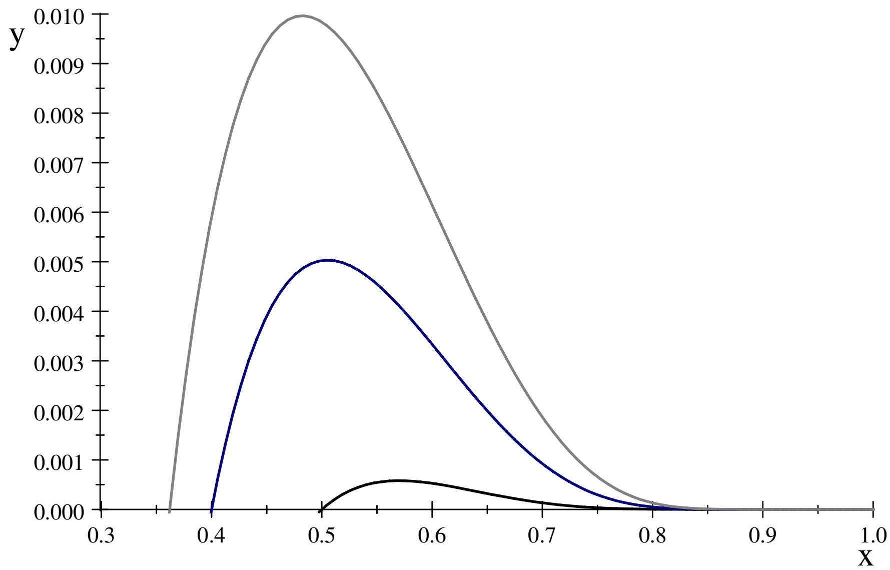


Figure 2: The x-axis corresponds to α_2, taking values in $[1 - \alpha_1, 1]$. The three curves correspond to $p_L(\alpha_1, \alpha_2)$ for $z = 0.1$, and are drawn for different values of α_1. From the curve at the bottom to the one at the top, α_1 takes values 0.5, 0.6, and 0.69941, respectively.

Second, $p_L(\alpha_1, \alpha_2)$ increases with α_1 (also see figure 2). Therefore, $\bar{\alpha}_1$ satisfies

$$p_L\big(\bar{\alpha}_1, \hat{\alpha}_2(\bar{\alpha}_1, z)\big) = z$$

For example, when $z = 0.1$, then $\bar{\alpha}_1 = 0.69941$. Alternatively, if $z = 0.4$ and $z = 0.01$, then $\bar{\alpha}_1 = 0.78811$ and $\bar{\alpha}_1 \approx 0.6382$, respectively.

B. Insuring Negotiators and Mitigating the Effects of Exit Threat

The previous section considered the default case (i.e., the *no-insurance case*). In this section, however, I will consider the *partial-insurance* case: in equilibrium, each negotiator will be compensated (only) when he receives the message c, and the negotiator's continuation payoff when he receives the message c and d will be identical. I prove the following results for this insurance plan, where the negotiators' compensations are minimal. However, the same results would hold when, for example, each negotiator i is ensured to receive a payoff no less than $1 - \alpha_j$ whenever he receives the message c.

Comparing with Theorem 2, the next result unambiguously shows that even if we insure both negotiators, the equilibrium share of the second negotiator, who is unable to use the exit threat, will increase.[10] Moreover, if negotiator 2 is insured by his principal, whose preferences are perfectly aligned with his agent's, then the second negotiator's principal prefers to insure his agent for sufficiently low values of z_2. Next, I will formally prove all these claims.

More formally, in any equilibrium where $\alpha_1 + \alpha_2 > 1$ and $K \geq 0$, the negotiator i's equilibrium game payoff is

$$U_i\big(F_i, F_j\big) = \alpha_i(1 - z_j)F_j(0) + (1 - \alpha_j)\Big[1 - (1 - z_j)F_j(0)\Big] \tag{6}$$

Therefore, the size of the minimal compensation in ex-ante terms is

$$w_i \equiv z_i\left[U_i(F_i, F_j) - \alpha_i \int_{y \in [0,K)} e^{-r_i y} dB_j(y)\right]. \tag{7}$$

Note that Theorem 1 holds whether there is partial insurance or not. The equilibrium strategies F_1 and F_2 do not change under partial insurance because these strategies represent the fourth-stage equilibrium behavior of the flexible negotiators. Furthermore, one may naturally suspect that negotiator 1's choice of K should depend on the existence of the partial insurance, but it does not. Recall from the arguments following Theorem 1: lowering K increases negotiator 1's equilibrium payoff in the game as long as negotiator 1's reputation reaches the level p_H prior to the time K. This observation holds whether or not there is partial insurance, so the optimal value for K is still given by Theorem 1. That is, K is simply calculated by Bayes' rule and flexible negotiator 1's concession game strategy F_1.

Theorem 1 and the discussions in the previous section also imply that regardless of the presence of the partial insurance, negotiator 1 will never choose a demand more than $\bar{\alpha}_1$ in equilibrium. If she does, then the second negotiator can profitably deviate to a demand that

[10] Therefore, one can immediately conclude that if negotiator 2 is insured but negotiator 1 is not, then the second negotiator's expected surplus will be even higher.

makes negotiator 1 weak. Therefore, in any equilibrium where negotiators' announced demands are α_1 and α_2, we must have $p_L(\alpha_1, \alpha_2) \leq z_1$.

Furthermore, if negotiators' announced demands α_1 and α_2 are such that $p_H(\alpha_1, \alpha_2) < z_1$, then negotiator 1 would be strong in any sequential equilibrium of the game G. In that case, however, negotiator 1's expected payoff (in the game G) would increase with α_1. That is, negotiator 1 would have incentive to deviate and announce $\alpha_1 + \epsilon$ for some $\epsilon > 0$ small enough so that the inequality $p_H(\alpha_1 + \epsilon, \alpha_2) < z_1$ holds, and thus, the strong negotiator 1 receives a higher expected payoff. For this reason, in any sequential equilibrium where negotiators' initial offers α_1 and α_2 are incompatible, the inequality $z_1 \leq p_H(\alpha_1, \alpha_2)$ must hold.

Thus, for the fixed values of the primitives and for any $\alpha_1, \alpha_2 \in [0, 1]$ such that $\alpha_1 + \alpha_2 > 1$ define the set

$$Z(\alpha_1, \alpha_2) = \Big\{ z \in (0,1) \Big| \quad p_L(\alpha_1, \alpha_2) \leq z \leq p_H(\alpha_1, \alpha_2) \Big\}.$$

Therefore, the previous arguments imply that $z_1 \in Z(\alpha_1, \alpha_2)$ must hold in equilibrium if the negotiators are partially insured. The next theorem characterizes the equilibrium demands in the first stage of the game G given that both negotiators are partially insured.

Theorem 3. *Suppose that both negotiators are partially insured. In any sequential equilibrium of the game G, the first-stage offers α_1^* and α_2^* must satisfy $\alpha_1^* + \alpha_2^* \geq 1$ and*

$$\alpha_1^* \in \underset{\substack{\alpha_1 \in [1-\alpha_2^*, \bar{\alpha}_1] \\ z_1 \in Z(\alpha_1, \alpha_2^*)}}{arg\ max} \left\{ \alpha_1 - z_2(\alpha_1 + \alpha_2^* - 1) \left(\frac{\alpha_1 + \alpha_2^* - 1}{z_1 \alpha_2^*} \right)^{\frac{r_1(1-\alpha_2^*)}{r_2(1-\alpha_1)}} \right\}.$$

Unlike the no-insurance case, there are multiple equilibria in the partial-insurance case. The unique efficient equilibrium is the one that negotiator 1 demands $\bar{\alpha}_1$. In all other equilibria, negotiator 1's initial offer (i.e., α_1^*) is less than $\bar{\alpha}_1$, and the second negotiator makes an incompatible offer in the first stage. In all these inefficient equilibria, negotiator 1 announces a positive exit-time (as characterized in Theorem 1). In particular, given the first-stage demands α_1^* and α_2^*, negotiator 1 announces

$$K^* = -\frac{\alpha_1^* + \alpha_2^* - 1}{r_2(1 - \alpha_1^*)} \ln \left(\frac{z_1 \alpha_2^*}{\alpha_1^* + \alpha_2^* - 1} \right).$$

It is easy to verify that K^* is an increasing function of α_1^*, α_2^* and z_1, but a decreasing function of r_2. Similar to the no-insurance case, negotiator 1 will be strong, and negotiator 2 will be weak in any sequential equilibrium under the partial-insurance case. Since negotiator 2 is partially compensated when he receives the message c, his expected payoff in the game will be $1 - \alpha_1^*$ regardless of his announcement (i.e., α_2^*). Negotiator 2's indifference among all possible demands in the set $[1 - \alpha_1^*, 1]$ is the reason for the multiplicity of equilibrium.

