Searching a Bargain: "Play it Cool" or Haggle*

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Abstract

This paper investigates the impacts of reputation (in contact with inflexibility) on imperfectly competitive search markets where the sellers announce their initial demands prior to the buyer's visit and the buyer directs his search for a better deal. The buyer facing multiple sellers can negotiate with only one at a time and can switch his bargaining partner with some cost. The introduction of commitment types that are inflexible in their demands, even with low probabilities, makes the equilibrium of the resulting multilateral bargaining game essentially unique. A modified war of attrition structure is derived in the equilibrium. If the sellers' initial demands are the same, then the buyer will never visit one seller more than once. If instead the demands are different, a given seller may be visited twice and the buyer may go first to the seller with the higher demand. The model unites and smooths out Bertrand and Diamond price competition models and eliminates their inexplicable predictions.

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1. Introduction

Consider markets in which buyer is either an investor who would like to buy a poorly managed but promising small business, a consumer who plans to purchase, for example, house or car, or a highly skilled worker searching for a job. As is the case for many more examples, behaviors of market participants aiming to buy (or sell) a good or service in these markets differ from those that Bertrand's (1883) price competition and Diamond's (1971) search model aim to explain. The common characteristics of the trading mechanism that we often observe in these markets are that there is more than one potential seller (in competition over the buyers) and each seller usually suggests or posts his price so that the buyer can pay the offered price, buy the good or the service, and finalize the trade.[1] However, the general practice in these markets is that the buyers negotiate with the sellers with the hope of getting a deal better than the solicited "buy-it-now" prices.

On the other hand, in many instances, even the competing sellers and buyers may be inflexible in their demands during the negotiation process. A used car dealer, for example, may be restricted by the owner of the dealership regarding how flexible he can be in his demands when negotiating with a buyer. An entrepreneur who owns a successful small business may be committed to a certain price even before negotiating with investors to sell his business or a franchise because of his overly optimistic expectations about the future of his business. A senior manager (or faculty) in good standing might be willing to accept the new job offer only when he finds the competing salary remarkably superior. Thus, the sellers' posted prices (or the buyers' initial demands) can naturally foster concern that the sellers (or the buyers) might not be willing to negotiate and might ultimately insist on the initial prices (demands) that they announce.

Given that the sellers and the buyers have the opportunity of building "*reputation*" on inflexibility, market participants' equilibrium behavior in imperfectly competitive search markets, such as the ones that are briefly exemplified in previous paragraphs, and the impacts of their behaviors on equilibrium outcome(s) are not known to us. Providing suggestive explanations in these lines, that may facilitate further applications in various other fields such as labor economics, industrial organization and market microstructure is the main motivation of this paper.

In order to illustrate the effects of reputation in imperfectly competitive search markets, I construct a simple set-up with two sellers and a buyer, in which flavors of Bertrand and Diamond price competition models coexist: the sellers bid their demands simultaneously, but the buyer believes that he can get a better deal if he searches for it. However, analysis suggests that equilibrium outcomes are in contrast to the predictions of Bertrand

[1] The buyers can solicit the sellers' posted prices through on-line search or by requesting quotes without visiting the sellers' stores or engaging in some negotiation process

and Diamond: Price undercutting is not an optimal strategy for the sellers even if the search friction approaches zero (thus marginal cost pricing is not necessarily the unique equilibrium outcome) and monopoly pricing does not occur for positive search frictions. Hence, the model I use in this paper unites and smooths out these celebrated price competition models and eliminates their inexplicable predictions.

More importantly, analysis establishes that each player uses his opponents' uncertainty about his commitment abilities to build reputation on inflexibility. Particularly, building up his reputation is an investment for the buyer that paves the way to convince the sellers that he is a stiff bargainer. Once the buyer attains sufficient level of reputation, he will leave his current bargaining partner unless his demand is accepted. Since the buyer can opt out of negotiation, this credible threat will increase the buyer's bargaining power against his current partner. On the other hand, the sellers can counterbalance the buyer's threat of opting out by posting sufficiently low prices (not necessarily zero). By doing so, the sellers can bring time pressure to bear on the buyer to ensure that he will not have enough time to build his reputation against his current partner. Namely, the sellers can "lock" the buyer in their stores and impel him to make an agreement promptly. As a result, the maximum payoff each seller can achieve in equilibrium will approximate to 1/6 of the total surplus (as the uncertainty about the players' rationality vanishes), whereas the minimum payoff to the buyer converges to 2/3 of the total surplus (assuming that players' time preferences are identical).

Another significant contribution of the analyses is that reputation concern of the players overwhelms their behaviors so that equilibrium has a war of attrition structure (each player is indifferent between accepting his opponents' initial demand and waiting for acceptance). As a result, equilibrium strategies and outcomes of the haggling process are robust in the sense that they are "independent" of the exogenously assumed bargaining protocols (unlike more familiar but relatively less sophisticated models). Moreover, the model and the outcomes could facilitate a fruitful ground to answer further questions such as how reputation on inflexibility would affect the structure of decentralized markets (in which no agent is assumed to be a price-taker and no auctioneer is assumed to be present).[2]

To be more specific, I consider the following simple market set-up: There are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[3] All players are impatient (that is, they discount time) and the valuation

[2]Specifically, if it is an option, do the sellers prefer to announce their prices before or after the buyer's arrival? Does the buyer prefer to be engaged in a haggling process or to invite the sellers for an auction? These questions and many more deserve comprehensive considerations and are beyond the scope of this paper. However, they constitute the immediate items of my research agenda.

[3]In Section 4, I consider the case where the number of sellers is some $N \geq 2$. I assume, without loss of generality, that there is a unique buyer since I presume that the sellers have a large number of goods

of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the players' valuations and time preferences. The buyer can learn the sellers' initial demands (or posted prices) for the good before visiting their stores (so the sellers compete in the spirit of Bertrand). The buyer can get the good from the seller who is asking the lowest price by paying the seller's demand, or he can negotiate with the sellers to receive a better offer. But to get a deal better than the posted prices, the buyer has to visit a seller. The buyer can move back and forth between the stores freely. However, switching from one seller to another incurs some (small but positive) cost.

Upon arriving at a store, the buyer and the seller can negotiate according to some predetermined bargaining protocol (in discrete time); whenever a player makes an offer his opponent immediately accepts (and finishes the game) or rejects the offer. Other than this assumption, I do not impose any restriction on possible bargaining protocols that might be used in the negotiation process. However, I am interested in equilibrium outcome(s) of the resulting multilateral bargaining game in the limit as the players can make increasingly frequent offers, with the interpretation that the players can make offers at any time they want.

There are two main challenges of analyzing this market setup as a two-stage game (I call it bargaining problem): The first stage is a price competition game between two sellers and the second stage is the negotiation phase between the buyer and the sellers. These two points make the analysis of such markets (in a game-theoretical framework) rather difficult. The first one is that the second-stage multilateral bargaining game has a continuum of subgame perfect equilibria (even in the limit as time between offers converge to zero), and this set depends on the fine details of the bargaining protocol.[4] This multiplicity problem makes the current bargaining models in the literature *ad hoc* and inconclusive when they are incorporated into a price competition game. The second challenge is integrating the first-stage price choices with the second-stage bargaining game. Posting a price has an unambiguous meaning in, for example, Bertrand price competition or in Diamond's search model because in these models the buyer knows that he cannot attain a price lower than the posted prices. Thus, he is not inclined to bargain with the sellers. However, the core presumption of my analysis in this paper is the players' uncertainty about others' persistence, so the buyer believes that he can actually get a better price through haggling with the sellers.

As I will discuss it in details in the following section, the literature does not seem to provide suggestive answers to the following critical questions. (1) Which bargaining protocol should we pick to model the multilateral bargaining game, and when we pick

to sell (so no competition between the buyers) and the buyers cannot convey information to one another (no interaction between the buyers).

[4]For related literature see Osborne and Rubinstein (1990), Muthoo (1999) and the references therein.

a particular one, how can we rationalize it over the others? (2) Furthermore, what is the feasible and tractable way of modeling the trading mechanism in such markets as a multilateral bargaining game between buyers and sellers? (3) How might the players interpret the posted prices when the sellers post price and when there is a belief that the sellers may not commit to these prices? (4) Given the uncertainty about the players' commitment abilities, how should the sellers and the buyer choose their initial demands? Finally, what would be the equilibrium outcome of the negotiation process?

The approach I am suggesting in this paper is a simple remedy to these questions. I show that a slight perturbation of the problem by introducing "obstinate" types, which allows players to build reputation for inflexibility, engenders an essentially unique equilibrium. Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), I assume that each of three players suspects that the opponents might have some kind of irrational commitment forcing them to insist on a specific allocation. Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991), and Abreu and Gul (2000) a commitment player always demands a particular share and accepts an offer if and only if it weakly exceeds that share. An obstinate seller, for example, never offers a price below his original posted price, and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount, and will never agree to pay more. Thus, a rational player must choose either to mimic an inflexible type, or reveal his rationality and continue negotiation with no uncertainty regarding his actual type.

Analogous to Abreu and Gul (2000) I show that, given the first-stage price selections and the presence of the obstinate types, the equilibrium outcomes of the second-stage discrete-time bargaining game (between the sellers and the buyer) converge to a unique limit, independent of the fine details of the bargaining protocols, as players can make increasingly frequent offers. This limit is the unique equilibrium outcome of the following continuous-time war of attrition game. Upon arrival at a store, the buyer and the seller start to play the *concession game*. At any given time, a player either accepts his opponent's demand or waits for his opponent's concession. At the same time, the buyer decides whether to stay or to leave the store.

In the unique equilibrium of the continuous-time bargaining game, the buyer does not visit a given store more than once as long as the sellers' posted prices are the same. Thus, in equilibrium, the buyer enters store 1, for example, at time 0, and starts playing the concession game with the seller until a specific finite time. If neither player concedes to his opponent, the buyer leaves store 1 at this time with a higher reputation (the posterior probability of being the obstinate type), and goes directly to store 2 to continue the concession game with the second seller. The negotiation in the second store ends at some finite time with certainty, at which point the players' reputations simultaneously reach one. The equilibrium strategy of a player in the concession game is some continuous and strictly increasing distribution function with a constant hazard rate. That is, each player concedes by choosing the timing of acceptance randomly with a constant (instantaneous) acceptance rate.

Since there are two sellers, building reputation on inflexibility by haggling with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. Having a higher outside option in return increases the bargaining power of the buyer in the first store. On the other hand, since the buyer discounts time, his expected payoff in store 1 is a decreasing function of his departure time from store 1. If the buyer needs more time to build his reputation before going to the other seller, then it is less likely that his opponent will concede to him earlier. Hence, the equilibrium departure time of the buyer from store 1 decreases with his own initial reputation and increases with the reputation of the sellers. When the sellers' posted prices are different, the structure of the equilibrium strategy dramatically changes. In this case, the buyer never negotiates with the seller whose posted price is higher, though he may visit this store at time 0 in order to make the "take it or leave it" ultimatum.

Furthermore, assuming that the sellers are identical and the initial reputations z_b and z_s (probabilities that the buyer and the sellers, respectively, are obstinate type) are small, I characterize the first-stage price selections of the sellers. In equilibrium both sellers must choose the same price. The set of equilibrium prices depends on the relative ratio of the players' initial reputations. However, for any given value in the interval $(0, 1)$ we can find z_b and z_s small enough to support it as an equilibrium outcome. Since the sellers are identical, it is not surprising that both sellers must choose the same price in equilibrium. On the contrary, price undercutting is not optimal for the sellers because a deviating seller cannot attain a payoff more than the minimal demand possible (within the set of obstinate types). This is true since the deviating seller will have to accept the buyer's "take it or leave it" ultimatum regardless of the buyer's demand.

If the players' discount rates are the same (as a special case), then the maximum equilibrium price would take values close to $\frac{1}{3}$ as players' initial reputations take decreasingly small values. Therefore, the minimum expected payoff the buyer may achieve will approximate to $\frac{2}{3}$, and the maximum expected payoff each seller can attain will converge to $\frac{1}{6}$. Namely, equilibrium outcomes are efficient in the limit. However, this is not the unique outcome; depending on the relative ratio between the players' initial reputations, the equilibrium prices will range in the set $[0, \frac{1}{3}]$ in the limit. Finally, I extend the results to $N(\geq 2)$ sellers and show that the highest equilibrium price approaches $\frac{r_b}{r_b+Nr_s}$ (where r_b and r_s are the time preferences of the buyer and the sellers, respectively) as the probabilities z_b and z_s converge to zero at the same rate. Thus, keeping the number of sellers fixed, the maximum payoff to the sellers attainable in equilibrium decreases with the impatience of the sellers but increases with the impatience of the buyer.

Section 2 explains the model and the bargaining problem in detail and motivates the assumptions. Equilibrium strategies of the second-stage bargaining game are characterized in Section 3. Section 4 examines the equilibrium prices and demands that the players would announce in stage 1. Finally, Section 5 makes some closing remarks.

Related Literature

Versions of the haggling game and the market environment that I analyze in this paper have been investigated in different contexts to explain various different phenomena. In this section, I would like to highlight the contribution of this paper by relating it to some of the brilliant works in the literature. Also, admitting that it is far from being a complete survey, this literature review underlines the directions that my work may complement our understanding on playing fields of negotiators.

To start with, this paper is directly related to reputation and bargaining literature initiated by Myerson (1991) on one-sided reputation building, as well as works by Abreu and Gul (2000) and Kambe (1999) with two-sided versions of it. Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in Abreu-Gul setting and explore the role of (exogenous) outside options. They show that if both agents' outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility and the unique equilibrium is again the Rubinstein outcome. The results I present in Section 3 of this paper suggest that in equilibrium, the buyer makes the "take it or leave it" ultimatum to seller 1 (the first seller the buyer visits) when the buyer's initial reputation is sufficiently high (that is, the value of the buyer's outside option of going to the other store to bargain with the other seller is high). Thus, analogous to Compte and Jehiel (2002), rational seller 1 immediately reveals his type, but the rational buyer does not. This strategy is an equilibrium in the setting that I analyze because the buyer's outside option depends on his reputation when he leaves the first store: If the buyer reveals his rationality in store 1, then he "loses" his outside option.

Another paper in the bargaining and reputation literature, that is closely related to my work, is Atakan and Ekmekci (2009). They consider a two-sided search market with a large number of buyers and sellers who wait to be matched (randomly) to an opponent to bargain over the unit surplus, so the bargaining parties' outside options are endogenous. Atakan and Ekmekci analyze the steady state of this market, and in agreement with my results, they show that the endogenous outside options of the rational agents are never large enough to deter the effect of commitment types. Atakan and Ekmekci (2009) examine "large" markets. That is, negotiations are anonymous, and thus market participants do not have incentive to invest on their reputations for future negotiations. However, I investigate "smaller" markets in this paper, where there are only a few competing sellers. Thus, negotiation process is used not only to reach an agreement with an opponent, but also serves as an investment for future negotiations.

The potential benefits of commitment are clear: Since one's opponent is convinced, his best strategy is to yield if possible. Shelling (1960) asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. However, an important question Shelling leaves unanswered is whether commitment can be rationalized in equilibrium. The answer is affirmative if revoking a commitment is costly, according to Crawford (1982) and Muthoo (1996), or if commitment is a costly action, according to Ellingson and Miettinen (2008). In contrast to these models, I assume in this paper that rational bargainers can mimic commitment types, that are not rational in the usual sense, if they find it optimal.[5] Parallel to the works of Abreu and Gul (2000), I show that "full" commitment does not need to be an equilibrium strategy. Proposition 3.1 suggests that each player will revoke his commitment with positive probability. Although I do not take commitment as a costly action for players, revoking commitment is costly because it means yielding to opponent's demand. However, this cost is endogenously determined and implied in equilibrium.

