Supplementary Appendix

for

Bargaining, Reputation, and Competition

by

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Proposition 1. *In any (sequential) equilibrium of the bargaining game G following a history h_T, where players' price announcements (bids) are $1 > \alpha_s > \alpha_1, \alpha_2$, the seller arrives at buyer i at time T and his actual type has not yet revealed, the players' concession game strategies are $F_s^{i,T}(t) = 1 - c_s^i e^{-\lambda_s^i (t-T)}$ and $F_i^T(t) = 1 - c_i e^{-\lambda_i (t-T)}$ for all $t \geq T$, where $c_i^s, c_i \in [0,1]$ and $F_s^{i,T}(T) F_i^T(T) = 0$.*

Proof of Proposition 1. First, I will study the properties of the equilibrium strategies (distribution functions) in concession games. For this purpose, take any $i \in \{1,2\}$ and history $h_{T_i} \in H^i$, and consider a pair of equilibrium distribution functions $(F_s^{i,T_i}, F_i^{T_i})$ defined over the domain $[T_i, T_i']$ where $T_i' \leq \infty$ depends on the seller's equilibrium strategy. Proofs of the following results directly follow from Ozyurt (2015), from the arguments in Hendricks, Weiss and Wilson (1988), and are analogous to the proof of Lemma 1 in Abreu and Gul (2000). Therefore, I skip the details.

Lemma A.1. *If a player's strategy is constant on some interval $[t_1, t_2] \subseteq [T_i, T_i')$, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *F_s^{i,T_i} and $F_i^{T_i}$ do not have a mass point over $(T_i, T_i']$.*

Lemma A.3. *$F_i^{T_i}(T_i) F_s^{i,T_i}(T_i) = 0$*

Therefore, according to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_s^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Recall that $U_s^i(t, F_i^T) = \alpha_s \int_0^{t-T} e^{-r_s y} dF_i^T(y) + \alpha_i [1 - F_i^T(t)] e^{-r_s(t-T)}$ denotes the expected payoff of rational seller who concedes at time $t \geq T_i$ and $U_i(t, F_s^{i,T}) = (1-\alpha_i) \int_0^{t-T} e^{-r_b y} dF_s^{i,T}(y) + (1-\alpha_s)[1 - F_s^{i,T}(t)] e^{-r_b(t-T)}$ denotes the expected payoff of the rational buyer i who concedes to the seller at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$. Then, it follows that $D^{i,T_i} := \{t | U_i(t, F_s^{i,T_i}) = \max_{y \in [T_i, T_i']} U_i(y, F_s^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_s^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $D^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_s^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_s^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda_i t}$ and $F_s^{i,T_i}(t) = 1 - c_s^i e^{-\lambda_s^i t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_s^i = 1 - F_s^{i,T_i}(T_i)$ such that $\lambda_i = \frac{\alpha_i r_s}{\alpha_s - \alpha_i}$ and $\lambda_s^i = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_i}$.

Q.E.D.

Proposition 2. *In any equilibrium where the players' initial bids satisfy the inequalities* (**??**) *and $\alpha_2 > \delta\alpha_1$, the rational seller visits each buyer at most once, and a rational buyer does not allow the seller to leave him without reaching an agreement. Moreover, in this equilibrium if the rational seller visits buyer 2 first, leaves 2 at time T_2^d and finalizes the game with buyer 1 at time T_1^e if the game has not yet ended before, then the players' concession game strategies must satisfy*

$$F_s^2(t) = 1 - c_s^2 e^{-\lambda_s^2 t} \quad F_2(t) = 1 - z_b e^{\lambda_2(T_2^d - t)}$$
$$F_s^1(t) = 1 - e^{-\lambda_s^1 t} \quad F_1(t) = 1 - z_b e^{\lambda_1(T_1^e - t)}$$

where

$$F_s^2(0)F_2(0) = 0 \qquad \text{and} \qquad F_s^1(T_1^e) = 1 - \frac{z_s}{1 - F_s^2(T_2^d)}$$