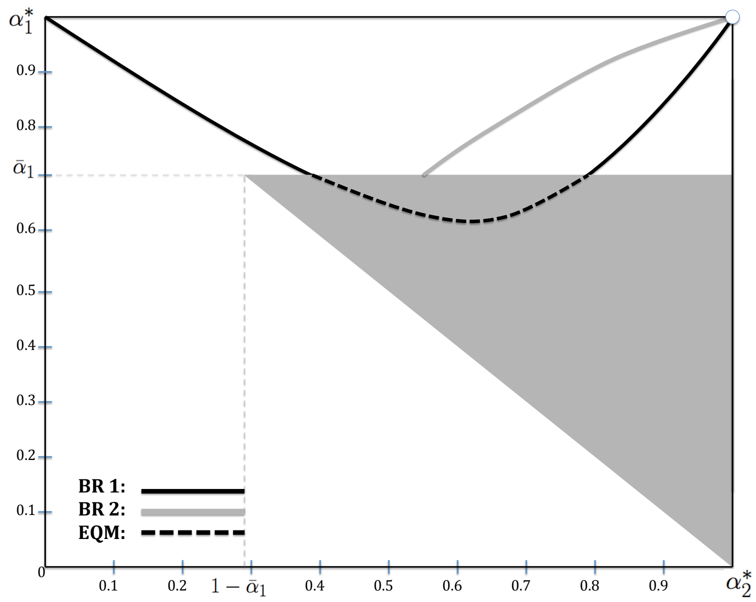


Figure 3: The first stage best response correspondences of the negotiators ($z_1 = z_2 = 0.1$ and $r_1 = r_2 = 0.7$).

We know from Theorem 2 that if negotiator 1's initial offer is α_1^* and if there is no insurance, negotiator 2's expected payoff in the game will be strictly less than $1 - \alpha_1^*$ when negotiator 2 makes an incompatible demand in the first stage. Given this, negotiator 2 will never make an incompatible demand in equilibrium. This is the reason for the uniqueness in Theorem 2. That is, the source of the multiplicity in Theorem 3 is due to negotiator 2's equilibrium payoff. Therefore, if only negotiator 2 would have been compensated, then there would be multiple equilibria (same as Theorem 3). However, the equilibrium demands would be different from those that are characterized by Theorem 3 because the first negotiator's expected payoff, if negotiator 1 is not compensated, is given by equation (5), not by equation (6).

In equilibrium where negotiators' initial offers α_1^* and α_2^* are incompatible, the second negotiator's principal should compensate (in ex-ante terms) his agent in the amount of w, which is given by equation (7).[11] If negotiator 2 is not insured, then his expected payoff in the game will be $1 - \bar{\alpha}_1$. But if he is insured, then his expected payoff will increase to $1 - \alpha_1^*$, and so the difference is $\bar{\alpha}_1 - \alpha_1^*$. This difference is greater than w whenever $\bar{\alpha}_1 - \alpha_1^* \geq z_2(1 - \alpha_1^*)$. Thus, for all values of $z_2 \leq \frac{\bar{\alpha}_1 - \alpha_1^*}{1-\alpha_1^*}$, the principal of the second negotiator prefers to compensate his agent. Clearly, *the principal of the first negotiator does not prefer to compensate her.*

Example 2 (an illustration of the equilibrium): Consider the following parameter values: $z_1 = z_2 = 0.1$ and $r_1 = r_2 = 0.7$. The optimization problems are solved by MATLAB. In figure 3, $BR_i(\alpha_j)$ denotes the best response correspondence of negotiator i given negotiator j's first-stage

[11]In equilibrium, $w = z_2\Big[1 - \alpha_1^* - \frac{1}{r_2+\lambda_1^*}\Big(1 - \frac{(z_1/p_H)^{(r_2+\lambda_1^*)/\lambda_1^*}}{r_2+\lambda_1^*}\Big)\Big]$, where $\lambda_1^* = \frac{r_2(1-\alpha_1^*)}{\alpha_1^*+\alpha_2^*-1}$.

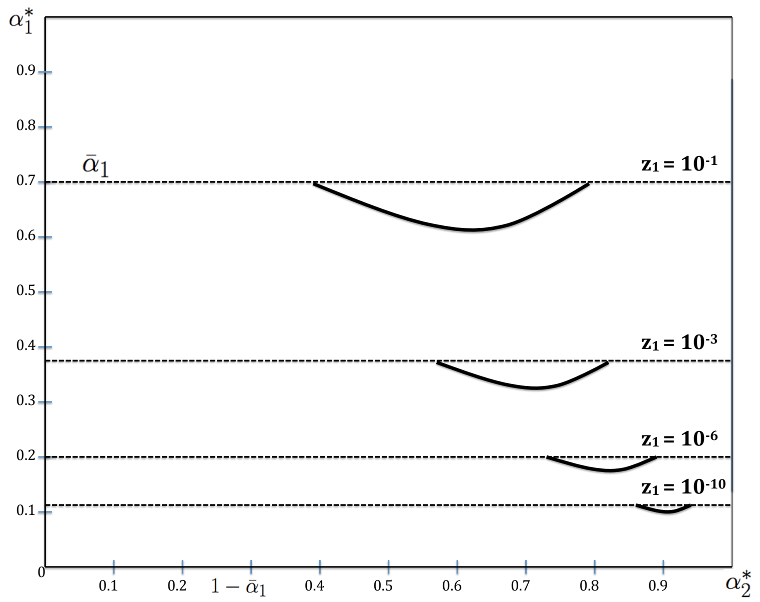


Figure 4: Equilibrium demands for different values of z_1.

demand α_j.

For each $\alpha_2 \in [0,1]$, $BR_1(\alpha_2)$ can be found by choosing α_1 satisfying $z_1 \in Z(\alpha_1,\alpha_2)$ and maximizing $u_1 = \left(1 - z_2\left(\frac{p_H}{z_1}\right)^{\frac{\lambda_2}{\lambda_1}}\right)\alpha_1 + z_2\left(\frac{p_H}{z_1}\right)^{\frac{\lambda_2}{\lambda_1}}(1-\alpha_2)$. Similarly for each $\alpha_1 \in [0,1]$, $BR_2(\alpha_1)$ can be found by choosing $\alpha_2 \in [0,1]$ that satisfies $p_L(\alpha_1,\alpha_2) \geq z_1$ and maximizes $u_2 = \big(1 - z_1 z_2^{\frac{-\lambda_1}{\lambda_2}}\big)\alpha_2 + z_1 z_2^{\frac{-\lambda_1}{\lambda_2}}(1-\alpha_1)$. Given α_1, if there is no $\alpha_2 \in [0,1]$ such that $p_L(\alpha_1,\alpha_2) \geq z_1$ holds, then $BR_2(\alpha_1) = [1-\alpha_1, 1]$. This is true because negotiator 2's expected payoff in this case is $1-\alpha_1$ regardless of his announcement (i.e., α_2). Note that there is no equilibrium in which $\alpha_i = 1$ for $i = 1,2$. This is true because negotiator j's best response correspondence is not well defined for $\alpha_i = 1$.

The dashed segment of the black line in figure 3 indicates the equilibrium demands in the first stage. One equilibrium demand profile is that $\alpha_1^* = 0.6143$ and $\alpha_2^* = 0.562$. In that case, $p_L = 0.0412$ and $p_H = 0.3137$, so $z_1 \in (p_L, p_H)$ as required. Then in equilibrium, we have $K^* = 0.7465$. Expected payoffs are $u_1 = 0.5497$ and $u_2 = 0.3857$. Since $u_1 + u_2 = 0.9354 < 1$, the expected utility loss is 0.0646.

Example 3: Figure 4 shows that when we fix the values of z_2, r_1, and r_2, the maximal demand choice of negotiator 1 (i.e., $\bar{\alpha}_1$) decreases as z_1 decreases to 0. In the figure, the dashed lines show $\bar{\alpha}_1$, and the solid lines give the equilibrium demands for each z_1. In this specific example, the parameters are such that $z_2 = 0.1, r_1 = 0.7$, and $r_2 = 0.7$, while z_1 takes four different values as indicated in the figure.

The Limiting Case of Complete Rationality: Both Abreu and Gul (2000) and Kambe (1999) show that as the initial probabilities of commitment types decrease to 0, the equilibrium demands converge to a compatible share that purely depends on negotiators' time preferences. The same conclusion holds for the game G as well.

For this purpose, I first fix the parameters r_b and r_s. I say the bargaining game $\mathrm{G}(z_1^m, z_2^m)$ converges to $\mathrm{G}(M)$ when the sequences $\{z_1^m\}$ and $\{z_2^m\}$ of initial priors satisfy

$$\lim z_1^m = 0, \lim z_2^m = 0 \text{ as } m \to \infty \text{ and } \log z_1^m / \log z_2^m = M \text{ for all } m \geq 0. \tag{8}$$

Corollary 1. *Whether or not the negotiators are partially insured for their loss, if the game* $G(z_1^m, z_2^m)$ *converges to* $G(M)$, α_1^m *is the equilibrium demand of negotiator 1 in the game* $G(z_1^m, z_2^m)$*, and if* $\alpha_1^* \in [0,1]$ *is a limit point of* α_1^m*, then we have* $\alpha_1^* = \frac{r_2}{Mr_1 + r_2}$.

Given that we must have $p_L(\alpha_1^m, \alpha_2^m) \leq z_1^m$ for all m, it is fairly straightforward to prove the last result.

Remark 2 In the limit where z_1^m and z_2^m converge 0, the equilibrium demand is uniquely determined under the no-insurance case. However, multiplicity of the equilibrium demands will still be the case under partial insurance. In that case, α_1^* is unique (as given by Corollary 1), but any $\alpha_2^* \geq \frac{Mr_1}{Mr_1 + r_2}$ is consistent with equilibrium.

4. Concession Game Strategies

In this section, I will characterize the flexible negotiators' fourth-stage sequential equilibrium strategies. Therefore, I take negotiators' demands α_1, α_2, and negotiator 1's deadline K as given. I suppose that $\alpha_1 + \alpha_2 > 1$ because the game finishes at the first stage when $\alpha_1 + \alpha_2 \leq 1$.

