Bargaining, Reputation and Competition*

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Abstract

This paper aims to understand whether or not reputation as tough/obstinate bargainer helps negotiators in competitive environments, where bargaining and searching for a better deal are the key elements. The analyses show that negotiators that have virtually no market power may possess significant share if they have the chance to build reputation for obstinacy. Somewhat surprisingly, having higher reputation for being a tough negotiator is bad for competing negotiators.

Keywords: Commitment, Obstinacy, Behavioral types, Heterogeneous priors, Overconfidence.

1. Introduction

Negotiators often use various bargaining tactics that are likely to lead inefficient outcomes. One example of such tactics is standing firm and not backing down from the last offer. One factor making this tactic effective is the negotiators’ uncertainty regarding the adversaries’ commitment (Schelling 1960 and Arrow et al. 1995). An entrepreneur (i.e., the seller) who aims to sell part of his growing and successful business in return for financing may simultaneously or sequentially negotiate with more than one business angel (i.e., the buyers), and depending on how the events unfold, each negotiator may follow a hard-line policy after some point of the negotiation. An investor may refuse to make further concessions after he discovers another, equally attractive investment opportunity. The entrepreneur, on the other hand, may show unwillingness towards additional concessions after receiving some favorable news regarding his

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loan or grant applications. A tough bargainer attempts to convince the other parties that he cannot change his offer and make further concessions because he is committed to a particular minimum position (Tedeschi et al. 1973). Likewise, a rational negotiator could mimic a tough bargainer and use similar arguments to make his obstinacy credible.

The growing literature on bargaining and reputation shows that in a bilateral negotiation, this tactic—playing a tough bargainer—benefits the party who has higher reputation for obstinacy (Myerson 1991, Kambe 1999, Abreu and Gul 2000, Compte and Jehiel, 2002). A negotiator’s reputation for obstinacy is the probability—attached to this negotiator by his opponents—of being the obstinate/commitment type. This paper investigates if this tactic will benefit a negotiator in a competitive environment, where searching and bargaining for a better deal are the key aspects. The analyses provide an affirmative answer when the competing negotiators are slightly “overconfident” about being a flexible negotiator. Somewhat surprisingly, having higher reputation for being the tough negotiator does not help a competing negotiator.

To examine this question, I construct a stylized model where the buyers, the long side, have virtually no market power since they compete in a Bertrand fashion. There are three defining features of the model. First, a single seller negotiates with two buyers over the sale of one item. Second, the buyers make initial posted-price offers simultaneously. The seller can accept one of these offers costlessly, or else visit one of the buyers and try to bargain for a higher price. Third, each of the three players suspects that his opponents might have some kind of commitment forcing them to insist on their initial offers. That is, each player would be a commitment type with some positive probability, providing incentive to the flexible (or noncommittal) players to build reputation for obstinacy. Obstinate types take an extremely simple form. Parallel to Myerson (1991) and Abreu and Gul (2000) an obstinate player always demands a fixed share and accepts an offer if and only if it weakly exceeds that share.

The main results of the paper are provided in Section 3. In an environment where the posterior beliefs about the negotiators’ types are common, the role of the players’ reputational concerns has no impact. That is, if the common prior assumption holds, then regardless of the size of these prior beliefs, the unique (sequential) equilibrium outcome is the Walrasian outcome, where the buyers post the monopoly price and the seller gets the whole surplus. However, even with a slightest perturbation of the common prior assumption (i.e. a small heterogeneity/divergence in prior beliefs), a large set of prices that are different from the monopoly price (i.e., non-Walrasian prices) can be supported in equilibrium. Therefore, playing the tough bargainer is a useful tactic for the buyers, who virtually have no market power due to competition, if each buyer believes that he is more flexible or less obstinate than the other buyer. Furthermore, we can support non-Walrasian prices even if the players’ reputational concerns are vanishingly small. It is not new in the literature that the standard conclusions would fall apart when common prior assumption is eliminated. However, what is new in this paper is that even if the
divergence in initial priors vanishes, then we still can support non-Walrasian prices, and this is largely due to the power of negotiators’ reputational concerns.

Section 2 explains the details of the three-stage, continuous-time, competitive (multilateral) bargaining game. Section 3 provides the main results of this paper. The first part of Section 3 presents a benchmark result: one-buyer-one-seller case that directly follows from Abreu and Gul (2000). The second part of Section 3 presents the secondary main result: If the prior probabilities of the players being obstinate are common, then the monopoly pricing is the unique equilibrium outcome of the game. The third part of this section provides the primary main results: A divergence in players’ prior beliefs leads to multiple equilibrium in which the buyers may possess significant (ex-ante) surplus, and this multiplicity is still the case when the heterogeneity of the initial priors are vanishingly small. As a technical part, Section 4 characterizes the equilibrium strategies of the third stage of the game, which are essential to prove the results of Section 3. All proofs are deferred to the Appendix.