Non-cooperative analyses of bargaining, using extensive form games, are sometimes criticized because the conclusions are sensitive to the exact description of the games, such as the timing for offers, counter-offers and exit options. On the contrary, real life negotiations seem very shapeless processes. In this regard, my paper adds to the literature aiming to understand whether credible commitment to certain promises or threats would wash out technical specifications of the bargaining procedures, and if so, how extensive forms of non-cooperative bargaining games would be. Caruana, Eirav and Quint (2007) consider a multilateral bargaining game with a fixed deadline and allow players to revise their demands often. However, revisions are costly and this cost increases as the deadline gets closer. Thus, earlier offers serve as a commitment mechanism because changing demand later becomes increasingly expensive. The unique equilibrium is "independent" of the bargaining protocols, where the agreement is reached immediately and the revision costs are avoided. Caruana and Einav (2008) work with a similar model where the number of players and their offers are restricted to two. The extensive form game takes a relatively simpler form (war of attrition), leaving the outcome independent of the bargaining procedures. Likewise, Chatterjee and Samuelson (1987) consider bilateral bargaining environment where each player has two types (regarding the valuation

[5] Abreu ad Sethi (2003) supports the existence of commitment types from evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such "irrational" types is not compatible with evolutionary stability in bargaining environment.

of the good) and knows his own but not his opponents' type. Moreover, players' offers are binary; the buyer, for example, either offers the demand of his weakest type or the demand of the seller's strongest type. Thus, the bargaining game is modeled as a war of attrition game, and so equilibrium strategies are similar to Abreu and Gul (2000). Samuelson (1992) investigates a similar case with an infinite number of buyers and sellers to explain why opting out would occur even though there is positive expected gain from continued bargaining. He shows, in Lemma 1, that a seller will opt out if the percentage of high-type buyers in the population is sufficiently high. On the other hand, I show in this paper that reputation concern of the players in a competitive-bargaining environment overwhelms their behaviors so that the unique equilibrium has a war of attrition structure, see Appendix C, and thus the equilibrium outcome is independent of the bargaining procedure.[6] Furthermore, unlike Samuelson (1992), I show that opting out (or disagreement) may occur in equilibrium for any values of the primitives.

This paper also adds to the literature initiated by Rubinstein and Wolinsky (1985). It is conventional wisdom that centralized and decentralized markets are Walrasian under frictionless conditions. Walrasian theory suggests that equilibrium will be achieved through a process of tâtonnement; given the supply and demand, the market will clear itself. This view implies that the institutional structure of a frictionless market that includes the particulars of the trading procedure has no or little impact on the market outcome. Since the Walrasian theory itself has nothing to say on this line, it remains an interesting and open question whether all frictionless markets are indeed Walrasian. Following the Rubinstein's seminal paper, varied contributions shed light on decentralized homogeneous goods markets where the price is determined as the perfect equilibrium of a bargaining game between sellers and buyers. My paper adds to this literature by noticing, in Section 4, that when players have reputation concerns, frictionless competitive markets need not be Walrasian.

Rubinstein and Wolinsky (1985) consider a market, in steady state, where at each period, finite (but large number of) buyers and sellers are matched with an exogenous matching mechanism to negotiate over the price, and new players enter as some leave the market after agreement. Their main result suggests that the unique outcome is not Walrasian even when search and bargaining frictions vanish. Gale (1986a,b) objects to this result by arguing that supply and demand in such market setups should be treated in terms of "flows" (not "stocks") of agents into the market at any period, and then shows that the bargaining approach indeed supports the Walrasian equilibrium. Binmore and Herrero (1989) support this point and show that frictionless markets will clear period by period. That is, the short side of the market will appropriate the whole surplus if

[6] Abreu and Gul (2000) and Atakan and Ekmekci (2009) reach similar conclusions in different settings.

and only if entry into the market is negligible relative to exit from it. Satterthwaite and Shneyerov (2007) reinforce this finding by achieving an analogous result when there is incomplete information (regarding the players' valuations) on both sides.

Rubinstein and Wolinsky (1990) show that the controversial result in their earlier paper does not occur if there is no new entry into the market. In this case, players' fear that they may not find a bargaining partner tomorrow if they reject their current offer today forces the long side of the market to compete fiercely, thus yielding a Walrasian outcome as frictions vanish. Bester (1988) employs a model similar to Rubinstein and Wolinsky (1985) with an infinite number of buyers and sellers and shows that if there is uncertainty regarding the sellers' product quality, then relative speed of convergence for bargaining friction and search friction determines whether the limit approaches Walrasian outcome. However, if the quality uncertainty is not in play along with the other frictions, then the market outcome is clearly Walrasian, as argued in Bester (1989).

In contrast, Shaked and Sutton (1984) examines a labor market with one firm and multiple workers (similar to the one I investigate in this paper), showing that the unique equilibrium outcome is non-Walrasian. This conclusion is correct under the assumption that the firm cannot switch its bargaining partner unless some time ($T > 1$ periods), which is exogenously set, passes. However, it is hard to motivate whether a firm would commit itself to such haggling protocols in a competitive environment. In this paper, however, I show that the buyer's reputation concern may lock him in with a seller. In equilibrium, when the buyer has a low initial reputation, he cannot leave his bargaining partner before his reputation reaches a certain point (optimal departure time). Moreover, Proposition 4.3 and 4.4 supports this point even stronger. As the uncertainty regarding the players' rationality vanish, i.e. probabilities z_b and z_s converge to zero at the same rate, the buyer has to continue to be "*weak*", implying that the optimal departure time is always positive in equilibrium. However, as the number of sellers approaches infinity, we cannot sustain non-Walrasian outcomes in equilibrium unlike Shaked and Sutton (1984).

Therefore, building a model of reputation with considerable sophistication is not only important to match some stylized facts but also overturns a number of conclusions reached in their absence. In this respect, my paper also contributes to market microstructure literature. Although negotiating over prices with sellers is common in many markets, it is not clear how a haggling price policy can help a firm gain a strategic advantage or whether it is even sustainable in a competitive market. Riley and Zeckhauser (1983), Bester (1993), Wong (1995), Desai and Purohit (2004), and Camera and Delacroix (2004) compare negotiated prices with posted prices and show that each argument has specific merits. It is common to all of these significant works that price posting requires irreversible commitment. That is, sellers either post price and act absolutely inflexible in their demands or do not post a price but behave completely flexible and bargain with each buyer. In this paper, however, I show that dedication to such extreme strategies (absolute flexibility or inflexibility) that postulate pure commitment is not optimal in a competitive market environment. Very roughly, rational players prefer to randomize (in a sense) these two strategies optimally. Clearly, the model I investigate in this paper facilitates a fruitful ground to answer further questions in this line of research.

This paper also builds on two approaches that are extensively used in competitive labor markets for wage determination (see Rogerson, Shimer and Wright (2005) and the references therein). In the first approach, *random matching and bargaining*, partners match according to some exogenous matching rule and the negotiation outcome is assumed to be Nash bargaining outcome where the workers' bargaining power λ is exogenously given and motivated as being the probability of making an offer. The second approach, *directed search and posting*, also referred to as competitive search, is that firms post wages and workers direct their search for the highest posted wage, so the wages are not negotiated. Plausibly, competitive search equilibrium theory provides a more precise explanation of the wage determination and of matching process than the bargaining model. However, a basic critique of the former models (indeed, of any model with posting) is the assumption that the agents are inflexible in their posted terms of trade. The combination of these two models, directed search and bargaining, is usually ignored mainly because there is nothing for workers to direct their search toward if the firms are homogeneous. However, my work (which combines directed search and bargaining) offers a new direction of research in the literature on this question by highlighting that slightest belief regarding firms' commitment abilities will direct the worker(s) to search for a better deal. The model I use also provides essential insights on equilibrium price determination and optimal posting strategies.

Finally, this paper also contributes to collusion literature where the static Bertrand game is the main building block and infinite repetition of this stage game is an approximation to "real life" price competition in the long run, as seen, for example, in Athey, Bagwell and Sanchirico (2004) and Athey and Bagwell (2008). Theoretical models are important for policy implications since they improve our understanding of instances where collusion (prices with positive mark-up) is sustainable in equilibrium. They are also used to identify price wars, that is, credible threats in case of deviation from equilibrium. However, the analyses rely heavily on the assumption that the Bertrand competition has a unique (marginal-cost pricing) prediction. On the contrary, this paper shows that if the sellers' and the buyer's commitment to their prices is not certain, then the buyer's reputation concern may destroy this unique Walrasian prediction. This result clearly would change the equilibrium predictions of the infinite repeated version of the Bertrand game.

In markets where the sellers negotiate with the buyers over the price, leaving it difficult to observe traded prices, an alternative benchmark for empirical studies is the Cournot competition.[7] In a theoretical perspective, Cournot model presumes that an outside auctioneer will determine the market-clearing price once the firms choose their outputs. However, in the absence of such market designers (which is usually the case), the sellers are the ones who determine their prices. Arguably, this bold assumption of the Cournot model is not so disruptive as Kreps and Scheinkman (1983) shows that price competition under output commitment yields identical results with the Cournot model. However, given the uncertainty of sellers' commitment powers, reputation concerns of the market participants will affect the equilibrium analysis, which may ultimately overturn the Kreps-Scheinkman result.

2. The Model

Consider a market where there are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit. All players are impatient (that is, they discount time) and the valuation of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the players' valuations and time preferences. The buyer can learn the sellers' initial demands (or posted prices) for the good before visiting their stores. He can get the good from the seller who is asking the lowest price by paying the seller's demand, or he can haggle with the sellers to receive a better offer. But to get a deal better than the posted prices, the buyer has to visit a seller. Switching from one seller to another incurs some (possibly small but positive) cost. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town while the buyer's position is midway in between the two. Thus, changing the bargaining partner is costly for the buyer because it takes time to move from one store to the other and the buyer discounts time.

Furthermore, each player might have some kind of irrational commitment forcing him to insist on a specific allocation. An obstinate (or commitment) type seller i is identified by a number $\alpha_i \in (0,1)$ and implementing the following strategy: He always offers α_i, rejects any price offer strictly below it and accepts any price offer weakly above it. The initial probability that a seller is obstinate is denoted by z_s. Without loss of generality, the initial prior, z_s, and the time preference, r_s, are common for both sellers.

Similarly, there is a small but strictly positive probability, z_b, that the buyer is a commitment type. Obstinate buyer with demand $\alpha_b \in (0,1)$ executes the following strategy: He always offers α_b to sellers, accepts any price offer less than or equal to α_b

[7] See for example, de Roos (2006) and the references therein.

and rejects any price offer strictly above it. The rational buyer's time preference is r_b. Assuming that the sellers' are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time, $\Delta > 0$, required to travel from one store to the other. That is, $\delta = e^{-r_b \Delta}$. Note that δ (the search friction) is the cost that the buyer incurs at each time he switches his bargaining partner.[8] Also note that, as the stores get very close to each other, δ converges to 1. Namely, the search friction vanishes.

I would like to determine the equilibrium outcomes of the bargaining problem (which will be specified comprehensively in the following section) when the search friction is insignificant and the set of obstinate types for each player is $[0, 1]$. Namely, any initial demand $\alpha \in [0, 1]$ chosen by a player will lead to a fear that this player might be the commitment type α. Therefore, for the sake of illustrational simplicity, I will first assume that the set of obstinate types for each player is a finite and reach set $C \subset (0, 1)$ and the search friction is sufficiently small but positive. Then, I will consider the limit where $C \to [0, 1]$ and $\delta \to 1$ simultaneously. Furthermore, to provide a concise presentation of the analysis, I will focus my attention to the case where the initial priors z_s and z_b are sufficiently small. Hence, I make the following assumptions throughout the paper.

Assumption 1 (***Small Search Friction***). *Suppose that δ is close enough to 1 so that for all $\alpha \in C$ and $\alpha' \in C$ such that $\alpha > \alpha'$ we have $1 - \alpha < \delta(1 - \alpha')$*

Assumption 2 (***Small Initial Priors***). *Suppose that z_b and z_s are small enough so that for all $\alpha, \alpha', \alpha'' \in C$ with $\alpha \geq \alpha' > \alpha''$ we have $z_s, z_b < \bar{z}$ and $\alpha z_s < \alpha''$ where $\bar{z} = \frac{1-\alpha''-\frac{1-\alpha}{\delta}}{1-\alpha'-\delta(1-\alpha'')}$.*

Finally, let $\pi(\alpha)$ denote the conditional probability that a player is type α given that he is obstinate. That is, π is a probability distribution on C.

The Bargaining Problem in Discrete-Time

The two-stage bargaining problem in discrete-time proceeds as follows. Initially (in stage 1), each seller i simultaneously chooses and reveals his demand $\alpha_i \in C$. If he is rational, this is a strategic choice. If he is the obstinate type, then he merely declares the demand corresponding to his type. After observing both sellers' demands, α_1 and α_2, the buyer immediately accepts α (the minimum of α_1 and α_2) and finishes the game strategically if he is rational or because he is obstinate and of type α_b such that $\alpha_b \geq \alpha$. Or the buyer visits one of the sellers and makes a counter offer $\alpha_b \in C_{(\alpha_1,\alpha_2)} = \{x \in C | x < \min\{\alpha_1, \alpha_2\}\}$. Again, this may be because the buyer is rational and strategically

[8] One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

demanding α_b or because the buyer is the obstinate type α_b. After the buyer declares his demand, the seller (who is currently visited by the buyer) can immediately accept the buyer's demand and finish the game or reject it, in which case the game proceeds to the second stage (the bargaining phase).

A player can convince his opponents that he is not the obstinate type by showing them that he is flexible in his demand. Therefore, players can reveal their rationality only in the negotiation phase, in stage two, by making offers different than their initial demands. In the bargaining phase, the buyer can negotiate only with the seller whom he is currently visiting. If the buyer wants to bargain with the other seller, he needs to visit that seller. The buyer can move back and forth between the sellers as much as he wants, but he will incur the travel cost at each time he switches his bargaining partner.[9] Throughout the game, both sellers can perfectly observe the buyer's moves.[10] Thus, the players' actual types remain to be the only source of uncertainty in the game.

The buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller $i \in \{1, 2\}$ is defined as $g^i : [0, \infty) \rightarrow \{1, 2, 3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[11] Moreover, $g^i(t) = 3$ implies a simultaneous offer. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller $i \in \{1, 2\}$ and for all $\bar{t} \geq 0$, the set $I^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1, 2, 3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non-stationary, non-alternating protocols.

An offer $x \in (0, 1)$ denotes the share the seller is to receive. If the proposer's opponent accepts his offer, the game ends with agreement x where $u_i(x, t, i) = xe^{-tr_s}$ denotes the payoff to the seller i, $u_j(x, t, i) = 0$ is the payoff to the seller $j \in \{1, 2\}$ with $j \neq i$ and finally $u_b(x, t, i) = (1 - x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the buyer decides whether

[9] The buyer does not need to visit the other seller's store to reenter the one that he previously visited. So, for example, the buyer may change his mind while he was going to the second store and turn back to the first one to continue negotiating with the first seller. However, the buyer will never behave that way in equilibrium.

[10] On the one hand, assuming that each seller is completely unaware of the buyer's move in negotiating with the other seller is in the extreme. When stakes are high, the negotiation becomes (to some degree) public and bargainers can scrutinize their opponents' moves throughout the negotiation process rather easily. YouTube's flirt with Google and Yahoo before Google has acquired YouTube for $1.65 billion and Yahoo's negotiation with Microsoft and AOL Time Warner are just two examples on this account. On the other hand, in this paper, I consider another extreme case where the buyer's actions are perfectly observable. Clearly, in some circumstances, the sellers may not be able to attain all the information nor will the buyer convey it perfectly. Extending the model to introduce some informational imperfections may naturally result in different equilibrium behaviors during the negotiation process. These issues deserve comprehensive considerations and transcend the focus of this particular paper.

[11] Time 0 denotes the beginning of the bargaining phase.

to stay or leave the store. If the buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage discrete-time bargaining problem is denoted by $\mathrm{G}\langle g, (C, z_i, \pi_i, r_i)_{i\in\{b,s\}}\rangle$ (or $\mathrm{G}(g)$ in short). The bargaining problem $\mathrm{G}(g)$ ends if the offers are compatible. In the event of strict compatibility the surplus is split equally.[12]

The Bargaining Problem in Continuous-Time

I am particularly interested in equilibrium outcome(s) of the bargaining problem $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time bargaining problem where the buyer and the sellers bargain, in stage two, according to the protocol $g_\epsilon = (g^1_\epsilon, g^2_\epsilon)$ such that for all $t \geq 0$ and $i \in \{1, 2\}$, both seller i and the buyer have the chance to make an offer, at least once, within the interval $[t, t + \epsilon]$ in the bargaining protocol g^i_ϵ.[13] In this sense, the discrete-time bargaining problem $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \to 0$.[14]

In Appendix C, I show that given the declared demands in stage 1, the second stage equilibrium outcomes of the discrete-time bargaining problem $\mathrm{G}(g_\epsilon)$ converge to a unique limit, independent of the exogenously given bargaining protocols, g_ϵ, as $\epsilon \to 0$, and this limit is equivalent to the unique outcome of the following continuous-time war of attrition game. Suppose, without loss of generality, that the buyer visits seller i and declares α_b in stage 1 such that α_b is incompatible with the sellers' demands. Upon the beginning of the second stage (i.e. seller i does not accept the buyer's demand), the buyer and seller i immediately begin to play the following concession game: At any given time, a player either accepts his opponent's demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. Concession of the buyer or seller i, while the buyer is in the store, marks the completion of the game. In case of simultaneous concession, surplus is split equally.[15] If the buyer leaves store i and goes to store j, the buyer and seller j start playing the concession game upon the buyer's arrival at that store. Both sellers can perfectly observe the buyer's moves throughout the game. Thus, the players' actual types are the only source of uncertainty in the game.[16] I denote the two stage bargaining problem in continuous-time by G.