Consider any sequential equilibrium in which $K = 0$. The obstinate type of negotiator 1 leaves the game at time 0_1. If the game does not end at time 0_1, then flexible negotiator 1 accepts α_2 at time 0_2 with probability one. This is true because (1) negotiator 1's type will be revealed if she does not leave at time 0_1, (2) flexible negotiator 1 receives 0 payoff if she leaves the game at time 0_1, (3) in equilibrium, negotiator 2's type is still not known to negotiator 1 at time 0_2, and thus, flexible negotiator 1 accepts α_2 at time 0_2 (if the game reaches this point) and finalizes the game for sure.[12] Hence, in equilibrium, the game will end at time 0 (either in stage 0_1 or in 0_2) for sure and $F_2(0_2) = 0$.

[12] This conclusion is a direct implication of Myerson (1991) and Abreu and Gul (2000); if negotiator 1's type is revealed but negotiator 2 is not known to be flexible, then in the unique equilibrium of he continuation game, negotiator 1 immediately concedes to negotiator 2.

Proposition 1. *In any sequential equilibrium where $K = 0$ and $\alpha_1 + \alpha_2 > 1$,*

1. *if $p_H = \frac{\alpha_1+\alpha_2-1}{\alpha_2} < z_1$, then negotiator 2 concedes at time 0_1 with probability 1 and negotiator 1 accepts α_2 only at time 0_2. That is, $F_1(0_1) = 0$, $F_1(0_2) = 1$ and $F_2(0_1) = 1$,*

2. *if $z_1 < p_H$, then negotiator 2 never concedes and negotiator 1 is indifferent between conceding at times 0_1 and 0_2. That is, $F_2(0_1) = F_2(0_2) = 0$, $0 \leq F_1(0_1) \leq 2\left(\frac{p_H-z_1}{p_H(1-z_1)}\right)$ and $F_1(0_2) = 1$,*

3. *if $z_1 = p_H$, then negotiator 1 concedes only at time 0_2 and negotiator 2 is indifferent between conceding at time 0_1 and 0_2. That is, $F_1(0_1) = 0$, $F_1(0_2) = 1$, $0 \leq F_2(0_1) \leq 1$ and $F_2(0_2) = F_2(0_1)$.*

Proof. It is deferred to Appendix.

□

Now, consider a sequential equilibrium in which $K \geq T_0 = min\{-\frac{\ln z_1}{\lambda_1}, -\frac{\ln z_2}{\lambda_2}\}$, where $\lambda_i = \frac{r_j(1-\alpha_i)}{\alpha_1+\alpha_2-1}$. Note that time $-\frac{\ln z_i}{\lambda_i}$ is the time at which negotiator i's reputation reaches 1 if $F_i(0) = 0$ (i.e., negotiator i does not concede at time 0). When negotiator i's reputation reaches 1, negotiator j stops playing the concession game. Therefore, in equilibrium, both negotiators' reputations reach 1 and the game G ends at time T_0 if negotiator 1 chooses $K \geq T_0$.

Proposition 2. *In any sequential equilibrium in which $K \in [T_0, \infty)$ and $\alpha_1 + \alpha_2 > 1$, we have*

$$F_i(t) = \begin{cases} \frac{1}{1-z_i}(1 - c_i e^{-\lambda_i t}), & \text{if } t \in [0, T_0] \\ 1, & \text{otherwise,} \end{cases}$$

where $c_i = z_i e^{\lambda_i T_0}$.

Proof. The proof follows from Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000). □

Finally, I will characterize the flexible negotiators' fourth-stage equilibrium strategies when negotiator 1's deadline K satisfy $0 < K < T_0$. For this purpose, first, consider a history in which no negotiator concedes prior to time $t = K$. Given the negotiators' equilibrium strategies, let $\hat{z}_1$ and $\hat{z}_2$ denote the posterior probabilities that negotiators 1 and 2, respectively, are obstinate types at time K. It must be the case that $\hat{z}_i$ is greater than or equal to z_i. Negotiator 2 believes that negotiator 1 will leave the game at time K_1 with probability $\hat{z}_1$, in which case flexible negotiator 2's instantaneous payoff is 0. On the other hand, negotiator 1 will concede to negotiator 2 either at time K_1 or time K_2 (depending on her equilibrium strategy) with probability $1 - \hat{z}_1$, in which case flexible negotiator 2's instantaneous payoff is α_2. Thus, in equilibrium, negotiator 2 concedes to negotiator 1 at time K_1 with a positive probability if and only if $\hat{z}_1$ is high enough.

More formally, given negotiator 1's equilibrium strategy F_1, let $U_2(Accept\ at\ K_1)$ and $U_2(Wait\ at\ K_1)$ denote flexible negotiator 2's instantaneous (expected) payoff of conceding and waiting, respectively, at time K_1. That is, these payoffs are calculated at time K_1. Thus, $U_2(Accept\ at\ K_1) \geq U_2(Wait\ at\ K_1)$ if and only if

$$\hat{z}_1(1-\alpha_1) + (1-\hat{z}_1)p_1(K_1)\left[\frac{1+\alpha_2-\alpha_1}{2}\right] + (1-\hat{z}_1)\Big[1-p_1(K_1)\Big](1-\alpha_1) \geq (1-\hat{z}_1)\alpha_2$$

where $p_1(K_1)$ indicates the probability that flexible negotiator 1 accepts α_2 at time K_1.

If $p_1(K_1) = 0$ is true, then we have $U_2(Accept\ at\ K_1) \geq U_2(Wait\ at\ K_1)$ if and only if $\hat{z}_1 \geq p_H$ holds. Moreover, if $\hat{z}_1 = p_H$, then

$$U_2(Accept\ at\ K_1) - U_2(Wait\ at\ K_1) = p_1(K_1)\Big[\frac{1-\alpha_1}{\alpha_2}\Big]\Big[\frac{\alpha_1+\alpha_2-1}{2}\Big]$$

is *non-negative* for all values of $p_1(K_1)$.

The last two observations imply that, in equilibrium, if negotiator 1's reputation at time K (i.e., $\hat{z}_1$) reaches p_H and if negotiator 1 concedes to negotiator 2 at time K_1 with some positive probability (i.e., $p_1(K_1) > 0$), then negotiator 2 should also concede to negotiator 1 at time K_1. However, if negotiator 2 concedes to negotiator 1 at time K_1 with a positive probability, then flexible negotiator 1 prefers to wait at time K_1.[13] Hence, in equilibrium, if negotiator 1's reputation at time K ever reaches p_H, then negotiator 1 will not concede to negotiator 1 at time K_1 and flexible negotiator 2 will be indifferent between conceding and waiting at this time.

Therefore, the question that we must answer is whether flexible negotiator 1 prefers to build his reputation to p_H prior to time K or not. The next result provides an affirmative answer.

Proposition 3. *In any sequential equilibrium in which $K \in (0, T_0)$ and $\alpha_1+\alpha_2 > 1$, the following must be true:*

1. *Equilibrium strategies F_1 and F_2 of stage 4 are continuous and strictly increasing over $[0, K_1]$. In particular, for all $t \in [0, K_1]$, $F_i(t) = \frac{1}{1-z_i}(1-c_i e^{-\lambda_i t})$ where $c_i \in [0,1]$ with $(1-c_1)(1-c_2) = 0$ and $\lambda_i = \frac{r_j(1-\alpha_i)}{\alpha_1+\alpha_2-1}$. Moreover, $F_2(t) = F_2(K_1)$ for all $t > K_1$ and $F_1(K_2) = 1$.*

[13] This argument is true because given F_2 (i.e., the second negotiator's strategy), $U_1(Accept\ at\ K_1) = (1-\hat{z}_2)p_2(K_1)\left[\frac{1+\alpha_1-\alpha_2}{2}\right] + \left[\hat{z}_2 + (1-\hat{z}_2)(1-p_2(K_1)\right](1-\alpha_2)$ is strictly smaller than $U_1(Accept\ at\ K_2) = (1-\hat{z}_2)p_2(K_1)\alpha_1 + \left[\hat{z}_2 + (1-\hat{z}_2)\big(1-p_2(K_1)\big)\right](1-\alpha_2)$ whenever $p_2(K_2) > 0$ (i.e., negotiator 2 concedes to negotiator 1 at time K_1 with a positive probability).

2. *Negotiator 1's reputation at time K_1 (i.e., $\hat{z}_1$) reaches exactly p_H whenever the second negotiator's reputation at time K_1 (i.e., $\hat{z}_2$) is strictly less than 1.*

Proof. It is deferred to Appendix.

□

According to Proposition 3, if negotiator 1 does not make a positive probabilistic concession at time 0, then in equilibrium, negotiator 1 should choose her deadline $K = -\frac{\ln(z_1/p_H)}{\lambda_1} \equiv K^*$. We get this term by solving the equation $\hat{z}_1(t) = p_H$ that must be satisfied in equilibrium. Note that if negotiator 1 does not make initial probabilistic concession, then $c_1 = 1$. Therefore, we have $\hat{z}_1(t) = \frac{z_1}{z_1+(1-z_1)(1-F_1(t))} = \frac{z_1}{e^{-\lambda_1 K}}$. Solving $\frac{z_1}{e^{-\lambda_1 K}} = p_H$ yields the desired term K^*.