The Related Literature

Shelling (1960) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996) and Ellingson and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if revoking it is costly. Kreps and Wilson (1982) and Milgrom and Roberts (1982) model commitment as behavioral types that exist in the society so that the rational players can mimic if they would like to do so. The approach I use in this paper is consistent with these two approaches: A negotiator may be forced to commit, possibly because his cost of revoking is very high, or he may choose to mimic the commitment type, so that he can build reputation for obstinacy.

This paper is directly related to the reputation and bargaining literature initiated by Myerson (1991). Myerson investigates the impacts of one-sided reputation building on bilateral negotiations. Abreu and Gul (2000), Kambe (1999), and Compte and Jehiel (2002) consider two-sided versions of it. Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in an Abreu-Gul setting and explore the role of exogenous outside options. They show that if both agents’ outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility, and the unique equilibrium is again the Rubinstein outcome. The work of Atakan and Ekmekci (2010) is more related to this paper as they study a market environment with multiple players. However, their main focus is substantially different. They show—in a market with large numbers of buyers and sellers—that the existence of commitment types and endogenous outside options provide enough incentive for the rational

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players to build reputation for obstinacy. On the other hand, in this paper, I aim to answer how reputational concerns affect the market participants’ pricing and search decisions.

Ozyurt (2014) is the most related paper to the current one. Although the two papers work with different underlying principles, both papers show that negotiators’ reputational concerns may provide them significant market power in a highly competitive environment. Ozyurt (2014) interprets obstinate types as types that are born with their demands. Given this interpretation, if negotiator \( i \) is rational and demanding \( \alpha_i \in [0, 1] \), then this is his strategic choice. If he is an obstinate type, then he merely declares the demand corresponding to his type. However, this interpretation leads to a substantially complex model in our setting when there is a large (in particular dense) set of types. In that regard, the current paper follow an approach leading to a fairly simpler model to work with, where a negotiator’s initial probability of being the obstinate type is independent of the demand the negotiators announce at the beginning of the game.

Ozyurt (2014) proves that deemed to be a tough bargainer (even if it is a less-greedy one) is bad for the competing negotiators. Being perceived as an obstinate buyer reduces the chance that his offer will be accepted by the seller because the rational seller prefers to visit first the buyer who is likely to be rational, and this restrains a rational buyer from overbidding his competitor. This observation contrasts with the predictions of the bilateral bargaining models of Kambe (1999), Abreu and Gul (2000), and Compte and Jehiel (2002). In their models, being perceived as an obstinate type is immediately followed by a concession from the rational opponent. This paper strengthens the result of Ozyurt (2014) by showing that having a higher reputation for obstinacy does not help a competing negotiator either.

Two other works that are closely related to the current one are Kambe (1999) and Abreu and Gul (2000). They consider a bilateral bargaining problem and demonstrate that if the players have “sufficiently high” reputational concerns\(^1\), then the outcome of the bilateral game would be very different than the alternating offer bargaining outcome of Rubinstein (1982). Interestingly, both of these papers show that in the limit where the players’ reputational concerns vanish, the equilibrium outcome converges to a unique outcome of Rubinstein (1982), where rational players choose to be virtually compatible and share the surplus in proportion to their impatience. This suggests that playing the tough bargainer would benefit a rational player only if his opponent has sufficiently high perception of him being obstinate (or high reputational concerns). Otherwise, players’ shares purely depend on their impatience, and so there is no point in using this tactic.

Building on those results, this paper provides an important example indicating that in a competitive environment, or in a case where the seller has endogenous outside option, the buyers (the long side of the market) can benefit from playing the tough bargainer tactic even if the players’ reputational concerns are minimal. This is true because Bertrand-like competition washes away the buyers’ bargaining power totally and the seller’s reputational concern—whether

\(^1\)That is, the prior probabilities that the buyers and the seller are obstinate types are sufficiently high.
it is large or small—can restore the buyers’ ability of getting a significant surplus. However, in a bilateral bargaining case, the negotiators already have a power, proportional to their impatience, and so, they do not bother to build reputation when these concerns are small. Therefore, we can conclude that we should expect more bargainers using the playing the tough bargainer tactic in competitive environments than in bilateral negotiations.

Wolitzky (2012) studies the effects of reputation on bilateral negotiations. However, his work differs from the rest of the bargaining and reputation literature, including this paper, in two fundamental points. First, Wolitzky allows bargainers to commit to a sequence of offers (posture), that is possibly changing by time, whereas commitment types in this paper always ask the same demand. Second, Wolitzky uses a weaker solution concept (first-order knowledge of rationality) rather than sequential equilibrium, and so characterizes the players' minmax payoffs. In Wolitzky (2012) the prior probability that a player will commit, after announcing his posture, is not necessarily common knowledge. A player only knows that his opponent will commit to her posture with some probability no less than $\epsilon > 0$. Given the value of $\epsilon$, Wolitzky characterizes the minimum payoff that players can achieve in the negotiation. This approach is fundamentally different from the one followed in this paper. First, as I mentioned earlier, I use sequential equilibrium as the solution concept. Second, I retain the common knowledge assumption over the initial priors and allow heterogeneous priors. Broadly speaking, the analysis in Wolitzky is closer to the analysis in, for example, Fudenberg and Levine (1989) and Battigalli and Watson (1997), whereas the approach that I use in this paper resembles the approach used in Yildiz (2003) and Feinberg and Skrzypacz (2005). Nevertheless, similar to Kambe (1999) and Abreu and Gul (2000), Wolitzky (2012) also concludes that the benefits of reputation vanish as the players’ reputational concerns vanish.