[12] This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[13] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t + \epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t + \epsilon]$.

[14] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

[15] This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[16] After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again.

Motivating the Obstinate Types

An obstinate player is a man of unyielding perseverance. Sellers may manifest such a steadfast attitude because they might be confined to do so. A company may be inflexible in a wage negotiation due to some regulations within the company. For example, a car dealer, a sales clerk or a realtor may be restricted by the owner regarding how flexible he can be in his demands while negotiating with a buyer. A fresh college graduate who is competing with other candidates for a specific job opening may commit to a certain salary because he wants to pay his student loan without too much financial difficulty.

Steady persistence in adhering to a course of action as assumed for an obstinate (type) buyer would be reasonable when, for example, the "buyer" is looking to advance his position. A worker (negotiating with more than one firm) may accept the new job offer if it provides a significant jump in his salary or title relative to the position he is already holding. On the other hand, a successful investor (venture capitalist) who only has assets that have high profit margins in his portfolio may commit to buy a house, a land or a small business only if it is a real bargain because otherwise it may not be worth including in his portfolio. An entrepreneur who is running a successful small business may commit to his initial demands while negotiating with investors to sell his business or a franchise because of overly optimistic expectations about the future of his business.

Therefore, I assume that the obstinate buyer (regardless of his demand) is not enthusiastic enough to haggle with the sellers and to exert great efforts in moving back and forth between the sellers. That is, the obstinate buyer is a man who "plays it cool." To be more specific, I assume that the obstinate buyer (1) does not discount time, (2) incurs a positive (but very small) switching cost ($\epsilon_b > 0$) every time he switches his bargaining partner, (3) understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate, and finally (4) visits each seller with equal probabilities to announce his demand if it is not compatible with the lowest price announced by the sellers.

According to (1), the time of an agreement is not a concern for the obstinate buyer, and thus he does not feel the need to distinguish himself from the rational buyer who wishes to reach an agreement as quickly as possible. Since the obstinate buyer does not discount time, i.e. $r_b = 0$, we have $\delta = 1$. Therefore, ϵ_b is the only search friction that the obstinate buyer is subject to and it has no crucial impact on our analysis.[17]

The statement in (3) can be interpreted as an implication rather than an assumption. Since the obstinate buyer does not value time (statement (1)), he is indifferent between

[17]The switching cost ϵ_b works as a tie-breaking device: When the buyer's continuation payoff of negotiating with each seller is the same, then the buyer will prefer to stay with his current bargaining partner.

staying with his current partner or visiting the other seller at any time (ignoring the switching cost). However, if he leaves his current partner before being convinced that he is obstinate, he will revisit this seller later if he exhausts all his hope to reach an agreement with the other seller. Therefore, since the switching cost ϵ_b is positive, the obstinate buyer will switch his partner just once and thus leaves a store when he is convinced that his opponent is the obstinate type.

Moreover, since the sellers share a common initial prior of being the commitment type, the obstinate buyer is initially indifferent about which store to visit first regardless of the sellers' announced demands. The assumption made in (4), however, is a simplification assumption that can be generalized with no impact on the main message of our results.[18]

Finally, one may think that coexistence of some other commitment types for the buyer (the ones who value time and wish to reach an agreement quickly) could change our results, but this is not necessarily the case. For example, consider a house owner who is negotiating with more than one person to sell his house in order to pay his urgent depth. The buyer (the house owner in this case) may have to commit to a certain price. But in this case, he will clearly not fit the model of the obstinate buyer I described above simply because he needs to reach an agreement as quickly as possible. If the buyer has some commitment as in this case, then he may wish to distinguish himself from the rational buyer, so he may go back and forth between the sellers multiple times. Thus, he may reach an agreement earlier. However, when time is a crucial factor for a bargainer, he usually needs to compromise between two things: time of the agreement and the share he will receive. Therefore, the commitment of the buyer who haggles fiercely with the sellers would be credible from the point of view of the sellers if he is committed to a demand relatively lower than the one who would "play it cool." Thus, coexistence of such commitment types with the ones I assume here will not alter our results[19] if we assume that the buyer's commitment to high demands is interpreted by the sellers such that the buyer must be the one who will "play it cool." For this reason, I restrict my attention only to those commitment types that I described above.

3. Second Stage Equilibrium Strategies

Since we have the convergence result, I will use the game G, the bargaining problem in continuous-time, in my analysis throughout the paper. In this section, I will examine the second stage equilibrium strategies of the bargaining problem G. Section 4 characterizes the equilibrium strategies and outcomes of the first stage. Finally, I will finish my analysis

[18] For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always choose a fix seller and some visit the sellers according to their announcements while the rest are possibly a combination of these two.

[19] In particular, the results in Section 4.

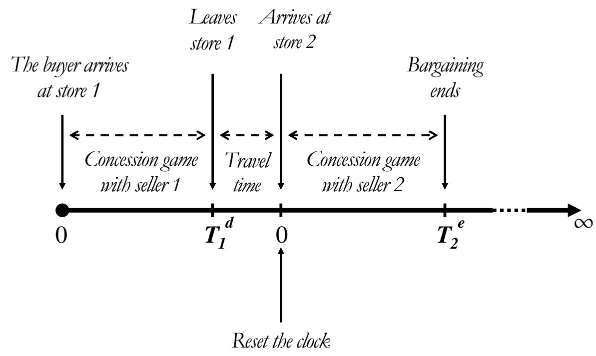

Figure 1: The time-line of the buyer's equilibrium strategy

by investigating the equilibrium outcomes of G (demand selections in stage 1) in the limit where the set of obstinate types converges to $[0, 1]$ and the search friction vanishes.

The Case Where the Sellers' Demands are the Same

Suppose now that the sellers choose the same demand, $\alpha \in C$, in stage 1 and it is incompatible with the buyer's demand, $\alpha_b \in C$. Since the sellers are initially identical, the rational buyer is indifferent about which store to visit first in stage 1. Therefore, I assume, without loss of generality, that the rational buyer chooses each seller with equal probabilities when the sellers' demands are the same.[20] Hence, regardless of the store visited in stage 1, the posterior probability that the buyer is obstinate is equal to his initial reputation z_b.

The equilibrium of the continuous-time bargaining problem in the second stage is unique. A short descriptive summary of the equilibrium strategy is as follows (see *Figure 1*). At time 0, the buyer enters store 1, for example, and starts playing the concession game with the seller until time T_1^d. At this time the buyer leaves store 1, if the game has not yet ended, and goes directly to store 2. Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, when both players' reputations reach 1. That is, by time T_2^e the game ends with certainty if one of the players is rational. Therefore, in equilibrium, the buyer visits each store at most once. The departure time of the buyer from store 1, T_1^d, is greater than or equal to zero depending on the primitives.

Each player's equilibrium strategy in the concession game is a continuous and strictly increasing distribution function. That is, in equilibrium both the buyer and the sellers

[20]This is the only equilibrium strategy of the rational buyer when he is strong (the definition of the term will be given in this section). Otherwise, it is one of the equilibrium strategies, all of which yield the same expected payoff to the buyer. However, picking this particular one does not affect our results in subsequent sections.

concede by choosing the timing of acceptance randomly with a constant hazard rate (or instantaneous acceptance rate). Therefore, at any moment the players are indifferent to either accepting the opponent's demand or waiting. The hazard rate of a player depends only on the demands chosen in stage 1 and his opponent's time preferences.

For the sake of simplicity in presentation and notation, I will focus for the moment on equilibrium strategy where the buyer visits each store at most once. Appendix A will consider more elaborate strategies to prove that all the results in this section hold without this restriction, which is a consequence of the equilibrium.

The buyer's strategy in the bargaining stage has two parts. The first part, σ_b, determines the buyer's location as a function of history. Assume, without loss of generality, that in equilibrium the buyer visits store 1 first and then store 2. Then let T_1^d denote the time that the buyer leaves store 1 if no agreement has been reached yet. Denote by ω_i the time that the buyer starts negotiating with seller i (if agreement has not been reached yet). That is, $\omega_1 = 0$ and $\omega_2 = T_1^d + \Delta$ where Δ is the travel time between the stores. For notational simplicity, I manipulate the subsequent notation and denote ω_2 by 0. That is, I reset the clock once the buyer arrives in store 2 (but not the players' reputations).[21]

The second part is a pair of right-continuous distribution functions $F_b^i : \mathbb{R}_+ \cup \infty \rightarrow [0, 1]$, $i = 1, 2$.[22] Thus, for each t, $F_b^i(t)$ is the probability that the buyer concedes to seller i by time t (inclusive). Similarly, seller i's strategy in the bargaining phase is a right continuous distribution function $F_i : \mathbb{R}_+ \cup \infty \rightarrow [0, 1]$ such that for all $t \geq 0$, $F_i(t)$ denotes the probability that seller i concedes to the buyer by time t (inclusive).[23]

Given the strategy of the buyer, let $z_b(t)$ denote the buyer's reputation (probability that the buyer is the obstinate type) at time $t \geq 0$. It is updated according to the Bayes' rule and is consistent with the buyer's strategy: For example, since the buyer visits seller 1 first, for any $t \geq 0$, we have $z_b(t) = z_b/(1 - F_b^1(t))$, and $z_b(t)$ is no less than the buyer's initial reputation z_b.[24] Furthermore, since the buyer leaves store 1 at time T_1^d, it must be that $F_b^1(T_1^d) \leq 1 - z_b$. On the other hand, the buyer visits store 2 if the players cannot reach an agreement in store 1, implying that $F_b^2(T_2^e) \leq 1 - z_b(T_1^d)$ where T_2^e denotes the time that the continuous-time bargaining problem ends in store 2. Since I consider the equilibrium strategies where the buyer visits the stores at most once, it must be that

[21] Thus, with some manipulation of the notation, I define each player's distribution function as if the concession game in each store starts at time 0.

[22] Since the buyer leaves store 1 at time T_1^d, $F_b^1(.)$ is defined over $[0, T_1^d]$ corresponding to the time frame that the buyer is in store 1 according to σ_b.

[23] Note that F_b^i is the sellers' belief about the buyer's play during the concession game with seller i. Hence, it is the strategy of the buyer from the point of view of the sellers. For this reason, the distribution function F_b^i never reaches 1 since the buyer is the obstinate type with probability z_b, implying that $\lim_{t\rightarrow\infty} F_b^i(t) = 1 - z_b$. Similar arguments are valid for the sellers' strategies F_1 and F_2.

[24] Note that the buyer's reputation, $z_b(t)$, reaches 1 when $F_b^1(t)$ reaches $1 - z_b$.

$F_1(T_1^d) \leq 1 - z_s$ and $F_2(T_2^e) \leq 1 - z_s$.[25]

Given F_b^i, seller i's expected payoff of conceding to the buyer at time $t \geq 0$ is

$$U_i(t, F_b^i) := \alpha \int_0^t e^{-r_s y} dF_b^i(y) + \frac{1}{2}(\alpha + \alpha_b)[F_b^i(t) - F_b^i(t^-)]e^{-r_s t} + \alpha_b[1 - F_b^i(t)]e^{-r_s t} \tag{1}$$

with $F_b^i(t^-) = \lim_{y \uparrow t} F_b^i(y)$.[26]

In a similar manner, given F_i, the expected payoff of the buyer who concedes to seller i at time $t \geq 0$ (conditional on not reaching a deal with seller j if store j was visited first) is

$$U_b^i(t, F_i) := (1 - \alpha_b) \int_0^t e^{-r_b y} dF_i(y) + \frac{1}{2}(2 - \alpha - \alpha_b)[F_i(t) - F_i(t^-)]e^{-r_b t} + (1 - \alpha)[1 - F_i(t)]e^{-r_b t} \tag{2}$$

where $F_i(t^-) = \lim_{y \uparrow t} F_i(y)$.[27]

The next result characterizes the equilibrium strategies of the continuous-time bargaining problem G in the second stage.

Proposition 3.1. *Suppose that the sellers declare the same demand, α, and the buyer chooses α_b in stage 1 such that $\alpha_b < \alpha$. The unique equilibrium of the continuous-time bargaining problem G in stage 2 (assuming that the buyer visits seller 1 first) is the following:*

(i) *The buyer visits each store at most once.*

(ii) *The buyer's strategy in the concession game with each seller is a continuous and increasing (cumulative) distribution function; $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$ and $F_b^2(t) = 1 - e^{-\lambda_b t}$ where*

$$c_b^1 = \begin{cases} \frac{z_b}{X_s} e^{\lambda_b T_1^d}, & \text{if } z_b < X_s \\ 1, & \text{otherwise,} \end{cases}$$

such that $X_s = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda}}$ and $A = \frac{1-\alpha_b-\frac{1-\alpha}{\delta}}{\alpha-\alpha_b}$. Moreover, the sellers' strategies are $F_1(t) = 1 - z_s e^{\lambda(T_1^d - t)}$ and $F_2(t) = 1 - z_s e^{\lambda(T_2^e - t)}$.

(iii) *The buyer leaves the first store when he is convinced that his opponent is the obstinate type. Therefore, the optimal time for the buyer to leave store 1 is*

$$T_1^d = \begin{cases} \min\{-\frac{\log z_s}{\lambda}, -\frac{\log(z_b/X_s)}{\lambda_b}\}, & \text{if } z_b < X_s \\ 0, & \text{otherwise,} \end{cases}$$

[25] In equilibrium, it must be that $F_1(T_1^d) = 1 - z_s$, and given that the buyer visits store 2 $F_2(T_2^e) = 1 - z_s$ and $F_b^2(T_2^e) = 1 - z_b(T_1^d)$.

[26] U_i is evaluated at time 0 in "real time".

[27] If the buyer visits seller i first, then U_b^i is evaluated at time 0 (in real time). Otherwise, it is evaluated at time $w_i + \Delta$ (in real time).

Moreover, if the game does not end before T_1^d, the buyer leaves store 1 at this time with probability 1.

(iv) *Finally, the concession game ends in store 2 at time*

$$T_2^e = \min\left\{-\frac{\log X_s}{\lambda_b}, -\frac{\log z_b}{\lambda_b}\right\}$$

and for each $i \in \{1, 2\}$, $\lambda = \frac{(1-\alpha)r_b}{\alpha-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha-\alpha_b}$.

I defer the proofs of all the results in this section to Appendix A.

Characterization of the distribution functions $(F_i, F_b^i)_i$ uses arguments in Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000). In equilibrium of the concession game between the buyer and a seller, if a player's strategy (that is, distribution function) has a discontinuity point at some time t, his opponent prefers to wait a little longer instead of conceding in some $\hat{\epsilon}$-neighborhood of t. Therefore, if $(F_i, F_b^i)_i$ are equilibrium strategies in the interval $[0, T]$ where T is either equal to T_i^d or T_i^e depending on which store the buyer visits first, then there cannot be common discontinuity point for these distribution functions on this interval.

On the other hand, if a player does not concede to his opponent during the time interval $[t, t'] \subset [0, T]$, his opponent prefers to wait in the interval $[t, t' + \hat{\epsilon}]$ for some small but positive $\hat{\epsilon}$. Along with the previous argument, in equilibrium a player's strategy cannot have a discontinuity point in $(0, T]$. Therefore, equilibrium strategies (F_i, F_b^i) must be strictly increasing, continuous and differentiable over $(0, T]$, implying that players are indifferent between conceding and waiting at any time of the concession game.[28] A simple manipulation in the utility functions given in equations (1) and (2) gives us the functional form of these distribution functions.

In equilibrium, the buyer's continuation payoff is no more than $1 - \alpha$ if he reveals his rationality.[29] Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is the commitment type, leaving the first seller "earlier" (or "later") than this time would reveal the buyer's rationality. Therefore, in equilibrium the rational buyer never leaves a seller as long as there is positive probability that this seller is a rational type, and he immediately leaves otherwise. Clearly the buyer does not revisit a seller once he knows that this seller is the obstinate type.

Since the buyer is indifferent between conceding and waiting at all times during the concession game with, for example, seller i, his expected payoff during the concession game with this seller is equal to what he can achieve at time 0 (the time that the buyer

[28] Notice that F_i or F_b^i (not both) may be discontinuous at 0.

[29] Arguments similar to the proof of Proposition A.2 in the Appendix yields this result.

