Randomness in Complex Geometry & Complex Analysis Workshop

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Rank one perturbations of hermitian random matrices

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Dumitriu and Edelman (2002) provided tridiagonal random models to compute eigenvalue distribution for Gaussian and Laguerre ensembles. By generalizing this method, Kozhan (2017) was able to find tridiagonal models corresponding to rank one additive non-hermitian perturbations of Gaussian and Wishart ensembles and computed the eigenvalue distribution for the mentioned random matrices.

I will survey the above results and discuss the recent results (joint work with Kozhan (2022)) on the eigenvalue distribution of rank one Hermitian and non-Hermitian perturbations of Chiral random matrices. I will also report new results on rank one multiplicative non-Hermitian perturbations of the ensembles mentioned above.

Optimal measures, Fekete points and polynomial interpolation

Len Bos
University of Verona

Suppose that $K \subset \mathbb{C}^d$ is a compact set. Fekete points of degree n are those that maximize the modulus of the polynomial Vandermonde determinant of degree n over points in K and are known to be an excellent set of points for the purposes of polynomial interpolation. An optimal measure of degree n for K is a probability measure for which maximum of the diagonal of the polynomial reproducing is as small as possible, i.e., minimize variance for polynomial least squares estimation. We will give a survey of the relationships between these two problems, give some examples, and discuss some conjectures and open problems.

Percolation, long-range correlations and critical exponents on transient graphs

Alexander Drewitz
University of Cologne

Percolation models have been playing a fundamental role in statistical physics for several decades by now. They had initially been investigated in the gelation of polymers during the 1940s by chemistry Nobel laureate Flory and Stockmayer. From a mathematical point of view, the birth of percolation theory was the introduction of Bernoulli percolation by Broadbent and Hammersley in 1957, motivated by research on gas masks for coal miners. One of the key features of this model is the inherent stochastic independence which simplifies its investigation, and which has lead to very deep mathematical results. During recent years, the investigation of the more realistic and at the same time more complex situation of percolation models with long-range correlations has attracted significant attention.

We will exhibit some recent progress for the Gaussian free field with a particular focus on the understanding of the critical parameters in the associated percolation models. What is more, we also survey recent progress in the understanding of the model at criticality via its critical exponents as well as the universality in the local geometry of the underlying graph.

On dynamics of asymptotically minimal polynomials

Melike Efe
Sabancı University

In this talk, we will focus on dynamical properties of asymptotically minimal polynomials associated with a non-polar planer compact set E. In particular, we shall prove that if the zeros of such polynomials are uniformly bounded then their Brolin measures converge weakly to the equilibrium measure of E. In addition, if E is regular and the zeros of such polynomials are sufficiently close to E then we prove that the filled Julia sets converge to polynomial convex hull of E in the Klimek topology. The talk is based on joint work with T. Bayraktar.

On amoebas of random plane curves

Ali Ulaş Özgür Kişisel
Middle East Technical University

Due to a Theorem of Passare and Rullgard, the area of the amoeba of a degree d algebraic curve in the complex projective plane is bounded above by $\frac{\pi^2 d^2}{2}$ and the curves attaining the bound - special Harnack curves - have been characterized by Mikhalkin. In this talk, reporting on joint work with Turgay Bayraktar, I will argue that the expected area of a randomly chosen complex algebraic curve, with respect to the Kostlan distribution, is bounded above by a constant times d. This result also generalizes in a natural way to half dimensional complete intersections in toric varieties with an arbitrary Newton polytope.

Bergman kernels and Poincare series

George Marinescu
University of Cologne

We show that the Bergman kernel of sections of a positive line bundle has off-diagonal exponential decay on a complete Kähler manifold with bounded geometry. As a consequence, the Bergman kernel on a compact Kähler manifold equals the Poincaré series of the Bergman kernel of L^2 holomorphic sections on a Galois covering.

Zeros of random elliptic polynomials: condition number and extremal energies

Jordi Marzo
University of Barcelona

Elliptic polynomials appeared in the mathematical physics literature in the 90's. Among its many interesting properties, especially relevant are the connections, revealed by Shub and Smale in 1993, with well conditioned polynomials and with minimal logarithmic energy points on the sphere. We will explain these connections and some of our recent related work.

Convergence theorems and asymptotically minimal polynomials

Bela Nagy
University of Szeged

In this talk, first, we recall some basic notions from logarithmic potential theory including energy, equilibrium measure, capacity, the principle of descent, and the lower envelope theorem. Then we discuss some examples about capacity and convergence. Moreover, we will see theorems of the last century (Wiener) and this century (Bloom-Levenberg, Piazzon, Kalmykov-Kovalev, Binder-Rojas-Yampolsky, and Totik) how the convergence of sets and convergence of equilibrium measures are connected. Then we apply this framework to investigate special, Bernstein-type extremal problems for polynomials and relate them with asymptotically minimal polynomials.

On optimal prediction measures

Franck Wielonsky
Aix-Marseille University

The aim of the talk will be to review some properties of optimal prediction measures (OPM), a notion related to optimal designs in statistics. The study of OPM's is connected with many classical notions in approximation and potential theory, such as the Bergman kernel, the Christoffel function, polynomials of extremal growth, Faber polynomials, the Szegö function and balayage of measures. We will try to describe some of these connections and give some hints on the asymptotic behavior of OPM's.

Geometry of Upper Level Sets of Lelong Numbers

Özcan Yazıcı
Middle East Technical University

Let T be a positive closed current of bidimension $(1,1)$ with unit mass on $\mathbb{P}^2$ and $\nu(T,x)$ denote the Lelong number of T at x. It is a remarkable result of Y.T. Siu that the upper level sets

$$E_\alpha(T) := \{x \in \mathbb{P}^2 | \nu(T,x) \geq \alpha\}$$

of Lelong numbers are analytic subvarieties of $\mathbb{P}^2$ of dimension at most 1 for any $\alpha > 0$. Later, it was shown that

$$V_\alpha(T) := \{x \in \mathbb{P}^2 | \nu(T,x) > \alpha\}$$

has certain geometric properties when α is large enough. In this talk, first we will go over these results. Then we will focus on our recent result with A.U.Özgür Kişisel that $|V_\alpha(T) \setminus C| \leq 2$ for some cubic curve when $\alpha \geq \frac{1}{3}$.