2. The Competitive (Multilateral) Bargaining Game

Here I define the continuous time, competitive multilateral bargaining game $G$.

The Players: There is a single seller having an indivisible good and two buyers who want to consume this good. The valuation of the good is 1 for the buyers and 0 for the seller. The rate of time preferences of the buyers and the seller are $r_b$ and $r_s$, respectively. All of this information is common knowledge among all three players.

The Timing of the Game: The bargaining game between the seller and the two buyers is a three-stage, infinite horizon, continuous-time game. Stage 1 starts and ends at time 0 and the timing within the first stage is as follows. Initially, the seller announces (posts) a demand (price) $\alpha_s$ from the set $[0, 1]$ and it is publicly observable. After observing the seller’s demand,
the buyers simultaneously announce their demands, \( \alpha_i \) for buyer \( i = 1, 2 \), from the set \([0, 1]\). The game finishes at this point by the seller’s acceptance of the highest offer if \( \max\{\alpha_1, \alpha_2\} \geq \alpha_s \). In case both buyers offer \( \alpha \) where \( \alpha \geq \alpha_s \), then the seller accepts each buyer’s offer with equal probabilities. However, if \( \alpha_s > \max\{\alpha_1, \alpha_2\} \), then the seller selects one of the buyers to visit and to negotiate up the price.

A player knows that he will never be forced to commit to his initial demand, but is uncertain about the other players. Therefore, each player believes that nature sends one of two messages \( \{c, d\} \) to his opponents in stage 2. A player who receives the message \( c \) “commit” is constrained to reject all shares that are less than what he initially claimed for himself. If a player receives the message \( d \) “don’t commit”, he will continue to play the game with no commitment to his initial share. The players share the same belief that the buyers and the seller receive the message \( c \) with probability \( z_b \) and \( z_s \), respectively, where \( z_b, z_s \in (0, 1) \).

Upon the beginning of the third stage (still at time 0) the seller and buyer \( i \), who is visited by the seller first, immediately begin to play the following bargaining game (i.e., the concession game): At any given time, a player either accepts his opponent’s initial demand or waits for a concession. At the same time, the seller decides whether to stay or leave buyer \( i \). If the seller leaves buyer \( i \) and goes to buyer \( j \in \{1, 2\} \) with \( j \neq i \), the seller and buyer \( j \) start playing the concession game upon the seller’s arrival. Concession of the seller or buyer \( i \), while the seller is with buyer \( i \), marks the completion of the game. If the agreement \( \alpha \in \{\alpha_s, \alpha_i\} \) is reached at time \( t \), then the payoffs to the seller, buyers \( i \) and \( j \) are \( \alpha e^{-r_s t}, (1 - \alpha) e^{-r_b t} \) and 0, respectively. In case of simultaneous concession, surplus is split equally.\(^2\)

Finally, one may wish to picture this haggling game as if it occurs in an environment where the buyers are located at opposite ends of a town. Therefore, changing the bargaining partner is costly for the seller because it takes time to move from one buyer to the other and the seller discounts time. Thus, supposing that the buyers are spatially separated, let \( \delta \) denote the discount factor for the seller that occurs due to the time, \( \Delta > 0 \), required to travel from one buyer to the other. That is, \( \delta = e^{-r_s \Delta} \). Note that \( 1 - \delta \) (the search friction) is the cost that the seller incurs at each time he switches his bargaining partner. It is equally acceptable to assume that the switching cost for the seller is independent of the “travel time” \( \Delta \). However, incorporating the search friction in this manner simplifies the equilibrium calculations substantially. I assume that the search friction is very small, and thus \( \delta \) is very close to 1. Although the search friction is important in equilibrium characterizations, the main results of this paper, which I present in Section 3, are independent of its value. \( I \) denote this three-stage bargaining game by \( G \).

A critical assumption of the model deserves explicit clarification here. The model adopts a war of attrition protocol in stage 3, disallowing counteroffers and permitting buyers only two

\(^2\)In this case, the seller’s and buyer \( i \)’s shares are \( \frac{\alpha_i + \alpha_s}{2} \) and \( 1 - \frac{\alpha_i + \alpha_s}{2} \), respectively. This particular assumption is not crucial because simultaneous concession occurs with probability 0 in equilibrium.
choices: concede or wait. The third stage of the game G is indeed a modified war of attrition game and it is partially justified by Ozyurt (2014), Atakan and Ekmekci (2010), Abreu and Gul (2000), Myerson (1991), and Abreu and Pearce (2007). Alternatively, for example, we could suppose that players can modify their offers at times \{1, 2, \ldots\} in alternating orders, but can concede to an outstanding demand at any \(t \in [0, \infty)\). But, modifying his offer would reveal a player’s type, and under certain conditions (such as, initial demand announcements of the buyers are the same), in the unique equilibrium of the continuation game he should concede to the opponent’s demand immediately (see Ozyurt 2014). That is, the third stage of the game G is outcome equivalent to a large class of (alternating offer) multilateral bargaining games when the buyers announce the same demand in stage 1.