Proposition 4. *In any sequential equilibrium in which $0 < K < \min\{K^*, T^0\}$ and $\alpha_1 + \alpha_2 > 1$, we have*

(i) $F_1(t) = \frac{1}{1-z_1}(1 - \frac{z_1}{p_H} e^{\lambda_1(K_1-t)})$ *for all* $t \in [0, K_1]$ *and* $F_1(t) = 1$ *for all* $t \geq K_2$.

(ii) $F_2(t) = \frac{1}{1-z_2}(1 - e^{-\lambda_2 t})$ *for all* $t \in [0, K_1]$ *and* $F_2(t) = F_2(K_1)$ *for all* $t \geq K_2$.

Proof. Negotiator 1 cannot build her reputation to p_H at time K_1 if she does not make a positive probabilistic concession. Therefore, $c_2 = 1$ must hold in equilibrium (since both negotiators cannot make concession at time 0 with positive probabilities). The last observation, together with Proposition 3, gives F_2. In order to find $F_1(t)$, we need to solve $\frac{z_1}{z_1+(1-z_1)(1-F_1(K_1))} = p_H$.

□

Proposition 5. *In any sequential equilibrium in which $K^* < K < T^0$ and $\alpha_1 + \alpha_2 > 1$, we have*

(i) $F_1(t) = \frac{1}{1-z_1}(1 - e^{-\lambda_1 t})$ *for all* $t \in [0, K_1]$ *and* $F_1(t) = 1$ *for all* $t \geq K_2$.

(ii) $F_2(t) = \frac{1}{1-z_2}(1 - z_2 e^{\lambda_2(K_1-t)})$ *for all* $t \in [0, K_1]$ *and* $F_2(t) = 1$ *for all* $t \geq K_2$.

Proof. Since $K^* < K$, negotiator 1 has more than sufficient time to build her reputation to p_H. In fact, reputation of negotiator 1 at time K_1 (i.e., $\hat{z}_1$) will be strictly higher than p_H because F_1 is strictly increasing on $[0, K_1]$. Thus, negotiator 2 strictly prefers to concede at time K_1. However, according to Proposition 3, F_2 must be continuous on $(0, K_1]$ (i.e., $p_2(K_1)$ must be 0). These arguments imply that in equilibrium $F_2(K_1) = 1$. The last equality implies the value of $F_2(t)$. Since $c_2 < 1$, we have $c_1 = 1$, which implies the value of F_1.

□

Proposition 6. *In any sequential equilibrium in which $0 < K = K^* < T^0$ and $\alpha_1 + \alpha_2 > 1$, we have* (*i*) $F_1(t) = \frac{1}{1-z_1}(1 - e^{-\lambda_1 t})$ *for all* $t \in [0, K_1]$ *and* $F_1(t) = 1$ *for all* $t \geq K_2$*, and*

(*ii*) $F_2(t) = \frac{1}{1-z_2}(1 - c_2 e^{-\lambda_2 t})$ *for all* $t \in [0, K_1]$ *and* $F_2(t) = F_2(K_1)$ *for all* $t \geq K_2$ *where* $c_2 \in \left[z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1},\ 1\right]$.

Proof. By the definition of K^*, if $F_1(0) > 0$, then $\hat{z}_1(K_1) > p_H$. But then, flexible negotiator 2 strictly prefers conceding at time K_1, implying that we must have $F_2(K_1) = 1$. Because $K_1 < T_0$, the last equality is possible only if $F_2(0) > 0$. However, in equilibrium, $F_1(0)F_2(0) = 0$ must hold. Thus, we must have $F_1(0) = 0$, which implies the value of F_1. We also have $c_2 \geq z_2 e^{\lambda_2 K_1}$ because $F_2(K_1) \leq 1$ is true. The last inequality gives the set of possible values for c_2.

□

Proposition 7. *If* α_1^* *and* α_2^* *are negotiators' demand choices in an equilibrium, then* $z_1 \in Z(\alpha_1^*, \alpha_2^*)$ *and* $1 - \alpha_2^* \leq \alpha_1^* \leq \bar{\alpha}_1$ *must hold.*

Proof. It is deferred to Appendix.

□

5. Some Extensions

One possible extension of the model is allowing negotiators to use mixed strategies in the first stage. However, this modification would add nothing new but additional technicalities. Theorem 1 and all the results in Section 4 clearly follow in this case. We know from Theorem 1 that in equilibrium, negotiator 1 will choose her deadline such that she is the strong negotiator in the game. Since the second negotiator is weak, he will be indifferent between all the demands in the set $[1 - \alpha_1, 1]$. The only difference is that negotiator 1 will be mixing over a subset of $[0, 1]$, which may include demands that are higher than $\bar{\alpha}_1$.

In what follows, I will consider some (relatively simpler) extensions of the model, where the negotiators sequentially choose their demands. Determining the set of equilibrium demands is not easy because the negotiators' payoff functions are highly complex and discontinuous. However, I will characterize the conditions that give the set of equilibrium demands.

It is crucial to note that the case where negotiator 1 chooses the deadline K before negotiator 2 chooses his demand α_2 has a completely different set of dynamics. Most of the results in Section 4 and the theorems in Section 3 will no longer be true in this case. Therefore, this specific extension is discarded not because it is unimportant, but because it will make Theorem 1 and the results in Section 4—the core of the equilibrium strategies—invalid. Finally, the following characterizations are done under the assumption that negotiators are partially compensated when they receive the message c. Therefore, negotiator i's expected payoff in the game is given by equation (6). For the case where the negotiators are not compensated, the utility functions will change as dictated by equation (5).

Sequential demand announcement (negotiator 2 is first): Suppose now that negotiator 2 makes his demand choice first (stage 1), and then negotiator 1 chooses both her demand α_1 and exit-time K (stage 2). The timing of the rest of the game (i.e., stages 3 and 4) are the same as before. In equilibrium, the optimal deadline will be determined according to Theorem 1. Equilibrium values of α_1^* and α_2^* are characterized as follows:

First, start with defining the negotiators' best response correspondences. Given $\alpha_2 \in (0,1)$, negotiator 1's best response correspondence is

$$BR_1(\alpha_2) = \underset{\substack{\alpha_1 \in [1-\alpha_2, 1) \\ z_1 \in Z(\alpha_1, \alpha_2)}}{\arg\max}\ u_1(\alpha_1, \alpha_2)$$

where $u_1(\alpha_1, \alpha_2) = \alpha_1 - z_2(\alpha_1 + \alpha_2 - 1)\left(\frac{\alpha_1+\alpha_2-1}{z_1\alpha_2}\right)^{\frac{r_1(1-\alpha_2)}{r_2(1-\alpha_1)}}$.

Hence, in equilibrium negotiator 2 chooses $\alpha_2^* \in (0,1)$ and negotiator 1 chooses a function $\alpha_1^* : (0,1) \to [1-\alpha_2, 1)$ such that

(*i*) $\alpha_2^* \in \underset{\alpha_2 \in (0,1)}{\arg\min}\ BR_1(\alpha_2)$, and

(*ii*) for any $\alpha_2 \in (0,1)$, $\alpha_1^*(\alpha_2) \in \underset{\substack{\alpha_1 \in [1-\alpha_2, 1) \\ z_1 \in Z(\alpha_1, \alpha_2)}}{\arg\max}\ u_1(\alpha_1, \alpha_2)$.

Sequential demand announcement (negotiator 1 is first): Suppose now that in stage 1 negotiator 1 makes her demand choice first, and then negotiator 2 chooses his demand. In stage 2, negotiator 1 chooses a deadline K. The timing of the rest of the game (i.e., stages 3 and 4) are the same as before. In equilibrium, the optimal deadline is determined by Theorem 1. Equilibrium values of α_1^* and α_2^* are characterized as follows.

If negotiator 1 demands $\alpha_1 > \bar{\alpha}_1$, then negotiator 2 will choose a correspondence $\alpha_2^* : (0,1) \to [1-\alpha_1, 1)$, satisfying that for any $\alpha_1 \in (0,1)$,

$$\alpha_2^*(\alpha_1) \in \underset{\substack{\alpha_2 \in [1-\alpha_1, 1) \\ z_1 < p_L}}{\arg\max}\ u_2(\alpha_1, \alpha_2)$$

where $u_2(\alpha_1, \alpha_2) = \alpha_2 - (\alpha_1 + \alpha_2 - 1) z_1 z_2^{-\lambda_1/\lambda_2}$.

However, if negotiator 1 demands $\alpha_1 \leq \bar{\alpha}_1$, then negotiator 2 will choose any correspondence, satisfying $\alpha_2^* : (0,1) \to [1-\alpha_1, 1)$.

Finally, negotiator 1 chooses her demand α_1^* such that $\alpha_1^* \in \underset{\alpha_1 \in (0,1)}{\arg\max}\ \bar{U}_1$ where

$$\bar{U}_1 = \begin{cases} 1 - \alpha_2^*(\alpha_1), & \text{if } \alpha_1 > \bar{\alpha}_1 \\ u_1(\alpha_1, \alpha_2^*(\alpha_2)) & \text{if } \alpha_1 \leq \bar{\alpha}_1 \text{ and } \alpha_1 \in Z(\alpha_1, \alpha_2^*(\alpha_1)) \\ 1 - \alpha_2^*(\alpha_1), & \text{if } \alpha_1 \leq \bar{\alpha}_1 \text{ and } \alpha_1 \notin Z(\alpha_1, \alpha_2^*(\alpha_1)). \end{cases}$$

and $u_1(\alpha_1, \alpha_2^*(\alpha_1)) = \alpha_1 - z_2(\alpha_1 + \alpha_2^*(\alpha_1) - 1)\left(\frac{\alpha_1 + \alpha_2^*(\alpha_1) - 1}{z_1 \alpha_2^*(\alpha_1)}\right)^{\frac{r_2(1-\alpha_2^*(\alpha_1))}{r_2(1-\alpha_1)}}$.