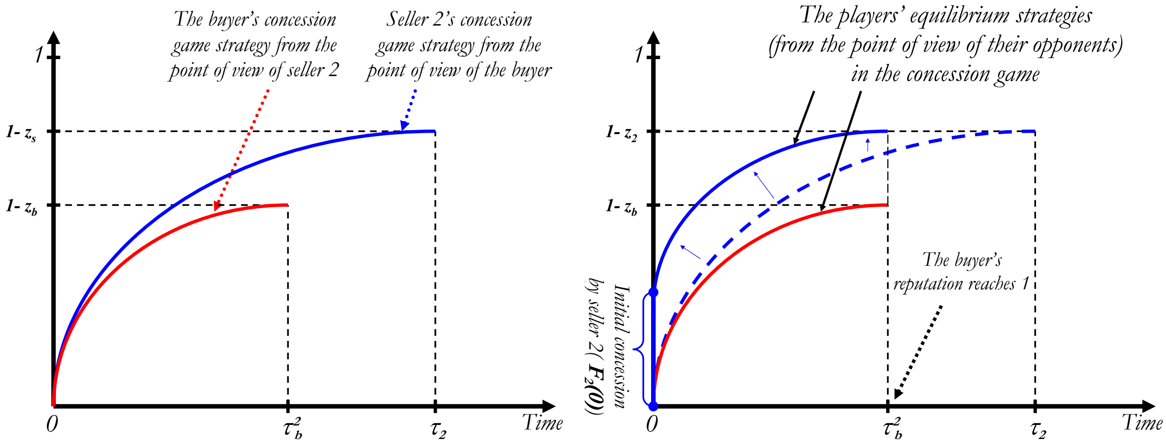

Figure 2: Concession game strategies of the buyer and seller 2 in equilibrium

enters store i), i.e.[30]

$$F_i(0)(1-\alpha_b)+(1-F_i(0))(1-\alpha) \tag{3}$$

Note that in equilibrium, at most one player makes an *initial probabilistic concession*, namely $F_i(0)F_b^i(0)=0$. I call the buyer ***strong relative to*** seller i if he receives this probabilistic gift from seller i and ***weak*** if he does not. Thus, if the buyer is weak relative to a seller, his expected payoff in the concession game with this seller is $1-\alpha$.

Players' strength in a concession game is determined within the equilibrium. Suppose for the moment that there is only one seller (seller 2) and one buyer, whose strategies in the concession game are as given in *figure 2*. If no player makes an initial probabilistic acceptance, then reputations of the buyer and seller reach 1 at time τ_b^2 and τ_2, respectively. However at time τ_b^2, seller 2 will be convinced that the buyer is the obstinate type. Thus, in equilibrium, seller 2 should also finish the concession game by this time, which implies that the seller's strategy (the distribution function) must reach $1-z_s$ at this time. However, since the shape of the distribution function is determined by the constant hazard rate, seller 2's reputation reaches one at time τ_b^2 only if the seller sets $F_i(0)>0$. Namely, seller 2 must concede to the buyer at time 0 with a positive probability, implying that in equilibrium, the buyer is strong relative to seller 2.

When there are two sellers, building reputation on inflexibility by haggling with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. Having a higher outside option in return increases the bargaining power of the buyer in the first store. More formally, the existence of the second store gives the buyer a valuable opportunity to threaten his opponent credibly so that seller 1 has to offer a probabilistic gift at time 0. The buyer can force seller 1 to adjust his strategy

[30] Similarly, seller i's expected payoff in the concession game is $F_b^i(0)\alpha+(1-F_b^i(0))\alpha_b$.

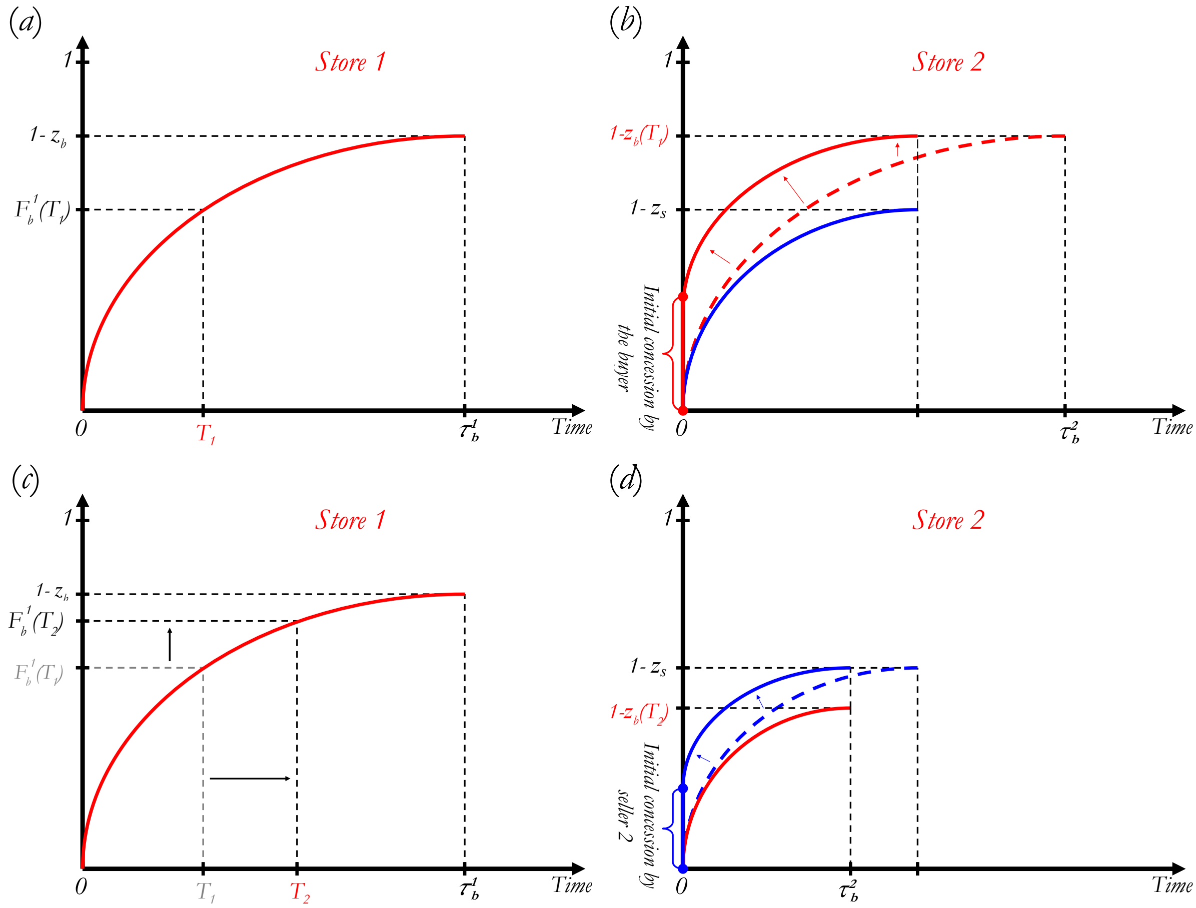

Figure 3: As the buyer continues to play the concession game in store 1, he builds up his reputation, which increases his continuation payoff in the second store. However, since the buyer is impatient, there should be an optimal departure time from store 1.

and increase the amount of this initial gift by choosing the departure time earlier. As the buyer is expected to leave store 1 earlier, seller 1 has to offer a bigger gift, and as the gift increases, the buyer's payoff increases. However, the buyer cannot impel seller 1 to increase this gift as much as he wants, because the buyer cannot credibly threaten seller 1 by leaving before T_1^d, equilibrium departure time, since his initial reputation is not high enough.

In equilibrium, the buyer would not threaten seller 1 to leave at time T_1, for example see *Figure 3-(a)*, because his initial reputation is not high enough to build up the required reputation to become the strong player in store 2. In this case, the buyer needs to haggle with seller 1 little longer. If the buyer leaves seller 1 at time $T_2 > T_1$, for example *Figure 3-(c)*, then the buyer arrives at store 2 with a reputation that is high enough to make himself strong relative to seller 2. Hence, the equilibrium departure time T_1^d resolves the rational buyer's trade off: He wants to leave seller 1 early in order to increase his (expected) payoff but cannot live too early because he may need to build up his reputation to make his outside option credible.

If the buyer's initial reputation is small, i.e. $z_b \leq z_s^{\lambda_b/\lambda} X_s$, then he cannot build enough reputation before time τ_1 –which is the time that seller 1's reputation reaches 1 if he does not concede to the buyer at time 0– to force seller 1 for probabilistic concession at time 0. Thus, the buyer's expected payoff during the entire bargaining phase, and thus at time zero, is $1-\alpha$. Therefore, I call the buyer ***weak*** if $z_b \leq z_s^{\lambda_b/\lambda} X_s$, and ***strong*** otherwise.

However, if the buyer's initial reputation is low so that the above inequality holds, then he may have to offer a probabilistic gift to seller 1 at time 0. The amount of this gift, c_b^1, is as given in Proposition 3.1. The gift cannot be less than this particular amount, because in such a case, the buyer strictly prefers accepting seller 1's demand to finish the game at time τ_1 instead of moving to the second store to play the concession game with seller 2. This contradicts the fact established in Proposition 3.1 that in equilibrium, the buyer's strategy, F_b^1, cannot have a discontinuity point in $(0, T_1^d]$. On the other hand, the initial gift cannot exceed this specific amount because in this case, at some time τ such that $\tau < \tau_1$, the buyer's reputation will reach to the point where it is optimal for the buyer to leave store 1. Then, seller 1 would have to set $F_1(0) > 0$ so that $F_1(t)$ would reach to $1 - z_s$ by the the time τ. That would contradict the fact that in equilibrium $F_1(0)F_b^1(0) = 0$ must hold.

On the other hand, when the buyer's initial reputation is high, i.e. $z_b \geq X_s$, then the buyer is strong relative to seller 2 even with his initial reputation z_b. In this case, the buyer prefers going to store 2 and playing the concession game with this seller over conceding to seller 1 at time 0. Thus, in equilibrium the buyer leaves store 1 immediately at time 0. Since the rational seller 1 knows that the buyer does not need to build reputation but rather plans to leave his store immediately, he accepts the buyer's demand at time 0. Therefore, I call the buyer ***distance-corrected strong*** when $z_b \geq X_s$. If the buyer is distance-corrected strong, then his continuation payoff in the game G evaluated at time 0 is given by

$$(1 - z_s)(1 - \alpha_b) + \delta z_s \left[(1 - z_s e^{\lambda T^e})(1 - \alpha_b) + z_s e^{\lambda T^e}(1 - \alpha)\right] \tag{4}$$

If the buyer is strong but not distance-corrected strong, then the buyer receives an initial probabilistic concession from the first seller he visits at time 0. That is, the concession game between the buyer and, for example, seller 1 lasts until the time of departure T_1^d, which is strictly positive in this case because the buyer needs to build his reputation in store 1 before it becomes optimal for him to go to store 2. In this particular case, the buyer's continuation payoff (in equilibrium) evaluated at time 0 is

$$\left(1 - \frac{z_s^2}{A z_b^{\lambda/\lambda_b}}\right)(1 - \alpha_b) + \frac{z_s^2}{A z_b^{\lambda/\lambda_b}}(1 - \alpha) \tag{5}$$

In equilibrium, the buyer is indifferent between conceding and waiting in the first store he visits until the departure time T^d and it is exactly at this time that he is indifferent between conceding to and leaving this seller. Moreover, according to Proposition 3.1 the buyer leaves the first store with probability 1 and from the time that the concession game starts in the second store to the time it ends, T^e, he is indifferent between conceding to the second seller and waiting in his store. As a result, the buyer's instantaneous payoff is $1-\alpha$ at all times. Therefore, the buyer's expected payoff is the same in each store and is equal to what he can achieve at time 0 in the first store, implying the functional form given in (5).

Finally, note that the unique equilibrium outcome in the second stage is always inefficient and this inefficiency is due to delay in agreement and uncertainty about the types of the players. For example, if the buyer is strong ($z_b > z_s^{\lambda_b/\lambda} X_s$), then seller 1's payoff (the seller who is visited by the buyer first at time 0) is simply α_b. Moreover, as the search friction vanishes, i.e. $\delta \to 1$, the buyer's continuation payoff in the bargaining phase converges to a limit that is strictly less than $1-\alpha_b$.[31]

The Case Where the Sellers' Demands are Different

This section characterizes the unique equilibrium strategy of the continuous-time bargaining problem G in stage 2 when the sellers choose different demands in stage 1. Without loss of generality, I assume that $\alpha_2 < \alpha_1$. In this case, the structure of the equilibrium strategy drastically changes (relating to the case where $\alpha_1 = \alpha_2$). In equilibrium, the bargaining phase never ends with the buyer's concession to the seller who has the higher demand (seller 1). If the buyer ever visits store 1, the rational seller 1 concedes to the buyer (upon the buyer's arrival at this seller) because the buyer has the tendency to opt out instantly from the concession game in store 1.

More formally, consider the case where the buyer is in store 1 and playing the concession game with this seller. This means that the buyer is indifferent between, on the one hand, accepting seller 1's demand, thus receiving the instantaneous payoff of $1-\alpha_1$, and on the other hand, waiting for the concession of the seller. However, if the buyer leaves (immediately) seller 1 and goes directly to the second store to accept the demand of seller 2, his discounted (instantaneous) payoff will be $\delta(1-\alpha_2)$. Thus, if the buyer ever visits store 1 in equilibrium, then he will never accept seller 1's demand because by Assumption 2, we have $(1-\alpha_1) < \delta(1-\alpha_2)$. Therefore, in equilibrium, the buyer does not concede to nor spend time with seller 1 given that he ever visits store 1. As a result, it must be the case that rational seller 1 instantaneously accepts the buyer's demand with

[31] Note that the second seller's expected payoff in the game is less than $z_s e^{-r_s T_1^d} \alpha_b$.

probability one upon his arrival, and the buyer immediately leaves store 1 if seller 1 does not concede to him.

Since the buyer and seller 1 play an equilibrium strategy that impels seller 1 to reveal his type immediately, the buyer's expected payoff of visiting this seller is $(1-z_s)(1-\alpha_b)+\delta z_s v_b^2$. I denote by v_b^2 the buyer's expected payoff in store 2 when he visits this store knowing that seller 1 is the obstinate type. Thus, if the buyer initially chooses to visit seller 2, then he concedes to this seller and receives the instantaneous payoff of $1-\alpha_2$, if and only if $1-\alpha_2 \geq \delta[(1-z_s)(1-\alpha_b)+\delta z_s v_b^2]$.

This inequality holds when $z_s \geq \bar{z}$ holds.[32] However, by Assumption 2 we have $z_s < \bar{z}$, implying that if the buyer visits seller 2 first in equilibrium, then the buyer strictly prefers leaving this seller immediately upon his arrival (given that seller 2 does not accept the buyer's demand and finish the game). Hence, rational seller 2 must concede to the buyer at time 0 with probability one. The next result characterizes the second stage equilibrium strategies of the bargaining problem G.

Proposition 3.2. *Suppose that the players' declared demands in stage 1 are such that $\alpha_b < \alpha_2 < \alpha_1$. Then the unique equilibrium of the continuous-time bargaining problem G in stage 2 is the following:*

(i) *If the buyer visits seller 1 first, then rational seller 1 immediately accepts the buyer's demand and finishes the game at time 0 with probability one. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is the obstinate type, so he immediately leaves store 1 and never comes back to this store again. The buyer goes directly to seller 2 to play the concession game with this seller. The concession game with seller 2 may continue until the time $T_2^e = \min\{-\frac{\log z_b^1}{\lambda_b}, -\frac{\log z_s}{\lambda_2}\}$ where $\lambda_2 = \frac{(1-\alpha_2)r_b}{\alpha_2-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha_2-\alpha_b}$ and z_b^1 is the posterior probability that the buyer is the obstinate type conditional on seller 1 is visited first.*[33] *Moreover, players concede according to the following strategies: $F_2(t) = 1 - z_s e^{\lambda_2(T_2^e - t)}$ and $F_b^2(t) = 1 - z_b^1 e^{\lambda_b(T_2^e - t)}$ for all $t \geq 0$.*[34]

(ii) *If the buyer visits seller 2 first, then rational seller 2 immediately accepts the buyer's demand upon his arrival. Otherwise, the buyer leaves seller 2 immediately at time 0 (knowing that seller 2 is the obstinate type), and goes directly to seller 1. Rational seller 1 instantly accepts the buyer's demand with probability one upon the buyer's arrival. In case seller 1 does not concede, the buyer immediately leaves this seller, directly returns to seller 2, accepts the seller's demand α_2 and finalizes the game.*

[32] See the proof of Proposition 3.2 in Appendix A

[33] Suppose that the rational buyer employs a strategy such that he visits seller 1 first with probability $\sigma_b^0(1) \in [0,1]$. Then $z_b^1 = \frac{1/2 z_b}{1/2 z_b + (1-z_b)\sigma_b^0(1)}$.

[34] Note that with some manipulation of the notation, I reset the clock once the buyer enters store 2.