**The Information Structure and Some Details on Obstinate Types:** The only source of uncertainty is the players’ actual types, which matters only in the third stage of the game. In the first stage, all players are flexible (rational)—in the sense that they choose their strategies, given their beliefs, to maximize their expected payoffs—and this is common knowledge. Following the second stage players are uncertain about their opponents. However, a player is either rational or obstinate (inflexible).

As is standard in the literature, the obstinate types follow a simple strategy: never back down from the initial offer. In particular, the obstinate type seller always demands his first stage price, \(\alpha_s\), accepts any price offer greater or equal to it and rejects all smaller offers. Likewise, obstinate type buyer \(i\) always demands his stage 1 offer, \(\alpha_i\), accepts any price offer smaller or equal to it and rejects all greater offers. Although the remaining assumption is dispensable, I give it for the sake of completeness. I assume that the obstinate type seller understands the equilibrium and leaves his bargaining partner permanently when he is convinced that his partner will never concede. One may wish to consider the case where the obstinate type seller is more strategic or aggressive in the sense that he leaves a buyer immediately in case his demand is not accepted by this buyer. This assumption would certainly not alter our results in Section 3.\(^3\)

**Strategies of the Rational (flexible) Players:** In the first stage of the bargaining game G, a strategy for the seller and buyer \(i\) is a pure action \(\alpha_s, \alpha_i \in [0, 1]\). Since the subsequent analysis is quite involved and the equilibrium outcome is unique, I will restrict my attention to pure strategies in the demand announcement phase of the first stage. Let \(\sigma_i\) denote the probability that the rational seller visits buyer \(i\) first, and so \(\sigma_1 + \sigma_2 = 1\). Although the buyers’ strategies \(\alpha_1, \alpha_2\) are functions of the seller’s announcement \(\alpha_s\), and \(\sigma_i\) is a function of all three players’ announcements, these connections are omitted for notational simplicity. The probabilities that

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\(^3\)This is true (by Theorem 2 and Lemma 4A–1) because if the rational buyers do not accept \(\alpha_s\) upon the seller’s visit, then in equilibrium the seller already leaves the buyers immediately.
the seller and the buyers are obstinate are $z_s$ and $z_b$, respectively, and this is true independent of the players’ strategies in the first two stages.

Third stage strategies are relatively more complicated. A nonterminal history of length $t$, $h_t$, summarizes the initial demands chosen by the players in stage 1, the sequence of buyers the seller visits and the duration of each visit until time $t$ (inclusive). For each $i = 1, 2$, let $\hat{H}_t^i$ be the set of all nonterminal histories of length $t$ such that the seller is with buyer $i$ at time $t$. Also, let $H_t^i$ denote the set of all nonterminal histories of length $t$ with which the seller arrives at the buyer $i$ at time $t$.\footnote{That is, there exits $\epsilon > 0$ such that for all $t' \in [t - \epsilon, t)$, $h_{t'} \notin \hat{H}_t^i$ but $h_t \in \hat{H}_t^i$.}

Finally, set $\hat{H}^i = \bigcup_{t \geq 0} \hat{H}_t^i$ and $H^i = \bigcup_{t \geq 0} H_t^i$.

The seller’s strategy in the second stage has three parts. The first part determines the seller’s location at any given history. For the other two parts, $\mathcal{F}_i^s$ for each $i$, let $\mathbb{I}$ be the set of all intervals of the form $[T, \infty]$ ($\equiv [T, \infty) \cup \{\infty\}$) for $T \in \mathbb{R}_+$, and $\mathcal{F}$ be the set of all right-continuous distribution functions defined over an interval in $\mathbb{I}$. Therefore, $\mathcal{F}_i^s : H^i \to \mathcal{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_{s,t}^i : [T, \infty] \to [0, 1]$ representing the probability of the seller conceding to buyer $i$ by time $t$ (inclusive). Similarly, buyer $i$’s strategy $\mathcal{F}_i : H^i \to \mathcal{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_{i,t}^T : [T, \infty] \to [0, 1]$ representing the probability of buyer $i$ conceding to the seller by time $t$ (inclusive).