The Case Where Both Negotiators Announce a Deadline: Another possible extension is the case where both negotiators (either sequentially or simultaneously) choose a deadline. Although this case might be interesting from an analytical perspective, it does not make much sense in practice. In a contract negotiation, for example, when the labor union threatens the management by announcing a deadline for the negotiations, the management is supposed to make a decision between (1) conceding to the union, (2) making an acceptable counteroffer or (3) allowing the labors to leave the bargaining table (e.g., strike). However, making a counter-threat by choosing an earlier deadline is not an expected move from the management.

On the other hand, from an analytical point of view, allowing both negotiators to announce a deadline and to leave the bargaining table gives rise to a significant amount of complexity. Suppose, for example, that negotiators simultaneously declare an exit-time, where $K_i \in [0, \infty)$ indicates negotiator i's deadline. If $K_1 < K_2$, then it is not clear whether flexible negotiator 1 will accept negotiator 2's demand prior to time K_1 with certainty.

We know from our analyses in Sections 3 and 4 that if only negotiator 1 chooses a deadline, then flexible negotiator 1 will concede to negotiator 2 before or at time K_1, and so the game will never go beyond this time. This behavior of flexible negotiator 1 affects both negotiators' equilibrium strategies. However, if both negotiators choose a deadline and the difference between K_2 and K_1 is sufficiently small, then flexible negotiator 1 may prefer to wait beyond time K_1. In equilibrium, negotiator 1's type will be revealed at time K_1 if she does not leave the bargaining table at this time. Likewise, the second negotiator's type will also be revealed at time K_2. Therefore, negotiator 1's behavior during the interval $[K_1, K_2]$ depends upon her continuation payoff in the subgame following time K_2, where both negotiators' types are common knowledge. That is, we first need to investigate the equilibrium of the concession (i.e., the war of attrition) game between two flexible negotiators. As we know from Hendricks, Weiss, and Wilson (1988), there are multiple equilibria of this game, depending on the negotiators' declared demands at the beginning of the game. Moreover, when both negotiators have the option of leaving the bargaining game, then there also are multiple equilibria even though the negotiators' outside options are 0 and there is no uncertainty on negotiators' types (Ponsati and Sakovics, 1998).

This multiplicity issue and the complexity/discontinuity of the negotiators' payoffs are likely to make equilibrium investigation of this particular case inconclusive.

6. Concluding Remarks and Related Literature

According to the equilibrium calculations, negotiator 1 always chooses the exit-time (i.e., the deadline) positive if the negotiators' initial offers are incompatible. The deadline is a function of both negotiators' initial demands, negotiator 1's commitment probability, and negotiator 2's time preferences. If negotiator 2 is more impatient, then negotiator 1 chooses a shorter deadline. That is, negotiator 1 applies more time pressure on her rival (by shortening the deadline) if her rival is impatient. There is a tradeoff between initial offers and the length of the deadline: If negotiator 1 makes a greedy offer, then she cannot choose a short deadline because it will not be a credible threat. In equilibrium, negotiators never choose extreme demands, and the exit threat always makes agent 1 advantageous.

In equilibrium, negotiator 1 demands $\bar{\alpha}_1$—a function of both negotiators' commitment probabilities and time preferences—and negotiator 2 offers $1 - \bar{\alpha}_1$, so the game ends in the first stage. Negotiator 2, who cannot choose a deadline, can mitigate the effects of his rival's threat by insuring himself against some of the utility loss he will suffer when he commits to his demand. There is multiple equilibria in the partial-insurance case. The upper bound for negotiator 1's equilibrium demands is $\bar{\alpha}_1$. When negotiator 2 is partially compensated, the temptation of reaching a better deal causes delay in equilibria. However, this temptation fades away when negotiator 2 is liable for all the utility losses that occur due to the commitment tactic.

As a possible extension and a new line for future research, it would be interesting to consider a setup where negotiator 1 chooses both her demand and the deadline before the second negotiator makes his offer. This structure allows the second negotiator to modify his demand given the first negotiator's deadline announcement. The equilibrium strategies in this case do not immediately follow from the analyses in the paper. However, intuition suggests that the second negotiator's equilibrium surplus should be higher because negotiator 1 will not be able to position her deadline according to the second negotiator's demand, and thus, she may not always be the strong negotiator in the game.

A model or a theory that helps us understand why and under what conditions bargainers use the "take it or leave it" offers at the first place would be a significant contribution to the literature. For this purpose, we need to study a model where the identity of the negotiator, using the exit threat, is determined in equilibrium endogenously. One theory on this account would be that negotiators may be uncertain about their valuations at the very beginning of the game, and so the size of the surplus that will be divided is incomplete information. Negotiators may prefer to make offers and counteroffers at the very beginning of the game in order to reduce the uncertainty about the rival's type. Once the negotiators' beliefs hit some level, which will be determined in equilibrium, one of them may want to make a "take it or leave it" offer to speed up the process. Since the bargainers discount time, reaching an agreement earlier is always preferable. However, using "take it or leave it" offers at the early stages of the game is risky for a negotiator because the expected loss in case his opponent calls his bluff is larger.

Commitment and exit threats are studied, to some extent, separately in the bargaining literature. Thus, the current work is the first one that combines these two threats. Shelling (1960) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996), and Ellingsen and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if revoking commitments is costly. The bargaining models in these papers are one-shot simultaneous-move games. Myerson (1991), Kambe (1999), and Abreu and Gul (2000) follow a reputational approach: parallel to Kreps and Wilson (1982) and Milgrom and Roberts (1982), commitments are modeled as behavioral types that exist in society so that rational players can mimic if they like to do so.[14] The bargaining models in the second group of papers are continuous-time, infinite-horizon games, which provide a fruitful platform to examine endogenous deadlines. A common message of the previous results is that commitment threat/tactic benefits a negotiator if this player is the only one who has the option to use it. However, if both negotiators are allowed to use the commitment tactic, then multiple equilibria arise, and inefficiency (that is caused by delay) is a very likely outcome.

The exit threat is studied less in the literature than the commitment threat. Among many others, Osborne and Rubinstein (1990), Vislie (1988), Shaked (1994), Ponsati and Sakovics (1998), and Ponsati and Sakovics (2001) model exit as the ability of opting out of negotiation and receiving an outside option. On the other hand, Fershtman and Seidmann (1993), Ma and Manove (1993), and Ponsati (1995), for example, model exit as a predetermined deadline for the negotiations. The treatment of the exit threat in this paper has resemblance to both of these approaches; announcing an exit time creates a deadline effect and the ability of choosing the time of exit provides strategic advantage for much the same reason as the ability of opting out does. However, the exit threat approach of this paper gives rise significantly different dynamics; unless the negotiators leave the bargaining table, the negotiations could continue beyond the announced deadline, and opting out is never a best response for the flexible negotiators.

Fudenberg and Tirole (1986), Chatterjee and Samuelson (1987), and Ponsati and Sakovics (1995), for example, study war of attrition (WOA) games with two-sided uncertainty. Kambe

[14] Abreu and Sethi (2003) supports the existence of commitment types from an evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with evolutionary stability in bargaining environments.

(1999) and Abreu and Gul (2000) take a step forward and add a pre-play to standard WOA games, where the negotiators simultaneously choose their demands that determine their strategy in the WOA phase. The current paper, however, adds another layer to Kambe (1999) and Abreu and Gul (2000), in which one of the negotiators announces an exit time.

Technically, this paper is closer to Kambe (1999) because in both models, negotiators choose a demand that they may have to commit to later. There are two reasons for adopting this approach. The first one is technical. Abreu and Gul (2000) interpret obstinate types as types that are born with their demands. Given this interpretation, if negotiator i is rational and demanding $\alpha_i \in [0, 1]$, then this is his strategic choice. If he is an obstinate type, then he merely declares the demand corresponding to his type.[15] However, this interpretation leads to a substantially complex model in our setting when there is a large (in particular dense) set of types. In that regard, both Kambe (1999) and the current paper follow an approach leading to a fairly simpler model to work with, where a negotiator's initial probability of being the obstinate type is independent of the demand (and the deadline) the negotiators announce at the beginning of the game.

The second reason for using this approach is that negotiators may (have to) commit to their initial demands or threats depending on how the events unfold during negotiation, and they may be uncertain about whether they will have to commit or not prior to their (demand and deadline) announcements. For example, an agent—who negotiates on behalf of his principal—may have unaligned preferences with his principal and may not know whether the principal will approve his tactic/bluff. In the Dr. Cuddy versus Atlantic Net negotiation, for instance, the board members make it clear to Dr. Cuddy that following a strategy leading to an impasse with the insurance company is not acceptable. However, Dr. Cuddy learns this after she makes her threats. On the other hand, a state leader (or an executive manager) may commit to his demand and deadline if revoking his commitments turns out to be very costly (cost of losing face or credibility of his rhetorics), and the leader may not know the size of this cost for a fact before seeing his audiences' attachment/reaction to his commitments.