Therefore, in equilibrium, when the buyer visits seller 1 first, he sends a *take it or leave it* ultimatum to this seller. If seller 1 does not accept the buyer's demand, then the buyer will go to the second seller. In this case, an agreement might be reached with seller 2, but possibly after some delay. On the other hand, when the buyer visits seller 2 first, he sends the same ultimatum to both sellers (first to seller 2 and then to 1). If no seller accepts the buyer's demand, then the buyer will come back to seller 2 and accept his demand α_2.[35] Hence, the buyer visits seller 1 first only when he is strong relative to seller 2 (according to the initial reputations z_b^1 and z_s) so that the initial probabilistic concession he will receive from seller 2 is high enough. This implies that z_b^1 (the posterior probability that the buyer is the obstinate type conditional on seller 1 is visited first) must be sufficiently high. The following result summarizes the last argument formally.

Proposition 3.3. *Suppose that the players' declared demands in stage 1 are such that $\alpha_b < \alpha_2 < \alpha_1$. In the unique equilibrium of the continuous-time bargaining problem G in stage 2, the buyer visits seller 1 first if and only if the buyer is sufficiently strong. That is, $z_b^1 \geq \left(\frac{\alpha_2-\alpha_b}{1-\alpha_b-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$.*

4. First Stage Equilibrium Demand Decisions

In this section, I first characterize the rational buyer's equilibrium strategy on store selection. Then I examine the set of equilibrium prices (or demands) that would be chosen by the players in the first stage of the bargaining problem G. Finally, I present some limit results regarding the cases where the initial probabilities of the obstinate types (z_s, z_b) vanish and the set of obstinate demands, C, approaches $[0,1]$.

For any $\alpha \in C$, let $C_\alpha = \{x \in C | x < \alpha\}$ denote the set of demands that are incompatible with α. I assume that the set of obstinate demands is finite and reach enough. Let α_{min} denote the minimum element of the set C, i.e. $\alpha_{min} = \inf\{x | x \in C\}$. Recall that π denotes the (common) probability distribution on C.

Assumption 3. *For any $\alpha \in C$ such that $\alpha_{min} < \alpha$ and $\{\alpha_{min}\} \neq C_\alpha$ we have $\alpha_{min} \leq \frac{1}{2}\sum_{x\in C_\alpha} x\pi(\alpha,x)$ where $\pi(\alpha,x) = \frac{\pi(x)}{\sum_{y\in C_\alpha}\pi(y)}$, and π is uniform on C.*

Assumption 3 holds with no hassle if α_{min} is sufficiently close to zero.[36] It implies, for example, that the minimal and the second minimal element of the set C are distant enough so that half of the (weighted) average of these two is no less than the minimal element of C. This assumption is not essential for the results (in particular the limit results), however it simplifies the subsequent analysis considerably.

[35] Seller 2's immediate concession to the buyer (and receiving the payoff of α_b) is optimal because otherwise the seller can achieve at most $\alpha_2 z_s$ (since the buyer revisits seller 2 only if seller 1 is the obstinate type) and we have $\alpha_2 z_s < \alpha_b$ by Assumption 2.

[36] In particular, I assume that $\alpha_{min} < \frac{r_b}{r_b+2r_s}$.

In the first stage of the continuous-time bargaining problem G, a strategy for seller i is some $\alpha_i \in C$ whereas a strategy for the buyer is defined as a collection (μ_b, σ_b^0). Here, given the announced demands (α_1, α_2), μ_b is a probability distribution over $C \cup \{Q\}$ describing the buyer's choice between Q (immediate acceptance) and $\alpha_b \in C$. Recall that $C_{(\alpha_1,\alpha_2)} = \{x \in C | x < \min\{\alpha_1, \alpha_2\}\}$ denotes the set of demands that are incompatible with α_1 and α_2. Without loss of generality, I require that $\mu_b(Q) = 0$ for all $\alpha_b \in C_{(\alpha_1,\alpha_2)}$ and $\mu_b(Q) = 1$ otherwise. That is, both conceding at $t = 0$ (the beginning of stage 2) and choosing Q (in stage 1) correspond to immediate concession. Finally, σ_b^0 is a probability measure over the sellers such that $\sigma_b^0(i)$ denotes the probability that the buyer visits seller i first.

Recall that the rational buyer chooses each seller with equal probabilities when the sellers' demands are the same. The next result characterizes the rational buyer's equilibrium strategy $\sigma_b^0(i)$ if the sellers choose different demands in stage 1.

Proposition 4.1. *Suppose that the sellers' declared demands in stage 1 are such that $\alpha_{min} < \alpha_2 < \alpha_1$. Then in equilibrium, the rational buyer declares his demand as α_{min} in both stores and visits seller 1 first with probability $\sigma_b^0(1) = \frac{z_b(1-\mu)}{2\mu(1-z_b)}$ where $\mu = \left(\frac{\alpha_2-\alpha_{min}}{1-\alpha_{min}-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$.*

Fix the search friction δ and the set of obstinate demands C satisfying Assumptions 1 and 3. For any $z_b, z_s \in (0,1)$, let $G(z_b, z_s)$ denote the continuous-time bargaining problem G where the initial reputations of the sellers and the buyer are z_b and z_s, respectively. Denote by $E(C) \subseteq C \times C$ the set of equilibrium prices (of the sellers). More formally, a pair of demands $(\alpha_1, \alpha_2) \in C^2$ is an element of $E(C)$ if there exit $z_b, z_s \in (0,1)$ small enough (i.e. satisfying Assumption 2) such that α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the bargaining problem $G(z_b, z_s)$.

Proposition 4.2. $E(C) = \{(\alpha, \alpha) | \alpha \in C\}$.

Since the sellers are *ex-ante* identical, it is natural to suspect that in equilibrium both sellers should choose the same demand. However, it is surprising that any obstinate demand in C can be supported in equilibrium, even though the sellers compete in the Bertrand fashion. Given that both sellers choose the same demand, α, that is higher than α_{min}, the buyer's strategy choosing each seller with equal probabilities and declaring a demand uniformly drawn from the set C_α is an equilibrium strategy when, for example, z_s and z_b are such that the buyer is weak at all demands incompatible with α. Therefore, assuming that the sellers are strong for all demand selections of the buyer within the set C_α while π and μ_b are uniform, each seller's ex-ante expected payoff of declaring the demand α, u_α, is greater than $\frac{1}{2m_\alpha} \sum_{x \in C_\alpha} x$, (where m_α is equal to the cardinality of C_α).

By Assumption 3, u_α is higher than α_{min}. Thus, undercutting is not an optimal strategy if the sellers deviate to the price of α_{min}.

On the other hand, if a seller price undercuts his opponent by demanding α' such that $\alpha_{min} < \alpha' < \alpha$, then the buyer would infer that this seller is the obstinate type with certainty. In this case, the buyer uses the deviating seller's price as an "outside option" to increase his bargaining power against the other seller. Thus, the buyer prefers to visit (first) the seller whom he knows he can negotiate and possibly get a much better deal. Hence, deviating to α' is not optimal for the sellers either because it would yield payoff strictly less than $z_s\alpha'$ (the buyer will visit deviating seller's store if the other seller is an obstinate type), and by Assumption 2 and 3 it is less than u_α.

Given the search friction δ and the set C, let $E^\infty(C)$ denote *the set of equilibrium prices of the bargaining problem* G *as initial priors vanish.* More formally, a pair of demands $(\alpha_1, \alpha_2) \in C^2$ is an element of $E^\infty(C)$ if for any $z_s, z_b \in (0, 1)$ small enough, where α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the bargaining problem $\mathrm{G}(z_b, z_s)$, we have the following: Take any sequences $\{z_s^n\}$ and $\{z_b^n\}$ (where $z_s^0 = z_s$, $z_b^0 = z_b$ and for all $n \geq 0$, $z_s^n = Kz_b^n$ for some finite $K > 0$) of the prior beliefs converging to zero.[37] Then α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the bargaining problem $\mathrm{G}(z_b^n, z_s^n)$ for all $n \geq 0$. For any $\alpha \in C$, define $\hat{x}(\alpha)$ to be the maximal element of the set C_α, i.e. $\hat{x}(\alpha) := \sup\{x \in C_\alpha | x < \alpha\}$. Finally, recall that the buyer and the sellers discount the time with the interest rates r_b and r_s, respectively. The following result characterizes the set of equilibrium prices as the initial priors vanish.

Proposition 4.3. $E^\infty(C) = \{(\alpha, \alpha) \in E(C) \mid 2\hat{x}(\alpha)r_s + \alpha r_b \leq r_b\}$.

Corollary 4.1. $\lim_{C\to[0,1]} E^\infty(C) = \{(\alpha, \alpha) \mid \alpha \in [0, \frac{r_b}{r_b+2r_s}]\}$.

It is easy to see that for any $\alpha \in C$ we have $\lim_{C\to[0,1]} \hat{x}(\alpha) = \alpha$.[38] Thus, depending on the relative ratio of the initial reputations (z_b and z_s), equilibrium prices of the bargaining problem, in the limit, range in $[0, \frac{r_b}{r_b+2r_s}]$.

As a special case, when the buyer's and the sellers' interest rates are common, i.e. $r_b = r_s$, then in the limit, $E^\infty(C)$ approaches $\{(\alpha, \alpha) | \ \alpha \in [0, \frac{1}{3}]\}$. Notice that higher impatience for the rational buyer (higher r_b) will increase the maximum price attainable in equilibrium. On the contrary, increasing impatience for the sellers (higher r_s) decreases the maximum price that can be supported in the limit.

In equilibrium (as initial priors converge to zero), if the sellers choose some $\alpha \in C$, which is sufficiently reach, then the buyer's expected payoff is $1-\alpha$. In other words, there

[37]That is, $\forall \epsilon > 0$, $\exists M > 0$ such that $|z_s^m - 0| < \epsilon$, $\forall m > M$.

[38]Note that we need $\delta \to 1$ as $C \to [0, 1]$ so that Assumption 1 continues to hold.

is no sufficiently small z_b and z_s such that α is the sellers' equilibrium demand selection in stage 1 and the buyer is strong with these priors for any demand he announces in C_α (see the proof of Proposition 4.3 in Appendix C). This is true because if the buyer is strong, then he could profitably deviate and declare his demand as α_{min} instead of randomizing over C_α as the values of z_b and z_s decrease to zero at the same rate (see the buyer's expected payoff given in equations (4) and (5) in Section 3). On the other hand, choosing α_{min} (only) cannot be an equilibrium strategy for the buyer either, because he would prefer to deviate to a demand slightly above α_{min} (when C is reach enough). Therefore, as $C \to [0,1]$, in equilibrium where both sellers choose $\alpha \in [0, \frac{r_b}{r_b+2r_s}]$ in stage 1, the expected payoff to the buyer is $1-\alpha$, whereas expected payoff to each seller converges to $\frac{\alpha}{2}$ (each seller is strong and so his expected payoff converges to α, but the buyer visits each seller with probability 1/2). Hence, the final outcome of the bargaining problem G is efficient in the limit.

The final result of this section examines a straightforward extension of the model to the case with $N \geq 2$ identical sellers. Namely, let G_N denote the continuous-time bargaining problem where the number of sellers is N. The game G_N is identical to G except the number of players. Symmetrically define $E^\infty(N,C)$ to be the set of equilibrium prices of the bargaining problem G_N as initial priors vanish. Therefore,

Proposition 4.4. $\lim_{C\to[0,1]} E^\infty(N,C) = \{(\alpha,\alpha)|\ \alpha \in [0, \frac{r_b}{r_b+Nr_s}]\}$.

5. Concluding Remarks

This paper develops a reputation-based model to highlight the influence of posted prices and bargaining postures on imperfectly competitive search markets. The introduction of obstinate types that are completely inflexible in their demands and offers, even with low probabilities, makes the equilibrium of the multilateral bargaining game essentially unique. The equilibrium allocation does not depend on the fine details of the bargaining protocols, nor do the sellers extract all the surplus of the buyer because of the positive search friction. Instead, it depends on the posted prices and initial reputations as well as the time preferences of the players. The equilibrium has a war of attrition structure that engenders inefficiency due to possible delay in reaching an agreement. Although the sellers compete in the spirit of Bertrand, the equilibrium outcomes are in contrast to Bertrand's prediction.

Appendix A

This section relaxes the restriction on strategies so the buyer can visit each store multiple times. The buyer's strategy in game G has two parts. The first part σ_b determines the buyer's location at any given history. The second part is a pair of right continuous distribution functions $F_b^{1,T}$ and $F_b^{2,T}$ such that for any $i \in \{1,2\}$, history h_T and interval $[T, T']$ where $T < T' \leq \infty$, we have $F_b^{i,T} : [T, T'] \to [0,1]$. Similarly, seller i's strategy in the game G is a right continuous distribution function F_i^T such that for any history h_T and interval $[T, T']$ where $T < T' \leq \infty$, we have $F_i^T : [T, T'] \to [0,1]$. Given time t, let $z_i(t)$ denote seller i's reputation (probability that seller i is the obstinate type) at time t. It then follows that $F_i^T(T') \leq 1 - z_i(T)$.

For any strategy profile $\sigma = \langle \mathbf{F}; \sigma_b \rangle$ and history h_t, denote by σ_b the continuation strategy after the history h_t. Then define $S_i^{\sigma_b}$ for each $i \in \{1,2\}$ as the set of intervals that the buyer is in store i and $I_i^{\sigma_b}$ as the collection of times that the buyer enters store i according to continuation strategy σ_b. For instance, if $T \in I_i^{\sigma_b}$, then there exists a $T' > T$ such that $[T, T'] \in S_i^{\sigma_b}$, which will be interpreted as the buyer enters store i at time T, stays there until time T', and leaves store i at T'. Therefore, the buyer's continuation strategy σ_b after a history h_t generates the sets $I_1^{\sigma_b}$ and $I_2^{\sigma_b}$ such that the sequence of distribution functions $\{(F_b^{i,T_i}, F_i^{T_i})\}_{T_i \in I_i^{\sigma_b}}$ for each $i \in \{1,2\}$ forms the continuation strategy profile of the strategy σ after a history h_t.

If σ is an equilibrium strategy, then after any history h_t, i and $T_i \in I_i^{\sigma_b}$ such that $[T_i, T_i'] \in S_i^{\sigma_b}$, F_b^{i,T_i} is a best response to $F_i^{T_i}$ within the set of all right-continuous distribution functions $\hat{F}_b^i$ defined over $[T_i, T_i']$ such that $\hat{F}_b^i(T_i') = F_b^{i,T_i}(T_i')$. Same holds for F_i.

Suppose that $\sigma = \langle \mathbf{F}; \sigma_b \rangle$ is a sequential equilibrium of the continuous-time bargaining problem. Pick an arbitrary history h_t. Then, for any $i \in \{1,2\}$ and $T_i \in I_i^{\sigma_b}$ consider the equilibrium continuation strategy profile $(F_b^{i,T_i}, F_i^{T_i})$. I next study the properties of these distribution functions on their domain $[T_i, T_i'] \in S_i^{\sigma_b}$. Finally, let $\tau_i = \inf\{t \geq 0 | \exists T_i \in I_i^{\sigma_b} \text{ s.t } F_i^{T_i}(t) = 1 - z_i(T_i)\}$ denote the time that seller i's reputation reaches 1. Similarly, let $\tau_b = \inf\{t \geq 0 | F_b^{i,T_i}(t) = 1 - z_b(T_i)$ for some $i \in \{1,2\}$ and $T_i \in I_i^{\sigma_b}\}$ denote the time that the buyer's reputation reaches 1. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *Consider the equilibrium continuation strategy profile $(F_b^{i,T_i}, F_i^{T_i})$. If a player's strategy is constant on some interval $[t_1, t_2]$ in the interior of its domain, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *Consider the equilibrium continuation strategy profile $(F_b^{i,T_i}, F_i^{T_i})$. Then the strategy of a player does not have a mass point over $(T_i, T_i']$.*

Lemma A.3. *Consider the equilibrium continuation strategy profile $(F_b^{i,T_i}, F_i^{T_i})$. Then*

$$F_i^{T_i}(T_i) F_b^{i,T_i}(T_i) = 0$$

Proof of Proposition 3.1. Consider the equilibrium continuation strategy profile $(F_b^{i,T_i}, F_i^{T_i})$. According to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_b^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Let

$$U_i(t, F_b^{i,T_i}) = \int_{T_i}^{t} \alpha e^{-r_s x} dF_b^{i,T_i}(x) + \alpha_b e^{-r_s t}(1 - F_b^{i,T_i}(t))$$

denote the expected payoff of rational seller $i \in \{1,2\}$ who concedes at time $t \geq T_i$ and

$$U_b(t, F_i^{T_i}) = \int_{T_i}^{t} (1-\alpha_b) e^{-r_b x} dF_i^{T_i}(x) + (1-\alpha) e^{-r_b t}(1 - F_i^{T_i}(t))$$

denote the expected payoff of the rational buyer who concedes to seller i at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$.