Proof of Proposition 2. Given the strategies in Proposition 1, the rational seller's expected payoff of playing the concession game with buyer i during $[T_i, T_i']$ is $[\alpha_s F_i^{T_i}(T_i) + \alpha_i(1 - F_i^{T_i}(T_i))]$, and buyer i's payoff is $[(1-\alpha_i)F_s^{i,T_i}(T_i) + (1-\alpha_s)(1 - F_s^{i,T_i}(T_i))]$. Moreover, by Lemma A.3, we know that $F_i^{T_i}(T_i)F_s^{i,T_i}(T_i) = 0$ for all i. Suppose for a contradiction that there exists equilibrium of the game G in which the rational seller visits, without loss of generality, buyer 1 twice. Suppose that the times at which the seller visits buyer 1 for the first and for the second time are denoted by t_1 and t_2, respectively. We must have $F_1^{t_2}(t_2) > 0$ (i.e., buyer 1 must concede to the seller immediately at time t_2). If this does not hold (i.e., $F_1^{t_2}(t_2) = 0$), then the seller's expected payoff of visiting buyer 1 for the second time would be α_1. However, if the seller would have accepted buyer 2's demand before coming to buyer 1, his payoff would be α_2. Because $\delta\alpha_1 < \alpha_2$, leaving buyer 2 and visiting buyer 1 for the second time would be inconsistent with the optimality of equilibrium.

Because $F_1^{t_2}(t_2) > 0$, we have $F_s^{1,t_2}(t_2) = 0$ by Lemma A.3. The last condition implies that buyer 1's continuation payoff, following the history where the seller visits buyer 1 for the second time, is $1 - \alpha_s$. However, buyer 1 would strictly prefer accepting the seller's offer when the seller first attempts to leave him to eliminate a further delay. Therefore, $F_1^{t_2}(t_2) > 0$ contradicts with the optimality of equilibrium. The same logic would apply to other cases where the seller visits buyer 2 (or buyer 1) multiple times. Thus, in equilibrium, a rational buyer will not allow the seller leave him without an agreement. Hence, the seller will not visit a buyer multiple times.

Now, I will use F_s^i and F_i to indicate the players' third stage strategies. Consider equilibrium strategies F_s^i and F_i, following the history where the rational seller visits buyer 2 first and leaves buyer 2 at time T_2^d. Note that, rational seller visits buyer 1 only if $F_1(0) > 0$. Suppose $F_1(0) = 0$. Then, the rational seller's discounted continuation payoff with buyer 1, $\delta[\alpha_s F_1(0) + \alpha_1(1 - F_1(0))]$, will be $\delta\alpha_1$. Since $\delta\alpha_1 < \alpha_2$, the rational seller prefers to concede to buyer 2 instead of visiting buyer 1, yielding the required contradiction. By lemma A.3. and $F_1(0) > 0$, we must have

$F_s^1(0) = 0$, implying that $c_s^1 = 1$. That is, $F_s^1(t) = 1 - e^{-\lambda_s^1 t}$. Furthermore, assuming that the rational seller leaves buyer 2 at time T_2^d and the concession game with buyer 1 ends at time T_1^e because no player concedes after this time, we must have $F_2(T_2^d) = 1 - z_b$ and $F_1(T_1^e) = 1 - z_b$. Thus we have $c_2 = z_b e^{\lambda_2 T_2^d}$ and $c_1 = z_b e^{\lambda_1 T_1^e}$ as required.

Finally, because buyer 1's reputation reaches 1 at time T_1^e, the rational seller will not continue the game G after this time. Thus, his reputation must also reach 1 at that time, implying that $F_s^1(T_1^e) = 1 - z_s^*$, where $z_s^* = \frac{z_s}{1-F_s^2(T_2^d)}$ is the seller's reputation at the time he arrives at buyer 1.

Q.E.D.

References

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[3] Ozyurt, S. (2015): "Searching for a Bargain: Power of Strategic Commitment," *American Economic Journal: Microeconomics*, 7(1): 320-353.