Although there are important similarities, the current paper is significantly different from earlier works, which also use the reputational approach, in two aspects. First, in this paper, I suppose that the obstinate type leaves the bargaining table, and this imposes a deadline effect on reputation building. Second, I consider two separate cases regarding negotiators' equilibrium payoffs: the no-insurance (e.g., Kambe, 1999) and partial-insurance cases. These two case studies are important because they are valuable steps towards more detailed investigations on the countermeasures of some bargaining tactics that negotiators extensively use in negotiations.

[15] We can easily extend this approach to the current setup. If negotiator i is rational and announcing a deadline $K \in [0, \infty)$, then this is his strategic choice. If he is an obstinate type, then he merely declares the deadline corresponding to his type.

Appendix

Propositions 1 — 6 deal with the flexible negotiators' equilibrium strategies in the concession game, so their proofs are independent of the compensation system. Thus, the negotiators' expected payoffs, that are used to prove these results, are calculated by Equation (2). Although Proposition 7 and Theorem 1 (in particular the optimal value of K) deal with negotiators' first stage choices, these two results hold regardless of the compensation system. I will provide the proofs of these two results only for the case where the negotiators are compensated when they receive the message c. I do this only for notational brevity/simplicity But, I will also provide the negotiators' expected payoffs if they are not compensated, and so one can easily check that both Proposition 7 and Theorem 1 will continue to hold regardless of the compensation system.

Proof of Proposition 1. Let $U_1(t, F_2)$ denote flexible negotiator 1's expected payoff of conceding at time $t \in \{0_1, 0_2\}$ given F_2. Therefore, $U_1(0_1, F_2) = F_2(0_1)(1-z_2)(\frac{1+\alpha_1-\alpha_2}{2}) + z_2(1-\alpha_2) + (1-z_2)(1-F_2(0_1))(1-\alpha_2)$ and $U_1(0_2, F_2) = (1-z_2)F_2(0_1)\alpha_1 + [(1-z_2)(1-F_2(0_1)) + z_2](1-\alpha_2)$.

Similarly, let $U_2(t, F_1)$ denote flexible negotiator 2's expected payoff of conceding at time t given F_1. Therefore, $U_2(0_1, F_1) = F_1(0_1)(\frac{1+\alpha_2-\alpha_1}{2})(1-z_1) + z_1(1-\alpha_1) + (1-z_1)(1-F_1(0_1))(1-\alpha_1)$. Let $U_2(F_1)$ denote flexible negotiator 2's expected payoff of not accepting negotiator 1's demand. Thus, $U_2(F_1) = (1-z_1)F_1(0_1)\alpha_2 + (1-z_1)(1-F_1(0_1))\alpha_2$.

1. Suppose now that $z_1 > p_H$. To show $F_1(0_1) = 0$, $F_1(0_2) = 1$ and $F_2(0_1) = 1$ are the equilibrium strategies, first show that $F_2(0_1) = 1$ is a best response to $F_1(0_1) = 0$, $F_1(0_2) = 1$. For this reason, we need to show $U_2(0_1, F_1) \geq U_2(F_1)$. The last inequality implies that $z_1(1-\alpha_1)+(1-z_1)(1-\alpha_1) \geq (1-z_1)\alpha_2$, which is true if and only if $z_1 > \frac{\alpha_1+\alpha_2-1}{\alpha_2} = p_H$. Similarly, to show that F_1 is a best response to $F_2(0_1) = 1$, we need to show $U_1(0_2, F_2) = \alpha_1(1-z_2) + z_2(1-\alpha_2) > U_1(0_1, F_2) = z_2(1-\alpha_2) + (1-z_2)(\frac{1+\alpha_1-\alpha_2}{2})$. The last inequality holds if and only if $\alpha_1 + \alpha_2 > 1$. Thus, these strategies constitute an equilibrium.

   To establish uniqueness, first observe that $U_1(0_2, F_2) > U_1(0_1, F_2)$ for all values of $F_2(0_1) > 0$. Thus, as long as $F_2(0_1) > 0$, we have $F_1(0_1) = 0$. But, if $F_1(0_1) = 0$, then flexible negotiator 2 prefers to accept at time 0_1. Hence, we need to show that $F_2(0_1) = 0$ cannot hold in equilibrium: Suppose for a contradiction that $F_2(0_1) = 0$. Then, we must have that $U_2(F_1) \geq U_2(0_1, F_1)$. However, the last inequality holds if and only if $F_1(0_1) \leq 2(\frac{p_H - z_1}{p_H(1-z_1)})$, which is never true since $z_1 > p_H$.

2. Given that $F_1(0_1) \leq 2(\frac{p_H-z_1}{p_H(1-z_1)})$, we have $U_2(F_1) \geq U_2(0_1, F_1)$. That is, negotiator 2's strategy is a best response to any $F_1(0_1)$ that satisfies the above inequality. Likewise, if $F_2(0_1) = F_2(0_2) = 0$, then $U_1(0_1, F_2) = U_1(0_2, F_2) = 1-\alpha_2$, and so $F_1(0_1)$ is a best response.

3. If $z_1 = p_H$ and $F_1(0_1) = 0$, then $U_2(0_1, F_1) = U_2(F_1)$, and so any $F_2(0_1) \in [0, 1]$ is a best response to F_1. Moreover, as $F_2(0_1) \in [0, 1]$, we have $U_1(0_2, F_2) \geq U_1(0_1, F_2)$.

Q.E.D. for the proof of Proposition 1.

Proof of Proposition 3. First, I will study the properties of the sequential equilibrium strategies (distribution functions) in the concession game, i.e. F_1 and F_2. For this purpose, given α_1, α_2 and K where $\alpha_1 + \alpha_2 > 1$ and $K \in (0, T_0)$, consider a pair of equilibrium distribution functions (F_1, F_2) defined over the domain $[0, \infty)$. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *If a negotiator's strategy is constant on some interval $[t_1, t_2] \subseteq [0, K_1)$, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *F_1 and F_2 do not have a mass point over $(0, K_1)$.*

The idea behind the proof of Lemma A.2 is that if negotiator i concedes with a positive probability at some time $t \in (0, K_1)$, then j prefers to wait during $[t - \epsilon, t]$ for some $\epsilon > 0$ and concede right after time t. This implies that negotiator j's strategy is constant on interval $[t - \epsilon, t]$. Therefore, by Lemma A.1 negotiator i's equilibrium strategy is also constant on this interval, contradicting with the initial assumption that i concedes with a positive probability at time t.

Lemma A.3. *$F_1(0)F_2(0) = 0$.*

Therefore, according to Lemma A.1 and A.2, both F_1 and F_2 are strictly increasing and continuous over $[0, K_1]$.

To prove the first part of Proposition 3, first note that there is no interval (t', t'') with $0 \le t' < t'' < K_1$ such that both F_1 and F_2 are constant. Assume on the contrary that $t^* < K_1$ is the supremum of the upper bounds of t'''s such that both F_1 and F_2 are constant. However, through lemma A.1, if F_i is constant on (t', t^*) for $i \in \{1, 2\}$, then F_j where $j \in \{1, 2\}, j \neq i$ is constant on $(t', t^* + \eta)$ for some small $\eta > 0$. Hence, both F_1 and F_2 are constant on this later interval, contradicting the definition of t^*.

Hence, if $0 \le t_1 < t_2 \le K_1$, then we have $F_i(t_2) > F_i(t_1)$ for $i = 1, 2$. Moreover, Lemma A.2 implies that both F_1 and F_2 are continuous over $[0, K_1)$. Finally, to show that both F_i's are continuous on $[0, K_1]$, suppose for a contradiction that F_2 has jump at time K_1. But, then negotiator 1 prefers to wait for some time before K_1 and concede at time K_2. However, this contradicts the fact that F_1 is strictly increasing over $[0, K_1]$. Likewise, F_1 cannot have a jump at time K_1. Suppose for a contradiction that $F_1(K_1) - F_1(K_1^-) = p_1 > 0$ where $F_1(K_1^-) = \lim_{t \uparrow K_1} F_1(t)$. Then let $U_2(t, F_1)$ denotes flexible negotiator 2's expected payoff of waiting until time t and accepting α_1 at that time. Then, we have

$$U_2(t = K_1 - \Delta, F_1) = \alpha_2(1 - z_1) \int_0^{K_1 - \Delta} e^{-r_2 y} dF_1(y) + (1 - \alpha_1)(1 - B_1(K_1 - \Delta))e^{-r_2(K_1 - \Delta)}$$

$$U_2(t = K_1, F_1) = \alpha_2(1 - z_1) \int_0^{K_1} e^{-r_2 y} dF_1(y) + e^{-r_2 K_1}[\frac{1}{2}(1 + \alpha_2 - \alpha_1)p_1 + (1 - \alpha_1)(1 - B_1(K_1))]$$

Therefore, $U_2(t = K_1, F_1) - U_2(t = K_1 - \Delta, F_1) > 0$ for small values of Δ because this difference is equal to $o_1(\Delta) + o_2(\Delta) + e^{-r_2 K_1}\frac{1}{2}(1 + \alpha_2 - \alpha_1)p_1$ where $o_1(\Delta) = \alpha_2(1 - z_1) \int_{K_1 - \Delta}^{K_1} e^{-r_2 y} dF_1(y)$,

$o_2(\Delta) = (1-\alpha_1)[(1-B_1(K_1)) - (1-B_1(K_1-\Delta))e^{r_2\Delta}]$ and both $o_1(\Delta)$ and $o_2(\Delta)$ approach 0 as Δ approaches 0.

