Problem 1 (6+6+2 pts).

a. For integers \(1 \leq k \leq n\), show that
\[
\binom{n+2}{k+1} - 2\binom{n+1}{k+1} + \binom{n}{k+1} = \binom{n}{k-1}
\]

b. What is the coefficient of \(x^4y^3\) in the expansion of \((3x + 2y)^7\)?

c. What is the coefficient of \(x^3y^2\) in the expansion of \((3x + 2y)^7\)?

Problem 2 (2 pts each). Using the numbers 1, 2, \ldots, 9, how many vectors of length 6 can be formed in each of the following cases?

a. vector starts with an even number and ends with an odd number,

b. vector starts with 5 or 7,

c. vector starts with an even number or ends with an odd number.