Then, it follows that $D^{i,T_i} := \{t | U_i(t, F_b^{i,T_i}) = \max_{s \in [T_i, T_i']} U_i(s, F_b^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_b^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $D^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_b^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_b^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda t}$ and $F_b^{i,T_i}(t) = 1 - c_b^i e^{-\lambda_b t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_b^i = 1 - F_b^{i,T_i}(T_i)$ such that $\lambda_b = \frac{\alpha_b r_s}{\alpha - \alpha_b}$ and $\lambda = \frac{(1-\alpha) r_b}{\alpha - \alpha_b}$.

Now, consider the equilibrium strategy where the buyer visits each store once. Then there is a unique $T_1 \in I_1^{\sigma_b}$ and $T_2 \in I_2^{\sigma_b}$. Moreover, given that the buyer visits store 1 first at time 0, we can find the values of c_2 and c_b^2 as follows (remark that with some manipulation of notation I also take $T_2 = 0$): By Lemma A.3, we know that $F_2(0) F_b^2(0) = 0$. First, suppose that $F_b^2(0) = 0$. It implies that $c_b^2 = 1$ and thus $F_b^2(t) = 1 - e^{-\lambda_b t}$. Therefore, $F_b^2(\tau_b) = 1 - z_b(T_1^d)$, implying that $1 - e^{-\lambda_b \tau_b} = 1 - z_b(T_1^d)$ if and only if $\tau_b^2 = -\frac{\log z_b(T_1^d)}{\lambda_b}$. On the other hand, if $F_2(0) = 0$, then we have $c_2 = 1$ and thus $F_2(t) = 1 - e^{-\lambda t}$. Therefore, $F_2(\tau_2) = 1 - z_s$, implying that $1 - e^{-\lambda \tau_2} = 1 - z_s$ if and only if $\tau_2 = -\frac{\log z_s}{\lambda}$. Hence, the game ends at time $T_2^e = \min\{\tau_2, \tau_b^2\}$.

Next we will find the buyer's equilibrium departure time from the first seller (i.e. seller 1). But first note that there are two critical times that we need to consider. These are $\tau_1 = -\frac{\log z_s}{\lambda}$ and $\tau_b^1 = -\frac{\log z_b(T_1^d)}{\lambda_b}$. The former denotes the time that seller 1's reputation reaches one and the latter denotes the time that the buyer's reputation reaches one in store 2 if he leaves store 1 at time T_1^d. If the buyer leaves store 1 at T_1^d, his continuation payoff in store 1 is $1 - \alpha$ (either because the buyer reveals his rationality or seller 1's reputation reaches one). However, if the buyer goes to store 2, his continuation payoff in store 2 will be $v_b^2(T_1^d) = 1 - \alpha_b - z_s e^{\lambda T_2^e}(\alpha - \alpha_b)$ where $T_2^e = \min\{\tau_1, \tau_b^1\}$.

Note that if the buyer is weak relative to seller 2 when the buyer's reputation is $z_b(T_1^d)$, that is if $\tau_2 > \tau_b^2$, then $v_b^2(T_1^d) = 1 - \alpha$. Thus, the buyer has no incentive to leave store 1 at time T_1^d since $\delta < 1$. However, as $z_b(T_1^d)$ is an increasing function of T_1^d, there exist some T_1^d such that the buyer becomes the strong player relative to seller 2. Therefore, we need to find T_1^d that yields $1 - \alpha = \delta v_b^2(T_1^d)$. Hence, given that the buyer is the strong player relative to seller 2 if the buyer leaves store 1 at time T_1^d, we have $T_2^e = -\frac{\log z_b(T_1^d)}{\lambda_b}$, implying that $v_b^2(T_1^d) = 1 - \alpha_b - z_s z_b^{-\frac{\lambda}{\lambda_b}}(\alpha - \alpha_b)$. Therefore, we have $A := \frac{1 - \alpha_b - \frac{1-\alpha}{\delta}}{\alpha - \alpha_b} = z_s z_b(T_1^d)^{-\frac{\lambda}{\lambda_b}}$ leading to $z_b(T_1^d) = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda}} := X_s$. Note that for large values of δ, we have $z_s^{\lambda_b/\lambda} < X_s < 1$. Moreover, since $z_b(T_1^d) = \frac{z_b}{1 - F_b^1(T_1^d)} = X_s$ and $F_b^1(T_1^d) = 1 - c_b^1 e^{-\lambda_b T_1^d}$, we have

$$c_b^1 e^{-\lambda_b T_1^d} = \frac{z_b}{X_s} \tag{6}$$

First, consider the case where $c_1 \neq 1$. Lemma A.3 implies that $c_b^1 = 1$ and Equation (6) yields

$$T_1^d = -\frac{\log(z_b/X_s)}{\lambda_b}$$

Remark that T_1^d is well defined, i.e. $T_1^d > 0$, whenever $z_b < X_s$. Note that $T_1^d < \tau_1$ whenever $-\frac{\log(z_b/X_s)}{\lambda_b} < -\frac{\log z_s}{\lambda}$ implying that $z_b > X_s z_s^{\lambda_b/\lambda}$. It is also true that $T_1^d < \tau_b^1$ because $X_s < 1$ for δ large enough.

However, if $c_1 = 1$, i.e. seller 1 does not make an initial probabilistic acceptance, it should be true that $T_1^d \geq \tau_1$. In equilibrium, the buyer does not bear the cost of delay and thus it must be true that $T_1^d = \tau_1$. Note that for $T_1^d = \tau_1$ to be true in equilibrium it must be that $\tau_1 \leq \tau_b^1$. Moreover, since $T_1^d = -\frac{\log z_s}{\lambda}$, Equation (6) implies that $c_b^1 = \frac{z_b}{X_s z_s^{\lambda_b/\lambda}}$. It is well defined whenever (i) $z_b \leq c_b^1$ implying $X_s z_s^{\lambda_b/\lambda} \leq 1$ which is true for δ large enough, and (ii) $c_b^1 \leq 1$ implying that $z_b \leq X_s z_s^{\lambda_b/\lambda}$. Note also that we need to have $X_s < 1$ and $A > 0$, which holds whenever $\delta > (1-\alpha)/(1-\alpha_b-(\alpha-\alpha_b)z_s)$.

Therefore, for all values of z_b satisfying $z_b < X_s z_s^{\lambda_b/\lambda}$, seller 1 does not make any initial concession, and the buyer leaves store 1 at $T_1^d = \tau_1$. Notice that for these values of z_b, we have $\tau_1 \leq \tau_b^1$ as required. Thus, for all $z_b < X_s$, the buyer leaves store 1 at time $T_1^d = \min\{-\frac{\log z_s}{\lambda}, -\frac{\log(z_b/X_s)}{\lambda_b}\}$. However, if $z_b \geq X_s$ then at time $t = 0$ the buyer's discounted expected payoff in store 2, $\delta v_b^2(0)$, is larger than $1-\alpha$. Therefore, for such values of z_b, the buyer immediately leaves store 1 at time 0. As a result, rational seller 1 concedes to the buyer with probability 1 at time 0.

Since $F_1(T_1^d) = 1 - c_1 e^{-\lambda T_1^d} = 1 - z_s$, we have $c_1 = z_s e^{\lambda T_1^d}$. Moreover, for $z_b \geq X_s$, we have $c_b^1 = 1$. Otherwise it must be that $c_b^1 = \frac{z_b}{X_s z_s^{\lambda_b/\lambda}}$. Therefore, it can be summarized that

$$c_b^1 = \begin{cases} \frac{z_b}{X_s} e^{\lambda_b T_1^d}, & \text{if } z_b < X_s \\ 1, & \text{otherwise} \end{cases}$$

On the other hand, we have $c_2 = z_s e^{\lambda T_2^e}$ and $c_b^2 = 1$ where $T_2^e = -\frac{\log z_b(T_1^d)}{\lambda_b}$.

Now, I want to argue that in equilibrium the buyer and seller 1 cannot play a strategy that extends the concession game in store 1 beyond the time T_1^d with some positive probability. Suppose on the contrary that each player chooses such a strategy, and these strategies establish an equilibrium. Conditional on players delaying the end of the concession game in store 1 for some extra $\hat{t}$ unit of time after T_1^d, the buyer should be indifferent between conceding to seller 1 and waiting for concession at any time $t \in [T_1^d, T_1^d + \hat{t}]$. That is, the buyer must concede to seller 1 with a constant hazard rate during this extra time.

However, since the buyer's expected payoff in store 2 is a continuous and increasing function of his own reputation, and of time, we have $1-\alpha < \delta v_b^2(t)$ for all $t \in (T_1^d, T_1^d + \hat{t}]$. That is, at any time $t > T_1^d$, the buyer's discounted continuation payoff in store 2 will be strictly higher than his instantaneous payoff in store 1. However, this contradicts the optimality of the equilibrium strategy.

Therefore, conditional on each player executing a strategy that extends the concession game in store 1 after the time T_1^d, and these strategies constitute an equilibrium, the buyer should not concede to seller 1 with a positive (constant) hazard rate after T_1^d. However, this requirement implies that rational seller 1 must accept the buyer's offer by the time T_1^d with probability 1, which contradicts the initial assumption. Moreover, since the buyer's equilibrium strategy F_b^1 cannot have a discontinuity point over the interval $(0, T_1^d]$, the buyer cannot make a positive concession at time T_1^d. Thus, the event that the buyer leaves seller 1 at time T_1^d must occur with probability one in equilibrium.

Proof of Proposition 3.2. Since we have $1-\alpha_1 < \delta(1-\alpha_2)$, in equilibrium the buyer finds it optimal to go to store 2 instead of conceding to seller 1 at any given time $t \geq 0$. Moreover, the buyer waits in store 1 if he believes that seller 1's concession will come after a short delay. But postponing concession is not optimal for rational seller 1 because the buyer will never accept the seller's demand. Thus, in equilibrium, the buyer leaves seller 1 immediately if seller 1 does not accept the buyer's demand α_b, and rational seller 1 concedes to the buyer with probability 1 upon his arrival.

Upon arrival at store 2 (after visiting seller 1), the buyer and seller 2 play the concession game, and Proposition 3.1 implies that the concession continues until time $T_2^e = \min\{\tau_b^2, \tau_2\}$. Moreover, players concede according to the strategies $F_2(t) = 1 - c_2 e^{-\lambda_2 t}$, $F_b^2(t) = 1 - c_b^2 e^{-\lambda_b t}$ where $c_2 = z_s e^{\lambda_2 T_2^e}$ and $c_b^2 = z_b e^{\lambda_b T_2^e}$. Note that the buyer's expected payoff (evaluated at time 0) in this subgame is $U_b^1 = (1-z_s)(1-\alpha_b) + \delta z_s u_b^2$ where $u_b^2 = (1-F_2(0))(1-\alpha_2) + F_2(0)(1-\alpha_b)$. In particular, if the buyer is strong relative to seller 2, i.e., $z_b^1 \geq z_s^{\lambda_b/\lambda_2}$ then $u_b^2 = z_s (z_b^1)^{-\lambda_2/\lambda_b}(1-\alpha_2) + (1 - z_s(z_b^1)^{-\lambda_2/\lambda_b})(1-\alpha_b)$. Otherwise, we have $u_b^2 = 1-\alpha_2$.

Now consider an equilibrium strategy after a subgame that the buyer visits seller 2 first at time 0. Again, by Proposition 3.1 we know that each player must concede with a constant hazard rate while the buyer is in store 2. If the buyer concedes to seller 2, his instantaneous payoff is $1-\alpha_2$. However, if the buyer leaves store 2 at some time $t \geq 0$ and goes to store 1, we know from previous arguments that concession in store 1 will immediately finish upon arrival of the buyer. So, the buyer would directly come back to store 2 if seller 1 is the obstinate type. Thus, the buyer's continuation payoff if he leaves store 2 at time t is $(1-z_s)(1-\alpha_b) + \delta z_s v_b^2$ where $v_b^2 = (1-F_2(0))(1-\alpha_2) + F_2(0)(1-\alpha_b)$ denotes the buyer's expected payoff in his second visit to store 2.[39]

$v_b^2 > 1-\alpha_2$ requires that seller 2 offers positive probabilistic gift to the buyer on his second visit. In this case, seller 2's expected payoff must be α_b. However, optimality of the equilibrium strategy implies that rational seller 2 should accept the buyer's offer with probability 1 when the buyer attempts to leave his store for the first time. Hence, it must be that in equilibrium $v_b^2 = 1-\alpha_2$. Therefore, the buyer's payoff if he leaves store 2 is $v_b^1 = \delta\left[(1-z_s)(1-\alpha_b) + \delta z_s(1-\alpha_2)\right]$.

If v_b^1 is larger than $1-\alpha_2$, then the buyer leaves store 2 immediately at time 0 instead of

[39]Notice that there is no equilibrium in which the buyer visits store 1 multiple times and store 2 more than twice.

conceding to seller 2. However, $v_b^1 > 1 - \alpha_2$ implies that $z_s < \frac{1-\alpha_b-\frac{1-\alpha_2}{\delta}}{1-\alpha_b-\delta(1-\alpha_2)} := \bar{z}$. Note that $\bar{z} > 0$ since by Assumption 1 we have $\delta(1-\alpha_b) > (1-\alpha_2)$. Moreover, since we have $z_s < \bar{z}$ by Assumption 2 the buyer finds it optimal to leave store 2 immediately at time 0.

On the other hand, if seller 2 is weak, i.e. $z_b^2 \geq z_s^{\lambda_b/\lambda_2}$, then optimality of the equilibrium strategy implies that rational seller 2 must accept the buyer's offer at time 0 with probability 1. However, if the buyer is weak relative to seller 2, i.e. $z_b^2 < z_s^{\lambda_b/\lambda_2}$, then seller 2's payoff when the buyer arrives at store 2 for the second time is no more than $\delta^2 z_s \alpha_2$. However, by assumption we have $\alpha_b \geq z_s \alpha_2$, implying that rational seller 2 immediately concedes to the buyer with probability 1, and gets α_b, upon the buyer's arrival at time 0.

Proof of Proposition 3.3. Suppose that the buyer is strong relative to seller 2, i.e. $z_b^1 > z_s^{\lambda_b/\lambda_2}$. If the buyer first visits seller 1, then his expected payoff is $U_b^1 = (1-z_s)(1-\alpha_b) + \delta z_s u_b$ where $u_b = z_s(z_b^1)^{-\lambda_2/\lambda_b}(1-\alpha_2) + (1 - z_s(z_b^1)^{-\lambda_2/\lambda_b})(1-\alpha_b)$. However, if he chooses to visit seller 2 first, his expected payoff is $U_b^2 = (1-z_s)(1-\alpha_b) + z_s\delta[(1-z_s)(1-\alpha_b) + \delta z_s(1-\alpha_2)]$. Hence, the buyer selects store 1 if, and only if $U_b^1 \geq U_b^2$, i.e. $(1-(z_b^1)^{-\lambda_2/\lambda_b})(1-\alpha_b) \geq (1-\alpha_2)(\delta - (z_b^1)^{-\lambda_2/\lambda_b})$ which yields the desired inequality.

Suppose now that the buyer is weak relative to seller 2, that is $z_b^1 \leq z_s^{\lambda_b/\lambda_2}$. If the buyer first visits store 1, then $U_b^1 = (1-z_s)(1-\alpha_b) + \delta z_s(1-\alpha_2)$. However, if the buyer first visits store 2, then the buyer's expected payoff, U_b^2, as given above. Thus, the buyer picks seller 1 to go to first if, and only if $U_b^1 \geq U_b^2$ which holds whenever $z_s \geq \frac{1-\alpha_b-(1-z_s)(1-\alpha_b)-\delta z_s(1-\alpha_2)}{1-\alpha_b-\delta[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)]}$. However, this inequality can be true only when $\alpha_b < \alpha_2$, yielding a contradiction.

Appendix B

Proof of Proposition 4.1. According to Proposition 3.2, rational sellers accept the buyer's demand (regardless of what it is) immediately after his arrival at their stores. Therefore, the buyer's optimal demand choice is α_{min} (supposing that z_s is sufficiently small). On the other hand, according to Proposition 3.3, the rational buyer's expected payoff of visiting store 1 and 2 at time 0 is equivalent if and only if the buyer's reputation after visiting store 1 first, μ, is $\left(\frac{\alpha_2-\alpha_{min}}{1-\alpha_{min}-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$. Since the Bayes' rule implies that $\mu = \frac{1/2z_b}{1/2z_b+(1-z_b)\sigma_b^0(1)}$, we have the desired result.