Thus we can conclude that if F_1 has jump at time K_1, then negotiator 2 prefers to wait for some time $[K_1-\Delta, K_1)$ for $\Delta > 0$ small enough and concede at time K_1, contradicting with the fact that F_2 cannot be constant over $[0, K_1]$. Recall that

$$U_i(t, F_j) = \alpha_j(1-z_j)\int_0^t e^{-r_i y} dF_j(y) + \alpha_i e^{-r_i t}[1-(1-z_j)F_j(t)]$$

denote the expected payoff of flexible negotiator i who concedes at time t. Therefore, the utility functions are also continuous on $[0, K_1]$.

Then, it follows that $D^i \equiv \{t | U_i(t, F_j) = \max_{s\in[0,K_1]} U_i(s, F_j)\}$ is dense in $[0, K_1]$. Hence, $U_i(t, F_j)$ is constant for all $t \in [0, K_1]$. Consequently, $D^i = [0, K_1]$. Therefore, $U_i(t, F_j)$ is differentiable as a function of t. The differentiability of F_1 and F_2 follows from the differentiability of the utility functions on $[0, K_1]$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i(t) = \frac{1}{1-z_i}(1-c_i e^{-\lambda_i t})$ for all $t \leq K_1$ where $c_i = 1 - F_i(0)$ and $\lambda_i = \frac{r_j(1-\alpha_i)}{\alpha_1+\alpha_2-1}$.

Finally, since obstinate type of negotiator 1 leaves the game at time K_1, negotiator 1's type will be revealed at time K_2. Hence, flexible negotiator 2 will never concede after time K_1 and flexible negotiator 1 will accept α_2 and finish the game before or at time K if the game has not ended before.

To prove the second part of Proposition 3, I suppose that $\hat{z}_2 < 1$. Because the equilibrium strategy F_2 is continuous on $[0, K_1]$, flexible negotiator 2 must be indifferent between conceding and waiting (not conceding) at time K_1. If negotiator 2 concedes at time K_1, then his instantaneous payoff will be $1-\alpha_1$. However, if he waits, then his expected payoff will be $(1-\hat{z}_1)\alpha_2$ because flexible negotiator 1 accepts α_2 at time K_2 for sure. These two payoffs are equal if and only if $\hat{z}_1 = \frac{\alpha_1+\alpha_2-1}{\alpha_2} = p_H$.

Q.E.D. for the proof of Proposition 3.

Proof of Theorem 1. As a result of the propositions 1 through 6, the negotiators' equilibrium payoffs in the game are as follows (assuming that $\alpha_1 + \alpha_2 > 1$ and negotiators are compensated when they receive the message c.)

CASE 1: If $K = 0$, then

1. if $z_1 > p_H$, then
$$u_1 = (1-z_2)\alpha_1 + z_2(1-\alpha_2) \quad \text{and} \quad u_2 = 1-\alpha_1$$
2. if $z_1 < p_H$, then
$$u_1 = 1-\alpha_2 \quad \text{and} \quad u_2 = (1-z_1)\alpha_2$$
3. if $z_1 = p_H$, then
$$u_1 = (1-z_2)F_2(0_1)\alpha_1 + [(1-z_2)(1-F_2(0_1)) + z_2](1-\alpha_2) \quad \text{and} \quad u_2 = 1-\alpha_1$$

where $F_2(0_1) \in [0,1]$.

CASE 2: If $0 < K \leq T_0$, then let $K^* = -\frac{\ln(z_1/p_H)}{\lambda_1}$

1. if $K < K^*$, then

$$u_1 = 1 - \alpha_2 \quad \text{and} \quad u_2 = (1 - \frac{z_1}{p_H} e^{\lambda_1 K})\alpha_2 + \frac{z_1}{p_H} e^{\lambda_1 K}(1 - \alpha_1)$$

2. if $K^* < K$, then

$$u_1 = (1 - z_2 e^{\lambda_2 K})\alpha_1 + z_2 e^{\lambda_2 K}(1 - \alpha_2) \quad \text{and} \quad u_2 = 1 - \alpha_1$$

3. if $K^* = K$, then

$$u_1 = (1 - c_2)\alpha_1 + c_2(1 - \alpha_2) \quad \text{and} \quad u_2 = 1 - \alpha_1$$

where $c_2 \in [z_2 \left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}, 1]$

CASE 3: If $T_0 < K$, then

1. if $z_1 > z_2^{\lambda_1/\lambda_2}$, then

$$u_1 = (1 - z_2 z_1^{-\lambda_2/\lambda_1})\alpha_1 + z_2 z_1^{-\lambda_2/\lambda_1}(1 - \alpha_2) \quad \text{and} \quad u_2 = 1 - \alpha_1$$

2. if $z_2^{\lambda_1/\lambda_2} > z_1$, then

$$u_1 = 1 - \alpha_2 \quad \text{and} \quad u_2 = (1 - z_1 z_2^{-\lambda_1/\lambda_2})\alpha_2 + z_1 z_2^{-\lambda_1/\lambda_2}(1 - \alpha_1)$$

3. if $z_1 = z_2^{\lambda_1/\lambda_2}$, then

$$u_1 = (1 - \alpha_2) \quad \text{and} \quad u_2 = 1 - \alpha_1$$

First note that according to Proposition 3, negotiator 1's reputation must reach p_H at time K. Hence, negotiator 1 needs K^* amount of time to build her reputation if $z_1 < p_H$ if she does not make any probabilistic concession at time 0. Clearly negotiator 1 can build her reputation to p_H much earlier than K^* if she makes a probabilistic concession at time 0. However, in this case, negotiator 1's expected payoff in the game will be the lowest payoff she can achieve in the game, that is it will be $1 - \alpha_2$. Therefore, in equilibrium, negotiator 1 will choose K no less than K^*.

To prove the first part of Theorem 1, suppose that $p_H \leq z_1$. In this case negotiator 1 does not need any time to build her reputation. Also, it is easy to see from the above payoff functions that payoff of negotiator 1 is the highest when she chooses $K = 0$. Hence, in equilibrium, negotiator 1 will pick $K = 0$, and the equilibrium strategies in the concession game will be as given in Proposition 1.

To prove the second part, suppose that $p_L \leq z_1 < p_H$. It immediately follows that K cannot be 0. Since $p_L \leq z_1$ holds, we have $K^* \leq T_0$. Thus, negotiator 1 will choose $0 < K \leq T_0$, and so her equilibrium payoff is one of those given above in CASE 2. Clearly, negotiator 1 does not select $K < K^*$ because she can achieve higher. However, for any K satisfying $K^* < K$, negotiator 1 can increase her payoff by choosing an exit-time shorter than K. Therefore, the optimal choice for negotiator 1 is $K = K^*$, and the equilibrium strategies are characterized by Proposition 6.

However, in equilibrium, the value of c_2 is uniquely determined and it is equal to $z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}$. The proof of the last argument is easy. First note that negotiator 1's payoff decreases as c_2 increases. Suppose for a contradiction that there is an equilibrium where negotiator 1 chooses $K^* = K$ and $c_2 > z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}$. In this case, negotiator 1 could increase her expected payoff by deviating to $K' = K^* + \epsilon$ for some sufficiently small $\epsilon > 0$. This is true because negotiator 1's payoff gets $\left(1 - z_2 e^{\epsilon\lambda_2}\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}\right)\alpha_1 + z_2 e^{\epsilon\lambda_2}\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}(1-\alpha_1)$, which is strictly higher than $(1-c_2)\alpha_1 + c_2(1-\alpha_2)$ for sufficiently small $\epsilon > 0$.

Finally to prove the third part of Theorem 1, suppose that $z_1 < p_L$. In this case, we have $T_0 < K^*$ and $z_1 < z_2^{\lambda_1/\lambda_2}$. Therefore, negotiator 1's expected payoff is as given under (2) in CASE 3. That says, negotiator 1's expected payoff is $1 - \alpha_2$, and is independent of the value of K she chooses. Thus, any $K \in [T_0, \infty)$ forms an equilibrium if F_1 and F_2 are given in Proposition 2.

REMARK: Negotiator 1's expected payoffs in the game, when the negotiators are *not* compensated, are as follow:

- If $K = 0$, then
  1. if $z_1 \geq p_H$, then $u_1 = (1 - z_1)\Big[(1 - z_2)\alpha_1 + z_2(1 - \alpha_2)\Big] + z_1(1 - z_2)\alpha_1$
  2. if $z_1 < p_H$, then $u_1 = (1 - z_1)(1 - \alpha_2)$
- If $0 < K \leq T_0$, then let $K^* = -\frac{\ln(z_1/p_H)}{\lambda_1}$
  1. if $K < K^*$, then $u_1 = (1 - z_1)(1 - \alpha_2) + z_1\alpha_1\lambda_2 \int_0^K e^{-(r_1+\lambda_2)t}dt$
  2. if $K^* < K$, then

$$\begin{aligned} u_1 = {} & (1 - z_1)\Big[(1 - z_2 e^{\lambda_2 K})\alpha_1 + z_2 e^{\lambda_2 K}(1 - \alpha_2)\Big] \\ & + (1 - z_2 e^{\lambda_2 K})z_1\alpha_1 + z_2 e^{\lambda_2 K} z_1\alpha_1\lambda_2 \int_0^K e^{-(r_1+\lambda_2)t}dt \end{aligned}$$

  3. if $K^* = K$, then

$$\begin{aligned} u_1 = {} & (1 - z_1)\Big[(1 - c_2)\alpha_1 + c_2(1 - \alpha_2)\Big] \\ & + z_1\alpha_1\Big[(1 - c_2) + c_2\lambda_2 \int_0^K e^{-(r_1+\lambda_2)t}dt\Big] \end{aligned}$$

  where $c_2 = z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}$ as we prove in the proof of Theorem 1.
- If $T_0 < K$, then

1. if $z_1 > z_2^{\lambda_1/\lambda_2}$, then

$$u_1 = (1-z_1)\Big[(1-z_2 z_1^{-\lambda_2/\lambda_1})\alpha_1 + z_2 z_1^{-\lambda_2/\lambda_1}(1-\alpha_2)\Big] + z_1\alpha_1\Big[1-z_2 z_1^{-\lambda_2/\lambda_1}\Big] + z_2 z_1^{-\lambda_2/\lambda_1}\lambda_2\int_0^{T_0} e^{-(r_1+\lambda_2)t}dt$$

2. if $z_2^{\lambda_1/\lambda_2} \geq z_1$, then $u_1 = (1-z_1)(1-\alpha_2)$

Q.E.D. for the proof of Theorem 1.

