The line and the translate problems for r-primitive elements

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Let q be a prime power and $n \geq 2$ an integer. We denote by $\mathbb{F}_q$ the finite field of q elements and by $\mathbb{F}_{q^n}$ its extension of degree n. An element of $\mathbb{F}_{q^n}^*$ of order $(q^n - 1)/r$, where $r \mid q^n - 1$, is called *r-primitive*, while, if $r = 1$, we simply call it *primitive*.

If θ is a *generator* of the extension $\mathbb{F}_{q^n}/\mathbb{F}_q$, i.e., is such that $\mathbb{F}_{q^n} = \mathbb{F}_q(\theta)$, then

$$\mathcal{T}_\theta := \{\theta + x \,:\, x \in \mathbb{F}_q\}$$

is the *set of translates* of θ over $\mathbb{F}_q$ and, if $\alpha \in \mathbb{F}_{q^n}^*$,

$$\mathcal{L}_{\alpha,\theta} := \{\alpha(\theta + x) \,:\, x \in \mathbb{F}_q\}$$

is the *line* of α and θ over $\mathbb{F}_q$. It is known that, given n, if q is large enough, every set of translates and every line contain a primitive element, while effective versions for these existence results are known for just a few small values of n.

In this work, we extend these existence results to r-primitive elements and we provide effective results for the case $r = n = 2$.

This work is still in progress and is in collaboration with Stephen D. Cohen.