Problem 1 (8+6 pts).

a. For an integer $n \geq 1$, show that
\[
\binom{2n+2}{n+1} = 2 \left( \binom{2n}{n+1} + \binom{2n}{n} \right)
\]

b. Evaluate the sum
\[
\sum_{k=0}^{n} 3^k \binom{n}{k}.
\]

Problem 2 (2 pts each). Using the numbers 1, 2, \ldots, 9, how many vectors of length 6 can be formed in each of the following cases?

a. no even number in the vector,

b. no even number in the vector and numbers cannot be repeated,

c. exactly 2 even numbers in the vector.