Proof of Proposition 4.2. First I will show that $\{(\alpha,\alpha)|\alpha \in C\} \subseteq E(C)$. For this purpose, take any $\alpha \in C$ and suppose that both sellers choose α in stage 1, the buyer chooses each seller with probability 1/2, and in the subgames following the first stage, each player uses the equilibrium strategies given in Propositions 2.1 and 3.1. Also assume that the buyer's strategy μ_b (randomization over the set C_α) is uniform. Then choose the initial priors z_b, z_s (sufficiently small) such that the buyer is weak at all $\alpha_b \in C_\alpha$. More formally, the posterior belief that the buyer is the obstinate type conditional on the buyer chooses $\alpha_b \in C_\alpha$, i.e. $z_b^{\mu_b}(\alpha_b) = \frac{(1/m)z_b}{(1/m)z_b+(1-z_b)(1/m_\alpha)}$ (where m and m_α are the cardinalities of the sets C and C_α, respectively), is strictly less than $(\frac{z_s^2}{A_{\alpha_b}})^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for all $\alpha_b \in C_\alpha$ with $A_{\alpha_b} = \frac{1-\alpha_b-\frac{1-\alpha}{\delta}}{\alpha-\alpha_b}$. Thus, the buyer is weak for any α_b in the support of his strategy μ_b, implying that the buyer's expected payoff is $1-\alpha$ in the game G. Since the buyer will never deviate to a demand above α, he has no profitable deviation.

On the other hand, the sellers do not have a profitable deviation either: If, for example, seller 1 deviates to a demand different than α, then the buyer will assign probability one that seller 1 is the obstinate type. In this case, the buyer will visit seller 1 first if the seller deviates to the demand α_{min}. Otherwise, the buyer will visit seller 1 after being convinced that seller 2 is the obstinate type as well.[40] Therefore, deviating to some α' such that $\alpha_{min} < \alpha' < \alpha$ is not a profitable deviation for seller 1 because we assume that z_s is very small. Likewise, deviating to α_{min} is not profitable either because the seller's ex-ante expected payoff of declaring the demand α is $u_\alpha = \frac{1}{2m_\alpha}\sum_{x\in C_\alpha}[x+(\alpha-x)F_b^1(0,x)]$ where $F_b^1(0,x) = 1 - z_b\left(\frac{A}{z_s^2}\right)^{xr_s/(1-\alpha)r_b}$ (since the sellers are strong for all $x \in C_\alpha$), and it is higher than α_{min} by Assumption 3.

Now I want to show that $E(C) \subseteq \{(\alpha,\alpha)|\alpha \in C\}$. Suppose for a contradiction that $(\alpha_1,\alpha_2) \in E(C)$, and without loss of generality that $\alpha_1 > \alpha_2 \geq \alpha_{min}$. According to Propositions 3.1 and 4.1, the buyer will visit seller 1 first with the probability $\sigma_b^0(1) \in (0,1)$, demand α_{min} and leave immediately if seller 1 does not concede to the buyer. This is the equilibrium strategy independent of the initial beliefs z_s and z_b. Therefore, at any $G(z_s,z_b)$ the payoff to seller 1 is no more than $\alpha_{min}\sigma_b^0(1) + z_s\alpha_{min}(1-\sigma_b^0(1))$. However, seller 1 could profitably deviate and demand α_{min} in stage 1, yielding a payoff more than $\frac{1}{2}\alpha_{min}$ that is higher than what seller 1 would get by posting α_1.

Proof of Proposition 4.3. First, I will show that there exist no sequences of $\{z_b^n\},\{z_s^n\}$ (converging to zero at the same rate) and $\alpha \in C$ with $\alpha_{min} < \alpha$ such that (α,α) is the equilibrium demand selection of the sellers in the game $G(z_b^n,z_s^n)$ and the buyer is strong for some $\alpha_b \in C_\alpha$ for each $n \geq 0$.

To see this, let $\hat{\alpha}_b$ be such that $\alpha_{min} \leq \hat{\alpha}_b \leq \alpha_b$. If α_b ($> \alpha_{min}$) is in the support of μ_b (the buyer's strategy in stage 1), so do the demands $\hat{\alpha}_b$; otherwise the buyer deviates and demands $\hat{\alpha}_b$ and the sellers accept this demand since they will infer that the buyer is the obstinate type with probability one. Moreover, if the buyer is strong for each $n \geq 0$ when he chooses α_b, (i.e. his payoff is close to $1-\alpha_b$ and strictly higher than $1-\alpha$), then the buyer must be strong for each $n \geq 0$ when he chooses $\hat{\alpha}_b$ so that $\hat{\alpha}_b$ could also be in the support of μ_b. Finally, assuming that C is reach enough, $\hat{\alpha}_b$ must be in the domain of the buyer's strategy μ_b^n (which may depend on n) for all n. This is true because if, for example, α_{min} is in the support of μ_b^n for some n but $\hat{\alpha}_b = \inf\{x \in C_\alpha | x > \alpha_{min}\}$ is not, then the buyer would profitably deviate and demand $\hat{\alpha}_b$ since the sellers will accept this demand because they will infer that the buyer is the obstinate type with probability one.

[40] If seller 1 deviates to α' such that $\alpha_{min} < \alpha' < \alpha$, then the buyer will visit seller 2 first for sure to make a "take α_{min} or I will leave you" offer. This choice is optimal for the buyer because rational seller 2 will immediately accept the buyer's demand α_{min} (since we have $\delta(1-\alpha') > 1-\alpha$ by Assumption 1).

To simplify the subsequent notation I will drop the n terms and denote the sequences by $\{z_b\}, \{z_s\}$. The buyer's expected payoff of declaring his demand as α_b (when he is strong) is either

$$\left(1 - \frac{z_s^2}{A(z_b^{\mu_b}(\alpha_b))^{\lambda/\lambda_b}}\right)(1-\alpha_b) + \frac{z_s^2}{A(z_b^{\mu_b}(\alpha_b))^{\lambda/\lambda_b}}(1-\alpha)$$

or

$$\left\{(1-z_s) + \delta z_s\left(1 - \frac{z_s}{(z_b^{\mu_b}(\alpha_b))^{\lambda/\lambda_b}}\right)\right\}(1-\alpha_b) + \frac{\delta z_s^2}{(z_b^{\mu_b}(\alpha_b))^{\lambda/\lambda_b}}(1-\alpha)$$

where $z_b^{\mu_b}(\alpha_b)$ is the posterior belief that the buyer is the obstinate type conditional on that he chooses the demand α_b. Remark that as z_b and z_s converge to zero at the same rate, both of the these payoffs converge to $1-\alpha_b$.[41] However, this implies that the buyer would strictly prefer to declare smaller demands for sufficiently smaller values of z_s and z_b, contradicting that in equilibrium there exists $\alpha_b > \alpha_{min}$ in the support of μ_b (for all n).

On the other hand, consider the case where α_{min} is the unique element in the support of μ_b and the buyer is strong at this demand. The price couple (α, α) cannot be supported in equilibrium because each seller's expected payoff will be $\frac{1}{2}\alpha_{min}$ (since the sellers are weak), and thus each seller would like to deviate and post his demand as α_{min} instead of α in the first stage.

Therefore, in equilibrium we must have that $z_b^{\mu_b}(\alpha_b) \leq \left(\frac{z_s^2}{A}\right)^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for each α_b in the support of μ_b and for each such $\{z_b, z_s\}$ (small enough) converging to zero at the same rate. (Otherwise, the unique equilibrium is such that both sellers choose α_{min}). Taking the log of both sides and rearranging the terms yields

$$\frac{2\alpha_b r_s}{(1-\alpha)r_b} - 1 \leq \frac{\ln[Kz_s\pi(\alpha_b) + (1-Kz_s)\mu_b(\alpha_b)]}{|\ln z_s|} - \frac{\ln(K\pi(\alpha_b))}{|\ln z_s|} - \frac{\alpha_b r_s \ln(A)}{(1-\alpha)r_b|\ln z_s|}$$

The limit of the right hand side as z_s converges to zero is 0, implying that we must have $2\alpha_b r_s \leq (1-\alpha)r_b$ for all $\alpha_b \in C_\alpha$ as required.

Proof of Proposition 4.4. The same arguments used in the proofs of Propositions 4.1 and 4.2 suffice to show that $E(N, C) = \{(\alpha, ..., \alpha) \in C^N | \alpha \in C\}$. That is, in equilibrium all the sellers will choose the same demand in stage 1 (retaining the model's specifications and assumptions that are modified to N sellers; for example, Assumption 3 should be modified so that $\alpha_{min} \leq \frac{1}{N}\sum_{x \in C_\alpha} x\pi(\alpha, x)$). Also, the arguments in the proof of Proposition 4.3 will show that there exist no sequences of prior beliefs $\{z_b^n\}, \{z_s^n\}$ (converging to zero at the same rate) and $\alpha \in C$ with $\alpha_{min} < \alpha$ such that $(\alpha, ..., \alpha)$ is an equilibrium demand selection of the sellers in the game $G_N(z_b^n, z_s^n)$ and the buyer is strong for some $\alpha_b \in C_\alpha$ (in the support of μ_b) for each $n \geq 0$.

[41] Remark that as the initial priors converge to zero at the same rate, the strength of the buyer may alter. For example, the buyer remains to be the strong player for sufficiently small priors if and only if $1 < \lambda/\lambda_b < 2$ holds. Therefore, the first equation gives the buyer's expected payoff for small values of z_s and z_b only when the last inequality holds, implying that the value of the first equation approaches to $1-\alpha_b$ for vanishing priors. Similar arguments apply for the second equation.

Recall that the claim of the Proposition 4.3 (i.e. $(\alpha, \alpha) \in E^{\infty}(C) \Leftrightarrow 2\alpha_b r_s \leq (1-\alpha)r_b$) relies solely on the fact that in equilibrium (when there are two sellers) we must have $z_b^{\mu_b}(\alpha_b) \leq \left(\frac{z_s^2}{A}\right)^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for each α_b in the support of μ_b and for each admissible sequence of $\{z_b, z_s\}$ converging to zero at the same rate. Namely, the buyer must be weak (and continues to be weak) as initial priors converge to zero. Therefore, we need to find the condition such that the buyer is weak under the initial priors z_b and z_s when the declared demands in stage 1 are $\alpha_b, \alpha \in C$ with $\alpha_b < \alpha$.

For the ease of exposition, I will derive this condition for the 3-sellers case, which can be extended to N-sellers case by iterating the same process (the following arguments are straightforward extensions of the approach that I use in the proof of Proposition 3.1). For this reason, suppose now that there are tree sellers all of which choose the same demand α in stage 1 and the buyer declares his demand as $\alpha_b < \alpha$. Without loss of generality, I assume that the buyer visits seller 1 first and seller 3 last (if no agreement have been reached with the sellers 1 and 2). Therefore, let T_i^d denote the time that the buyer leaves seller $i \in \{1, 2\}$.

The buyer leaves seller 2 when his discounted continuation payoff in store 3, i.e. $v_b^3(T_2^d) = 1 - \alpha_b - z_s(z_b(T_2^d))^{-\lambda/\lambda_b}(\alpha - \alpha_b)$, equals to $1 - \alpha$. This equality implies that the buyer must leave the second store when his reputation reaches to X_s, namely $z_b(T_2^d) = X_s = (\frac{z_s}{A})^{\lambda_b/\lambda}$. Therefore, the buyer's expected payoff in store 2 at the time he enters this store is $v_b^2(T_1^d) = 1 - \alpha_b - z_s(\frac{X_s}{z_b(T_1^d)})^{\lambda/\lambda_b}(\alpha - \alpha_b)$. Note that if the buyer is weak, then $z_b(T_1^d) < X_s$ is always the case. If, on the contrary, $z_b(T_1^d) \geq X_s$ holds, then at the time the buyer leaves store 1, his expected payoff in store 2 will be strictly higher than his continuation payoff in store 1 (i.e., $1 - \alpha$). This is possible in equilibrium only if $T_1^d = 0$, implying that the buyer is strong.

Thus, similar to the previous arguments, the buyer leaves seller 1 when his discounted continuation payoff in store 2, i.e. $v_b^2(T_1^d)$, equals to $1 - \alpha$. Then we have $z_b(T_1^d) = \left(\frac{z_s^2}{A}\right)^{\lambda_b/\lambda}$. Since $z_b(T_1^d) = \frac{z_b^{\mu_b}(\alpha_b)}{1 - F_b^1(T_1^d)}$, the previous equality implies that $\frac{z_b^{\mu_b}(\alpha_b)}{c_b^1 e^{-\lambda_b T_1^d}} = \left(\frac{z_s^2}{A}\right)^{\lambda_b/\lambda}$. If the buyer does not make an initial acceptance, i.e. $c_b^1 = 1$, then $\frac{z_b^{\mu_b}(\alpha_b)}{(z_s^2/A)^{\lambda_b/\lambda}} = e^{-\lambda_b T_1^d}$ which implies that $T_1^d = \frac{-\log(z_b^{\mu_b}(\alpha_b)/(z_s^2/A)^{\lambda_b/\lambda})}{\lambda_b}$. Therefore, if the buyer is weak, then we must have $T_1^d \geq \frac{-\log z_s}{\lambda}$ implying that

$$z_b^{\mu_b}(\alpha_b) \leq \left(\frac{z_s^3}{A}\right)^{\lambda_b/\lambda} \tag{7}$$

The last inequality implies that (as z_b and z_s converge to zero at the same rate) an equilibrium demand, α, chosen by the sellers must satisfy $3\hat{x}(\alpha)r_s + \alpha r_b \leq r_b$. Therefore, as $C \to [0,1]$ we will have $E^{\infty}(N, C) \to [0, \frac{r_b}{r_b + 3r_s}]$ which proves the claim for $N = 3$. Iterating the above arguments suffice to prove the claim for any finite N.

Appendix C

In this section I assume, without loss of generality, that in the first stage each seller chooses some $\alpha \in C$ and the buyer's demand $\alpha_b \in C$ is incompatible with α, i.e. $\alpha > \alpha_b$. Next, I will show that the equilibrium outcomes of the discrete-time bargaining problem $G(g_\epsilon)$ in the second stage converges to the unique equilibrium outcome of the continuous-time bargaining problem I described in Section 3 as $\epsilon \to 0$.

Proposition A.1. *As ϵ converges to zero, in any sequential equilibrium of the discrete-time bargaining problem $G(g_\epsilon)$ in stage two after any history h_t such that the buyer is in store $i \in \{1, 2\}$ and unknown to be rational while seller i is known to be rational, the payoff to the buyer is no less than $1 - \alpha_b - \epsilon$ and the payoff to seller i is no more than $\alpha_b + \epsilon$ (payoffs are evaluated at time t).*

The proof of this result is the same with the proof of Theorem 8.4 in Myerson (1991) and Lemma 1 in Abreu and Gul (2000) and very similar to the proof of Proposition A.2: I show that the payoff to the buyer if he continues to stay in store i and mimics the obstinate type converges to $1-\alpha_b$ as ϵ converges to zero. Given this, we can conclude that in any sequential equilibrium, the buyer chooses not to reveal his type and he stays in store i unless his expected payoff of doing the opposite exceeds $1 - \alpha_b$. I first show that the game ends (by seller i's acceptance of the buyer's offer α_b) with probability 1 in finite time, given history h_t, if the buyer continues to stay in store i and mimics the obstinate type. Finally, I show that as players make offers frequent enough ($\epsilon \to 0$), the game ends immediately with (almost) no delay. Therefore, I skip the proof.

With a similar spirit, Proposition A.2 claims that as ϵ converges to 0, at any sequential equilibrium of the game G(g_ϵ) after the history h_t such that the buyer is in store $i \in \{1, 2\}$ and known to be rational while both sellers are not known to be rational, the buyer makes immediate agreement with seller i, and the payoff to seller i (which depends on the details of the bargaining protocol g_ϵ) cannot be lower than α in the limit.

Before presenting the proof of Proposition A.2, I prove two Lemmas that I use extensively later:

Lemma A.4. *Let $\epsilon \to 0$ and let h_t be a history such that the buyer is in store $i \in \{1, 2\}$, known to be rational, seller i is unknown to be rational and seller $j \in \{1, 2\}$, $j \neq i$ is known to be the obstinate type. Then, for any sequential equilibrium of the game $G(g_\epsilon)$ in stage two after the history h_t, the payoff to the buyer is no more than $1 - \alpha + \epsilon$ and the payoff to the seller i is no less than $\alpha - \epsilon$ (payoffs are evaluated at time t).*

Proof. Given that seller j is the obstinate type, the buyer's continuation payoff in store j is at most $1 - \alpha$. Therefore, the buyer has no incentive to leave store i to get a price better than α. Given this, seller i does not reveal his type unless he gets a payoff higher than α by doing the opposite. Hence, the payoff to the buyer is no more than $1 - \alpha$ as ϵ converges to zero. □

Lemma A.5. *Let ϵ converge to 0 and let h_t be a history such that the buyer is in store $i \in \{1, 2\}$ and known to be rational while both sellers are unknown to be rational. Then in any sequential equilibrium of the game $G(g_\epsilon)$ in stage two after the history h_t it cannot be the case that seller i finishes the game at time t at some price $x < \alpha - \epsilon$ with probability one.*

Proof. Suppose for a contradiction that rational seller i makes a deal with the buyer at some price $x < \alpha$ at time t with probability 1. Given that this is an equilibrium strategy, both seller j and the buyer assign probability 1 to the event that seller i is the obstinate type if the seller does not accept the buyer's offer. But then, according to Lemma A.4, the buyer accepts the price α and finishes the game immediately at time t^* where $t < t^* \leq t + \epsilon$.