Player $n \in \{1, 2, s\}$’s reputation $\hat{z}_n$ is a function of histories and $n$’s strategies, representing the probability that the other players attach to the event that $n$ is the obstinate type. It is updated according to the Bayes’ rule. For example, given a history $h_0$ where the seller announces $\alpha_s$ and visits buyer $i$ first, the seller’s reputation at the time he visits buyer $i$ (i.e., $\hat{z}_s(h_0)$) is $z_s$. Following the history $h_0$, if the seller plays the concession game with buyer $i$ until some time $t > 0$, and the game has not ended yet (call this history $h_t$), then the seller’s reputation at time $t$ is $\frac{\hat{z}_s(h_0)}{1 - F_{s,t}^i(t)}$, assuming that the seller’s strategy in that concession game against buyer $i$ is $F_{s,t}^i$. Note from the last arguments that the seller’s reputation at time $t$ reaches 1 when $F_{s,t}^i(t)$ reaches $1 - \hat{z}_s(h_0)$. This is the case because $F_{s,t}^i(t)$ is the buyers’ belief about the seller’s play during the concession game with buyer $i$. That is, it is the strategy of the seller from the point of view of the buyers. More generally, the upper limit of the distribution function $F_{s,t}^i$ is $1 - \hat{z}_s(h_T)$ where $\hat{z}_s(h_T)$ is the seller’s reputation at time $T \geq 0$, the time that the seller (re)visits buyer $i$. Some arguments apply to the buyers’ strategies.

Given $F_{i,t}^T$, rational seller’s expected payoff of conceding to the buyer $i$ at time $t$ (conditional on not reaching a deal before time $t$ where $T \leq t$,) is

$$U_{s,t}^i(t, F_{i,t}^T) := \alpha_s \int_0^{t-T} e^{-r_s y} dF_{i,t}^T(y) + \frac{1}{2}(\alpha_t + \alpha_s)[F_{i,t}^T(t) - F_{i,t}^T(t^-)]e^{-r_s(t-T)}$$

+ $\alpha_t[1 - F_{i,t}^T(t)]e^{-r_s(t-T)} \quad (1)$

\footnote{That is, there exits $\epsilon > 0$ such that for all $t' \in [t - \epsilon, t)$, $h_{t'} \notin \hat{H}_t^i$ but $h_t \in \hat{H}_t^i$.}
with $F_i^T(t^-) = \lim_{y \uparrow t} F_i^T(y)$.

In a similar manner, given $F_s^{i,T}$, the expected payoff of rational buyer $i$ who concedes to the seller at time $t$ is

$$U_i(t, F_s^{i,T}) := (1 - \alpha_i) \int_0^{t-T} e^{-r_s y} dF_s^{i,T}(y) + \frac{1}{2}(2 - \alpha_i - \alpha_s)[F_s^{i,T}(t) - F_s^{i,T}(t^-)]e^{-r_s(t-T)}$$

$$+ (1 - \alpha_s)[1 - F_s^{i,T}(t)]e^{-r_s(t-T)}$$

(2)

where $F_s^{i,T}(t^-) = \lim_{y \uparrow t} F_s^{i,T}(y)$. Expected payoffs are evaluated at time $T$, and are conditional on the event that the buyer visits seller $i$ at time $T \geq 0$.

### 3. The Main Results

#### A Simple Benchmark Result

Suppose for now that there is only one buyer, denoted by $b$, with a unique demand to announce $\alpha_b \in (0, 1)$, which is incompatible with the seller’s demand $\alpha_s \in (0, 1)$. The timing of the modified version of the game $G$ goes as follows. In stage 1, the seller and then the buyer announce their demands. Since each player has a unique demand, this stage has no strategic content. In the third stage (still at time 0), players begin to play the concession game as described in Section 2 with one important difference; the seller has no outside option of leaving the buyer. This version of the model is identical to the single-type setup of Abreu and Gul (2000), and so the equilibrium strategies are characterized by the following three conditions:

$$F_n(t) = 1 - c_n e^{-\lambda_n t} \text{ for all } t \leq T^e$$

$$c_n \in [0, 1], (1 - c_b)(1 - c_s) = 0, \text{ and }$$

$$F_n(T^e) = 1 - z_n \text{ for all } n \in \{b, s\}$$

(3)

where $\lambda_b = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_b}$ and $\lambda_s = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$.

During the concession game, the rational buyer and seller concede by choosing the timing of acceptance randomly with constant hazard (or instantaneous acceptance) rates $\lambda_b$ and $\lambda_s$, respectively. They play the concession game until $T^e$, when both players’ reputations simultaneously reach 1. Since rational player $n$ is indifferent between conceding and waiting at all times, his expected payoff during the concession game $v_n$ is equal to what he can achieve at time 0. Therefore, by Equations (1) and (2) we have

$$v_b = F_s(0)(1 - \alpha_b) + (1 - F_s(0))(1 - \alpha_s), \text{ and }$$

$$v_s = F_b(0)\alpha_s + (1 - F_b(0))\alpha_b$$

(4)
Note that $1 - c_n$ indicates the probability of player $n$’s initial concession (or player $n$’s initial probabilistic concession), and the second condition in (3) implies that only one player can make concession at time 0. In equilibrium, Abreu and Gul (2000) call a player **strong** if his opponent makes an initial probabilistic concession at time 0 and **weak** otherwise. Therefore, equilibrium payoff of the rational buyer (seller) is $1 - \alpha_s (\alpha_b)$ when he is the weak player.