Proof of Proposition 7. Consider a sequential equilibrium where negotiators' demands α_1^* and α_2^* are incompatible. Suppose for a contradiction that $z_1 \notin Z(\alpha_1^*, \alpha_2^*)$. First, suppose that $p_H(\alpha_1^*, \alpha_2^*) < z_1$. According to Theorem 1, in equilibrium, negotiator 1 selects $K = 0$, which leads to expected payoff of $(1-z_2)\alpha_1^* + z_2(1-\alpha_2^*)$. However, if negotiator 1 demands $\alpha_1^* + \epsilon$ where $\epsilon > 0$ is small enough so that we still have $p_H(\alpha_1^*+\epsilon, \alpha_2^*) < z_1$, then negotiator 1's expected payoff increases to $(1-z_2)(\alpha_1^*+\epsilon)+z_2(1-\alpha_2^*)$, contradicting the optimality of the equilibrium.

Now suppose that $z_1 < p_L(\alpha_1^*, \alpha_2^*)$. Then, according to Theorem 1, negotiator 1 selects any $K > T_0$ and her expected payoff in the game is $1-\alpha_2^*$. However, if negotiator 1 demands $1-\alpha_2^*+\epsilon$ where $\epsilon > 0$ is small enough so that $\frac{\epsilon}{\alpha_2^*} = p_H(1-\alpha_2^*+\epsilon, \alpha_2^*) < z_1$, then negotiator 1's expected payoff increases to $(1-z_2)(1-\alpha_2^*+\epsilon) + z_2(1-\alpha_2^*)$, contradicting the optimality of the equilibrium.

Next, I will show that we must have $\alpha_1^* \leq \bar{\alpha}_1$. Suppose for a contradiction that $\alpha_1^* > \bar{\alpha}_1$. That is, given the definition of $\bar{\alpha}_1$, there must exist some $\alpha_2 > 1-\alpha_1^*$ such that $p_L(\alpha_1^*, \alpha_2) > z_1$. Note that α_2 is different from α_2^* because, as we just proved, the equilibrium prices α_1^* and α_2^* must satisfy $z_1 \in Z(\alpha_1^*, \alpha_2^*)$. Finally, note that in equilibrium, negotiator 1 will choose $K = \frac{-\ln(z_1/p_H)}{\lambda_1}$ because $z_1 \in Z(\alpha_1^*, \alpha_2^*)$ holds, and so negotiator 2 will be weak. That is, $u_2(\alpha_1^*, \alpha_2^*) = 1-\alpha_1^*$

However, if negotiator 2 deviates to α_2, then according to Theorem 1, negotiator 1 will choose $K \in [T_0, \infty)$ and negotiator 2 will become strong. In this case, negotiator 2's expected payoff in the game will be $u_2(\alpha_1^*, \alpha_2) = (1-z_1 z_2^{-\lambda_1/\lambda_2})\alpha_2 + z_1 z_2^{-\lambda_1/\lambda_2}(1-\alpha_1^*)$, which is larger than $u_2(\alpha_1^*, \alpha_2^*)$ because $\alpha_2 > 1-\alpha_1^*$, contradicting with the optimality of equilibrium. Thus, in equilibrium, we must have $\alpha_1^* \leq \bar{\alpha}_1$. Furthermore, since the optimality of equilibrium implies $\alpha_1^* + \alpha_2^* \geq 1$, we must have $1-\alpha_2^* \leq \alpha_1^* \leq \bar{\alpha}_1$.

Q.E.D. for the proof of Proposition 7.

Proof of Theorem 2. Suppose now that the negotiators are not compensated for their loss that may occur due to commitment. In this case, the negotiators' expected payoffs are calculated by Equation (4). Note that Propositions 1—6 must still hold because these results characterize the flexible negotiators' equilibrium strategies. One can easily check the proofs of Theorem 1 and Proposition 7 that these results will continue to hold.

For any $\alpha_1 \in (0,1)$, the best response correspondence of negotiator 2 is as follows:

$$BR_2(\alpha_1) = \begin{cases} 1-\alpha_1, & \text{if } \alpha_1 \leq \bar{\alpha}_1 \\ \underset{\substack{\alpha_2 \in [1-\alpha_1, 1) \\ z_1 < p_L}}{\arg\max} \; \bar{U}_2, & \text{otherwise.} \end{cases}$$

where $\bar{U}_2 = (1-z_2)\left(\alpha_2 - z_1(\alpha_1+\alpha_2-1)z_2^{-\lambda_1/\lambda_2}\right) + z_2(1-z_1)\int_0^K e^{-r_2 y} dF_1(y)$. In the best response correspondence of the second negotiator, the critical change is that negotiator 2 prefers to choose a compatible demand $(1-\alpha_1)$ and finish the game at the very beginning of the game. The reason for this is the following. If negotiator 1 chooses a price α_1 that is less than $\bar{\alpha}_1$, then there is no price that negotiator 2 can deviate and make himself strong. However, because negotiator 2 is not compensated when he receives the message c, the weak negotiator 2's expected payoff in the game is strictly less than $1-\alpha_1$ if he chooses an incompatible demand. Therefore, optimality implies that negotiator 2 will finish the game by choosing a compatible demand $1-\alpha_1$ in stage 1.

No price $\alpha_1 > \bar{\alpha}_1$ can be sustained in equilibrium because if $\alpha_1 > \bar{\alpha}_1$, then negotiator 2 prefers to deviate to a demand $\alpha_2 > 1-\alpha_1$ to make himself strong. No price $\alpha_1 < \bar{\alpha}_1$ can be supported in equilibrium because negotiator 1 can increase her expected payoff by increasing her price to some $\alpha_1 + \epsilon$. If negotiator 1 chooses $\alpha_1 < \bar{\alpha}_1$, then in equilibrium the second negotiator will choose $1-\alpha_1$. However, by choosing $\alpha_1+\epsilon$, the first negotiator guarantees that $z_1 > p_H = \frac{\epsilon}{\alpha_2}$, and so, she increases her expected payoff over α_1. Hence, the unique equilibrium is that first negotiator chooses $\bar{\alpha}_1$.

Q.E.D. for the proof of Theorem 2.

Proof of Theorem 3. The best response correspondences of the negotiators are as follows: For any $\alpha_1 \in (0,1)$,

$$BR_2(\alpha_1) = \begin{cases} [1-\alpha_1, 1), & \text{if } \alpha_1 \leq \bar{\alpha}_1 \\ \underset{\substack{\alpha_2 \in [1-\alpha_1, 1) \\ z_1 < p_L}}{\arg\max} \left\{\alpha_2 - z_1(\alpha_1+\alpha_2-1)z_2^{-\lambda_1/\lambda_2}\right\}, & \text{otherwise.} \end{cases}$$

For any $\alpha_2 \in (0,1)$, the best response correspondence of the first negotiator is,

$$BR_1(\alpha_2) = \underset{\substack{\alpha_1 \in [1-\alpha_2, \bar{\alpha}_1] \\ z_1 \in Z(\alpha_1,\alpha_2)}}{\arg\max} \left\{\alpha_1 - z_2(\alpha_1+\alpha_2-1)(p_H/z_1)^{\lambda_2/\lambda_1}\right\}$$

Note that for any α_2 given, as α_1 decreases down to $1-\alpha_2$, then p_H decreases to 0. Hence, for any α_2 given, there always exists some α_1 strictly higher than $1-\alpha_2$ such that $z_1 \in Z(\alpha_1,\alpha_2)$.

By proposition 7 we know that the equilibrium demands α_1^* and α_2^* must satisfy $1-\alpha_2 \leq \alpha_1^* \leq \bar{\alpha}_1$ and $z_1 \in Z(\alpha_1^*, \alpha_2^*)$. Given that we must have $z_1 \in Z(\alpha_1^*, \alpha_2^*)$, Theorem 1 implies that negotiator 1 is strong, and hence her expected payoff is $(1 - z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1})\alpha_1^* + z_2\left(\frac{p_H}{z_1}\right)^{\lambda_2/\lambda_1}(1-\alpha_2^*)$ where the parameters p_H, λ_1 and λ_2 are calculated with α_1^* and α_2^*.

Q.E.D. for the proof of Theorem 3.

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