However, for arbitrarily small ϵ, rational seller i would prefer to deviate from his equilibrium strategy and wait until time t^* by mimicking the obstinate type so that he can get the payoff of α which is higher than x.[42] Hence, in equilibrium after the history h_t seller i delays the game with a positive probability. □

Proposition A.2. *As ϵ converges to zero, in any sequential equilibrium of the discrete-time bargaining problem $G(g_\epsilon)$ in stage two after any history h_t such that the buyer is in store $i \in \{1, 2\}$ and known to be rational while both sellers are not known to be rational, the payoff to the buyer is no more than $1 - \alpha + \epsilon$ and the payoff to the seller i is no less than $\alpha - \epsilon$ (payoffs are evaluated at time t).*

Proof. Without loss of generality, suppose that the buyer is in store 1 at time t after the history h_t. I will show that as seller 1 continues to mimic the obstinate type, the payoff to the buyer converges to $1 - \alpha$ and the payoff to seller 1 converges to α, as ϵ converges to zero. For the remainder of this proof, assume that seller 1 continues to mimic the obstinate type, while the buyer and seller 2 execute their equilibrium strategies.

For $i \in \{1, 2\}$, let z_i^t denote the probability that seller i is the obstinate type at time t after the history h_t. By Bayes' rule, z_i^t is either zero or higher than z_s. By our assumption, however, for each i, we must have $z_i^t \geq z_s$.

If the buyer continues to stay in store 1 for long enough according to his equilibrium strategy while seller 1 continues to act irrationally, we know by proposition A.1 that the payoff to the buyer converges to $1 - \alpha$ as ϵ converges to zero, and this proves the claim of the proposition. It is, however, possible that in equilibrium the buyer does not stay in store 1 long enough if seller 1 continues to mimic the obstinate type. This implies that the buyer leaves store 1 at some time $t' \geq t$. Note that $z_s^t = z_s^{t'}$.

The buyer's decision of leaving store 1 at time t' implies that $z_s^t \leq \frac{\delta + \alpha - 1}{\delta \alpha} = \hat{\rho} < 1$ This is true because, if the buyer goes to store 2 and seeks an agreement with seller 2, the highest payoff he could achieve is $\delta[1 - z_s^t + (1 - \alpha) z_s^t]$. But leaving store 1 and going to store 2 at time t' is optimal for the buyer only if $1 - \alpha \leq \delta[1 - z_s^t + (1 - \alpha) z_s^t]$ which implies the desired result.

According to his strategy, if the buyer continues to stay in store 2 long enough, conditional on seller 2 mimicking the obstinate type, we know again by Proposition A.1 that the payoff to the buyer converges to $1 - \alpha$ as ϵ converges to zero. This implies that $1 - \alpha$ is the highest payoff the buyer can attain in store 2. If this is the case, however, the buyer does not leave store 1 at time t', which contradicts our supposition. Therefore, it must be the case that the buyer leaves

[42] Receiving α at time t^* is equivalent to receiving $\alpha e^{-r_s(t^* - t)}$ at time t, which is arbitrarily close to α as ϵ converges to 0.

store 2 as well, conditional on seller 2 continuing to mimic the obstinate type, at some time t'' where $t'' > t'$.

According to his equilibrium strategy, seller 2 may be playing a strategy that ends the game while the buyer is in store 2. However, according to Lemma A.5, we know that seller 2 will not play a strategy that will end the game with a price less than α (in the limit) with probability one. If rational seller 2 is playing a strategy which ends the game with a price higher than α, then he buyer does not leave store 1 at time t', which contradicts our supposition. Therefore, it must be the case that seller 2 is playing a strategy that extends the game, i.e. seller 2 will mimic the obstinate type with a positive probability, until time t''.

Conditional on the buyer arriving to store 1 once more, the same arguments show that the buyer shall leave store 1 once again as seller 1 continues to mimic the obstinate type (because otherwise, the payoff to the buyer will be at most $1 - \alpha$ and this contradicts our supposition that the buyer leaves store 2 when seller 2 continues to mimic the obstinate type).

Therefore, conditional on both sellers extending the game and the buyer leaving store 1 twice, we have $z_s^t \leq \hat{\rho}^2$, so that extending the game by going back and forth between the sellers (twice) is more profitable for the buyer than seeking an immediate agreement with irrationally behaving seller 1. Similarly, for the game that lasts until the k^{th} departure of the buyer, it must be true that $z_s^t \leq \hat{\rho}^k$. Choosing k such that $\hat{\rho}^k < z_s$ establishes contradiction since, as argued earlier, $z_s^t \geq z_s$.

Therefore, as seller 1 continues to mimic the obstinate type, seller 2 will continue to play a strategy which extends the game with positive probability (immediate consequence of Lemma A.5). The buyer, however, will travel back and forth between the sellers only for some finite time in order to get a deal better than α. This implies that the buyer will end up at some store $i \in \{1, 2\}$ at some finite time $\bar{t}$. That is, the buyer does not leave store i after time $\bar{t}$ while seller i continues to mimic the obstinate type.

This implies that the buyer's continuation payoff in store i is at most $1 - \alpha$, evaluated at time $\bar{t}$. This leads to a contradiction because, given that the buyer's continuation payoff in his final destination is less than $1 - \alpha$, the buyer should not have left store j when seller j continues to act irrationally. Hence, repeating this argument backward, we can conclude that the buyer does not delay the game, but instead seeks an immediate agreement with seller 1 at time t. $\square$

Now, let σ_ϵ denote a sequential equilibrium of the discrete-time bargaining problem $\mathrm{G}(g_\epsilon)$ in stage 2. Denote by σ^0 the buyer's strategy on store selection in stage 1.[43] Given σ^0, the random outcome corresponding to σ_ϵ is a random object $\theta_\epsilon(\sigma^0)$ which denotes any realization of an agreed division as well as a time and store at which agreement is reached.

The final result in this section shows that in the limit as ϵ converges to zero $\theta_\epsilon(\sigma^0) \to \theta(\sigma^0)$ in distribution, where $\theta(\sigma^0)$ is the unique equilibrium distribution of the game G (given the buyer's initial choice of seller σ^0). Therefore, the second stage outcomes of the discrete-time

[43] It is a probability measure on two pure actions; seller 1 and seller 2.

bargaining problem, independent of the bargaining protocol g_ϵ, converge in distribution to the unique equilibrium outcome of the continuous-time bargaining problem analyzed in Section 3.

Proposition A.3. *As ϵ converges to 0, $\theta_\epsilon(\sigma^0)$ converges in distribution to $\theta(\sigma^0)$.*

Proof. This proof is adapted from the proof of Proposition 4 in Abreu and Gul (2000). Let $\mathrm{G}(g_{\epsilon_n})$ be a sequence of (second stage) discrete-time bargaining problems and σ^n (drop the term ϵ to ease the notation) be the corresponding sequence of sequential equilibria. For each σ^n, let σ_b^n denote the buyer's strategy that determines his location at any given history. Then, given a history h_t, define $S_i^{\sigma_b^n}$ and $I^{\sigma_b^n}$ as in the proof of Proposition 3.1.[44] Then, for each $i \in \{1,2\}$ and $T \in I_i^{\sigma_b^n}$ with $[T,T'] \in S_i^{\sigma_b^n}$ define $F_n^{i,T} : [T,T'] \to [0,1]$, where $F_n^{i,T}(t)$ is the cumulative probability that seller i takes an action not consistent with the obstinate type in the interval $[T,t]$, conditional on the buyer and the other seller having acted like a obstinate type until time t.

Similarly, define $F_n^{b,i,T} : [T,T'] \to [0,1]$ where $F_n^{b,i,T}(t)$ is the cumulative probability that the buyer takes an action not consistent with the obstinate type in the interval $[T,t]$, conditional on the buyer is in store i in this time interval according to σ_b^n and both sellers having acted as if they are obstinate until time t.

To prove the Proposition, arbitrarily choose some $\bar{n} \geq 0$, an equilibrium strategy $\sigma^{\bar{n}}$ and a history h_t to fix the sets $S_i^{\sigma_b^{\bar{n}}}$ and $I^{\sigma_b^{\bar{n}}}$. Then, for each $i \in \{1,2\}$ and $T \in I_i^{\sigma_b^{\bar{n}}}$ with $[T,T'] \in S_i^{\sigma_b^{\bar{n}}}$, I show that as $n \geq \bar{n}$,

Step (1) Every subsequence of $F_n^{i,T}$ and $F_n^{b,i,T}$ have a convergent subsequence: Similar to Steps 1 and 2 in the proof of Proposition 4 in Abreu and Gul (2000), for any $T \in I_i^{\sigma_b^{\bar{n}}}$ and $[T,T'] \in S_i^{\sigma_b^{\bar{n}}}$ where $T < T^*$ (the time that the bargaining game finishes), define G_n^i such that

$$G_n^{i,T}(t) = \frac{F_n^{i,T}(t)}{F_n^{i,T}(T')} \quad \text{whenever} \quad F_n^{i,T}(T') \neq 0$$

for all $t \leq T'$ where $F_n^{i,T}(T') = 1 - z_i^T / z_i^{T'}$. Note that $\{F_n^{i,T}(T')\}$ is a bounded real sequence, which is bounded below by 0 and above by $1 - z_s$ for all n. The same arguments hold for the buyer. Moreover, by Helly's selection Theorem (See Billingsley (1986)), the sequence $G_n^{i,T}$ has a subsequence $G_{n_k}^{i,T}$ which converges to a right continuous, non-decreasing function G_i^T at every continuity point of G_i^T. Let $F_{n_k}^{i,T} = F_{n_k}^{i,T}(T')G_{n_k}^{i,T}$. Since the real sequence $F_{n_k}^{i,T}(T')$ is bounded below 0 and bounded above $1 - z_s$ for any n, there must exist a subsequence $F_{n_{k_j}}^{i,T}(T')$ which converges to some real number $F_i^T(T')$. Therefore, $F_{n_{k_j}}^{i,T} = F_{n_{k_j}}^{i,T}(T')G_{n_{k_j}}^{i,T}$ implies that $F_i^T = F_i^T(T')G_i^T$. Apply the same arguments to the buyer and renumber the sequence n_{k_j} will yield the desired result.

Step (2) the limit points of $(F_n^{i,T}, F_n^{b,i,T})$ do not have common points of discontinuity in the domain $[T,T']$; The proofs of this claim utilizes the exact methods used in the proof of steps 3-6

[44] For the sake of simplicity, I manipulate the notation and denote the continuation strategy following the history h_t by σ_b^n.

of Proposition in Abreu and Gul (2000). Therefore, I do not represent the proof here to prevent duplication.

Step (3) if $(F_n^{i,T}, F_n^{b,i,T})$ converges to $(F_i^T, F_b^{i,T})$ and if the limit functions do not have common points of discontinuity then $(F_i^T, F_b^{i,T})$ is an equilibrium of the continuous-time bargaining problem in the interval $[T, T']$.

The following arguments prove Step 3 and complete the proof of the Proposition A.3. Recall that σ^n is the equilibrium strategy of the game g_{ϵ_n}. For any $t > 0$ and $\hat{\epsilon} > 0$ define a strategy $\tilde{\sigma}_n^i$ to be a strategy of seller i within the interval $[T, T']$ as follows: Seller i behaves according to σ_n^i until time t_n where t_n is the last time the buyer makes an offer prior to $t + \bar{\epsilon}$ (for some $\bar{\epsilon} > 0$) and at time t_n seller i accepts the buyer's offer α_b. Let U_n^i denote the utility function of seller i in the game g_{ϵ_n}. Then there exist finite integers N_1, N_2, N_3 and $\bar{\epsilon} > 0$ sufficiently close to 0, such that $t + \bar{\epsilon}$ is a continuity point of $F_b^{i,T}$ and

$$U^i(t, F_b^{i,T}) - U^i(t + \bar{\epsilon}, F_b^{i,T}) < \hat{\epsilon}, \tag{8}$$

$$U^i(t + \bar{\epsilon}, F_b^{i,T}) - U^i(t_n, F_n^{b,i,T}) < \hat{\epsilon} \quad \forall n \geq N_1, \tag{9}$$

$$U^i(t_n, F_n^{b,i,T}) - U^i(\tilde{\sigma}_n^i, \sigma_n^b) < \hat{\epsilon} \quad \forall n \geq N_2, \tag{10}$$

$$U_n^i(\tilde{\sigma}_n^i, \sigma_n^b) - U_n^i(\sigma_n^i, \sigma_n^b) \leq 0 \qquad \forall n, \tag{11}$$

$$U_n^i(\sigma_n^i, \sigma_n^b) - U_n^i(F_n^{i,T}, F_n^{b,i,T}) < \hat{\epsilon} \quad \forall n \geq N_2 \tag{12}$$

$$U^i(F_n^{i,T}, F_n^{b,i,T}) - U^i(F_i^T, F_b^{i,T}) < \hat{\epsilon} \quad \forall n \geq N_3. \tag{13}$$

Equation (8) follows immediately from the definition of U^i. That is, $U^i(., F_b^{i,T})$ is continuous at continuity points of $F_b^{i,T}$. If t is not a continuity point of $F_b^{i,T}$, for $\bar{\epsilon}$ small enough the left-hand side of (8) is strictly negative (similar logic to the proof of step 3 of Abreu and Gul (2000): if the buyer makes a mass acceptance at time t, seller i would prefer conceding at time $t + \bar{\epsilon}$ over conceding at time t). Since $t + \bar{\epsilon}$ is a continuity point of $F_b^{i,T}$, (9) follows from Step 6 of Abreu and Gul (2000). Equation (10) follows from the definition of $\tilde{\sigma}_n^i$ and Proposition A.2. Equation(11) is the consequence of the fact that (σ_n^i, σ_n^b) is equilibrium. Equation (12) is an application of Proposition A.1; seller i can never get more than α_b after revealing his rationality. Moreover, in equilibrium, since his opponent makes offers frequently, he can reveal himself to be rational in a manner that guarantees α_b. Equation (13) follows from Steps 3-6 of Abreu and Gul (2000).

Choosing $n \geq \max\{N_1, N_2, N_3\}$ and adding Equations (8)-(13) will yield

$$U^i(t, F^{b,i,T}) - U^i(F^{i,T}, F^{b,i,T}) < 5\hat{\epsilon}$$

Since this inequality is true for any $\hat{\epsilon} > 0$, it must be the case that

$$U^i(t, F^{b,i,T}) - U^i(F^{i,T}, F^{b,i,T}) \leq 0.$$

Hence, F_i^T is a best response to $F_b^{i,T}$. Symmetric arguments imply that $(F_i^T, F_b^{i,T})$ is a Nash equilibrium of the continuous-time game within the interval $[T, T']$. Note that if seller i is the first to reveal his type, he can guarantee α_b by accepting the buyer's offer. This would yield the buyer a payoff of $1-\alpha_b$. If seller i reveals his type in some other way, then by Proposition A.1 he is still, in the limit, guaranteed α_b. This happens only if agreement is reached immediately at these terms. Analogous arguments are valid for the buyer. Therefore, convergence in expected payoffs implies convergence in distribution within the interval $[T,T']$.

After an arbitrary history h_t and continuation strategy $\sigma_b^{\bar{n}}$, I proved the convergence in each interval $[T,T'] \in S_i^{\sigma_b^{\bar{n}}}$. So, for given $\sigma_b^{\bar{n}}$ let $\mathbf{F}_n$, the distribution function profile of the discrete-time bargaining problem $\mathrm{G}(g_{\epsilon_n})$, converge to $\mathbf{F}$, the distribution function profile of the continuous-time bargaining problem G, history by history (i.e., interval by interval) in the product topology.

Given that $\mathbf{F}_n$ converges to $\mathbf{F}$ history by history, similar arguments in the proof of Proposition 3.1 suffice to show that for sufficiently large n, the buyer visits each store at most once according to the equilibrium strategy of the game $\mathrm{G}(g_{\epsilon_n})$. As a result, convergence in distribution in all subgames implies that the buyer's timing and location decisions together with the distribution functions, $\mathbf{F}_n$, converge to the unique equilibrium of the continuous-time problem (given the buyer's store selection in stage 1). $\square$

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