### The Unrestricted Model

Now I resume the unrestricted version that I presented in Section 2. The equilibrium strategies of the game $G$ are more complicated than the equilibrium of the restricted version presented in the previous section. The characterization of the equilibrium strategies is postponed to Section 4. In this section, I will provide the equilibrium price selections of the buyers in the first stage of the game. The main conclusion is that the unique equilibrium of the game $G$ is the Walrasian outcome (i.e., the monopolist seller gets the whole surplus). The proof of this particular result does not immediately follow from standard Bertrand-like price competition models. In this sub-section I will give a sketch of the proof. In the next sub-section I prove that this uniqueness result may be very sensitive to small perturbations of the common prior assumption.

Equilibrium strategies in the third stage depend on the primitives of the model and the demands declared in stage 1. Given that the buyers’ announcements are $\alpha_1$ and $\alpha_2$ and that $\alpha_s > \alpha_1 \geq \alpha_2$, there are two main cases to consider. The first case is that $\delta \alpha_1 > \alpha_2$. That is, the buyers’ posted prices are significantly distinct from each other. In equilibrium where the demands satisfy $\delta \alpha_1 > \alpha_2$, the seller never plays the concession game with the greedy buyer (i.e., buyer 2). The seller plays the concession game with buyer $i$ when the seller (1) is indifferent between, on the one hand, accepting buyer $i$’s demand, thus receiving the instantaneous payoff of $\alpha_i$, and on the other hand, waiting for the concession of buyer $i$, and (2) prefers accepting buyer $i$’s demand over his endogenous outside option—visiting the other buyer and playing the concession game with that one. Since $\delta \alpha_1 > \alpha_2$ holds, accepting buyer 1’s demand is strictly better than conceding to buyer 2. Therefore, the game never ends with the seller’s concession to buyer 2.

The distinct demands give the seller strong incentive to make a take-it-or-leave-it ultimatum to buyer 2. If his ultimatum is not accepted, then the seller immediately leaves buyer 2 and goes to the other buyer to play the concession game with him. In equilibrium, both the seller and buyer 1 concede by choosing the timing of acceptance randomly with constant hazard rates, which are slightly modified versions of the ones given in the previous sub-section (i.e., $\lambda_1$ and $\lambda_2^1$). Depending on the parameters, the seller may visit buyer 1 more than once, but never more than twice.
The second case is that $\alpha_2 \geq \delta \alpha_1$ (i.e., the buyers’ posted prices are close to one another or equal). In equilibrium where the buyers’ initial demands are close to one another, the seller visits each buyer at most once. Furthermore, if $z_s$ is sufficiently small relative to $z_b$ and if the seller’s initial demand is sufficiently high relative to the buyers’ demands, then in equilibrium, the seller will have weak bargaining power relative to the buyers, and thus, leaving a buyer will never be optimal action unless the seller builds his reputation for obstinacy. In this case, the seller visits both buyers with a positive probability. The seller first visits a buyer, say buyer 1, and plays the concession game with this buyer until some deterministic time $T^d_i \geq 0$, which depends on the primitives and the first stage prices. Unless buyer 1 or the seller concedes prior to time $T^d_i \geq 0$, the seller leaves buyer 1 at this time and immediately goes to buyer 2 to play the concession game with him. Since the equilibrium in the concession games are in mixed strategies, the seller builds up his reputation for obstinacy while negotiating with the buyers. Thus, the seller’s reputation for obstinacy will be higher when the seller leaves buyer 1. As noted earlier, the seller visits each buyer at most once, and so, a rational buyer will never allow the seller leave him without having an agreement. Put it differently, when the seller leaves a buyer, he will be convinced that this buyer is obstinate.

Since the players’ demands are chosen in a dense set, there are various other cases where the equilibrium strategies are somewhat different from the ones prescribed above. The reader should refer to Section 4 for full characterization of the equilibrium. I call buyer $i$ strong if the rational seller concedes to buyer $i$ with a positive probability at the time the seller visits buyer $i$ first at time 0 and weak otherwise. Similarly, the seller is called strong against buyer $i$ if rational buyer $i$ concedes to the seller with a positive probability at the time the seller visits buyer $i$ first at time 0 and weak against buyer $i$ otherwise.

FACTS: Given the equilibrium strategies that are characterized in Section 4, the following observations must hold in any equilibrium where the players’ realized prices satisfy $1 > \alpha_s > \alpha_1, \alpha_2$.

1. If a buyer is strong, then the seller must be weak against this buyer. Equivalently, if the seller is strong against a buyer, then this buyer must be weak.

2. All three players can be weak, but cannot be strong.

3. If buyer $i$ is weak, then buyer $i$’s expected payoff (in the game) is at most $1 - \alpha_s$. Moreover, this payoff can be achieved only if the seller plays a strategy in which he visits the buyer first with probability 1.

4. If the seller is weak against buyer $i$, then the seller’s continuation payoff, following a history where the seller visits buyer $i$ first at time 0 to play the concession game, is $\alpha_i$. 

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5. If $\delta \alpha_1 > \alpha_2$ holds (i.e., the buyers’ bids are sufficiently distinct), then the seller visits buyer 2 only once, whereas he visits buyer 1 twice.

6. If $\delta \alpha_1 > \alpha_2$ holds, then buyer 2’s expected payoff can be at most $1 - \alpha_s$, and this payoff can be achieved only if the seller plays a strategy in which the seller visits buyer 2 first with probability 1. Therefore, buyer 2 is weak if $\delta \alpha_1 > \alpha_2$ holds.

7. If $\alpha_1 \geq \alpha_2$ and $\alpha_2 \geq \delta \alpha_1$ hold, then the seller visits and plays the concession game with each seller at most once.

8. If $\alpha_1 \geq \alpha_2$ and $\alpha_2 \geq \delta \alpha_1$ hold, then buyer $i$’s continuation payoff, following a history where the seller visits buyer $i$ after he negotiates with buyer $j$ first ($j \in \{1, 2\}, j \neq i$) must be $1 - \alpha_s$. This payoff is evaluated at the time that the seller and buyer $i$ begins the concession game.

9. If buyer $i$ is weak, then buyer $i$’s expected payoff in the game must be exactly $1 - \alpha_s$.

The last observation is important. It is true because if buyer $i$’s expected payoff is $1 - \alpha_s - \epsilon$ for some $\epsilon > 0$, then buyer $i$ can profitably deviate. If buyer $i$ bids $\alpha_s + \epsilon/2$, then the seller immediately accepts buyer $i$’s demand and finishes the game at time 0 regardless of the value of $\alpha_j \leq \alpha_s$. Hence, by observations #3 and #9 we can conclude that if a buyer is weak in equilibrium, then the seller must visit this seller first with probability 1.

**Theorem 1.** There always exists a sequential equilibrium of the game $G$, in which the seller posts the price of 1 and both buyers announce 1 regardless of the seller’s announcement, and thus, the game ends at time 0. Moreover, the monopoly price is the unique equilibrium outcome of the game $G$. That is, there does not exist a sequential equilibrium where the players’ realized price announcements in stage 1 satisfy $1 \geq \alpha_s \geq \alpha_1, \alpha_2$ such that at least one of the inequalities is strict.

The existence part is simple. Given that buyer 2 announces his demand as 1, there is no profitable deviation for buyer 1. However, the uniqueness of the equilibrium outcome is not straightforward. The reason for this is that overbidding is not always an optimal deviation strategy for the buyers since it does not always attract the seller. That is, for some parameter values, the seller may prefer to visit the buyer who bids the lower price. Here, I will provide a sketch of the proof. The detailed version is deferred to the Appendix.

It is easy to show that there does not exist an equilibrium where $1 > \alpha_s$ and $\alpha_i = \alpha_s$ for some $i \in \{1, 2\}$. Suppose for a contradiction that there is such an equilibrium. Particularly, let $\alpha_1 = \alpha_s > \alpha_2$. In this case, the game ends at time 0 by the seller’s acceptance of the first buyer’s price, and thus, buyer 2’s payoff is 0. However, buyer 2 would profitable deviate to a
price \( \alpha_s + \epsilon \), where \( \epsilon > 0 \) is small, contradicting with the optimality of the equilibrium strategies. The case where \( \alpha_s = 1 > \alpha_1, \alpha_2 \) is deferred to the Appendix.

Now, suppose for a contradiction that there exists an equilibrium in which the players’ price announcements in stage 1 satisfy \( 1 > \alpha_s > \alpha_1, \alpha_2 \). I reach a contradiction in four main steps. The first step shows that at least one of the buyers must be weak. The second step shows that if one buyer is weak, then the buyers’ announcements must be different. The third step shows that the first buyer (who makes the highest bid) cannot be the weak buyer. As a result of these three steps, we can conclude that in equilibrium we must have \( 1 > \alpha_s > \alpha_1 > \alpha_2 \), buyer 1 is strong, and 2 is weak. However, the fourth step shows that there is no such equilibrium.

In order to prove the first claim (i.e., “at least one buyer must be weak”) I suppose, for a contradiction, that both buyers are strong. This claim implies that (with the first condition above) the seller must be weak against both sellers, and thus his expected payoff in the subgame following a history where he visits buyer \( i \) first is \( \alpha_i \). Therefore, optimality implies that the seller strictly prefers to visit first the buyer who posts the higher price. Therefore, the buyers have incentive to overbid their opponents, and this implies that the buyers would deviate unless \( \alpha_1 \) and \( \alpha_2 \) are equal to 1, contradicting with our starting assumption that \( 1 > \alpha_s > \alpha_1, \alpha_2 \). Hence, we can conclude that at least one buyer must be weak.

To prove the second step, I suppose that one buyer is weak, and for a contradiction, that the buyers’ announcements are equal. In this case, since the buyers are identical, both buyers must indeed be weak. However, by facts #3 and #9, the seller must visit both buyers in equilibrium with probability 1, which is not possible.

For the third case, I assume for a contradiction that the realized announcements satisfy \( 1 > \alpha_s > \alpha_1 > \alpha_2 \) and buyer 1 is weak. Then, by facts #3 and #9 we can conclude that the seller shall visit buyer 1 first with probability 1. Therefore, facts #6 and #8 implies that the second buyer’s expected payoff in the game is strictly less than \( 1 - \alpha_s \) (as the seller will visit buyer 2 after visiting buyer 1). However, 2 can profitably deviate by posting a price \( \alpha_s + \epsilon \), where \( \epsilon \geq 0 \) is sufficiently small, contradicting with the optimality of the equilibrium.

The last part is trickier. I suppose for a contradiction that the realized announcements satisfy \( 1 > \alpha_s > \alpha_1 > \alpha_2 \) and buyer 2 is weak. Therefore, by facts #3 and #9 the seller shall visit buyer 2 first with probability 1 and buyer 2’s expected payoff in the game must be \( 1 - \alpha_s \). Then, I show that buyer 2 can deviate to a price very close to \( \alpha_s \), but higher than \( \alpha_1 \), so that the seller strongly prefers to visit buyer 2 first, and thus, buyer 2 achieves a payoff strictly higher than \( 1 - \alpha_s \). This achieves the desired contradiction and finalizes the proof.
As the arguments in the previous section highlight, the main idea behind the proof of Theorem 1 is that if there exists an equilibrium where the players post prices different from the monopolist price (i.e., 1), then two incompatible conclusions directly follow. The first conclusion is that the seller cannot be weak against the buyers. The reason for this is that when the seller is weak against both buyers, then the (rational) buyer who announces the highest price certainly makes an agreement with the seller. Thus, each buyer can profitably deviate by overbidding his opponent. Since the buyers’ choice set is dense, this deviation is unavoidable unless both buyers post the monopoly price. The second conclusion is that the buyers cannot be weak. This is the case because in equilibrium the seller must visit the weak buyer with probability 1, implying that the other buyer has strong incentive to deviate from its initial demand by accepting the seller’s initial offer and finishing the game at the first stage of the game.

It is clear that the first conclusion is unavoidable unless we restrict the set of strategies (bids) the players are allowed to choose. On the other hand, the second conclusion is avoidable: if, for example, each buyer believes that the seller will visit him first at time 0 with probability 1, then the buyers may not have incentive to deviate. However, the Aumann’s “Agreeing to Disagree” result (Aumann 1976) shows that the rational buyers will never believe that the seller will certainly visit them first at time 0 if they have common priors.

The last argument is the starting point of the inquiry that is laid down in detail in this section. The following results show that relaxing the common prior (not the common knowledge) assumption allows the buyers a possibility to use the seller’s reputational concern so that they can reinforce non-Walrasian outcome, and this logic retains to be the case even if the deviation from the common prior assumption is arbitrarily small.

Therefore, in this section, *I assume that each buyer conjectures that his opponents are tougher negotiators*. More formally, each player knows that he will never be forced to commit to his initial demand, but is uncertain about the other players. Each player believes that nature sends one of two messages \{c, d\} to his opponents in stage 2. Let \(z_h, z_l \in (0, 1)\) where \(z_h \geq z_l\). The new twist in this section is the following: Each buyer \(i\) has a conjecture that all the players share the same belief that buyer \(j\) and the seller receive the message \(c\) with probability \(z_h\), whereas buyer \(i\) receives the message \(c\) with probability \(z_l\). Therefore, in the sense of Harsanyi (1967/68), rational seller has two types: type–i seller has the same model of the world with buyer \(i\).

This assumption provides an informational structure that is very similar to the one used in Gossner, Kalai and Weber (2009). Each buyer has a different model of the world, but believes

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5Harsanyi (1967/68) proposed that a player’s type should be a description of his beliefs about states of the world (payoff-relevant events), his beliefs about other players’ beliefs about the states of the world, and so on.
that all the other agents share his own model. That is, each agent believes that what he believes is common knowledge. Note that when \(z_h - z_l = \epsilon = 0\), then we go back to our initial case; common priors. However, as long as \(\epsilon > 0\), the initial priors are heterogeneous. In this section, I will show that a large set of prices can be supported in equilibrium even if \(\epsilon\) or both \(z_h\) and \(z_l\) are vanishingly small. There may be alternative deviations from the common prior assumption, but I aim to pick the simplest perturbation that is sufficient to prove that the uniqueness of the monopoly price (i.e., Theorem 1) is not robust.

One can consider the homogeneous–product Bertrand duopoly model (a single buyer and two sellers case) to conceive why heterogeneous priors soften competition so much. In the bare-bones version of that model, the buyer wants exactly one unit, valued at \(v\). The two identical sellers who has an opportunity cost of \(c < v\), simultaneously set prices, and the buyer purchases at the lowest price. The unique equilibrium price equals the sellers’ cost. Suppose now instead that each seller believes that the buyer “prefers trading with me”. Specifically, let each seller believe that the buyer values “my product” at \(v\) and “the opponent’s product” at \(v - \epsilon\), where \(\epsilon\) is small compared to \(v\). In this case, although the priors can be very close to the truth (when \(\epsilon\) is close to 0), the game has an equilibrium in which both buyers set the monopoly price \(v\). The reason is that both buyers are convinced that the seller will buy from them rather than from the competitor if the two prices are the same.

One may argue that, in reality, buyers’ divergent beliefs will eventually converge one another as these buyers engage in repetitive trade because the one that cannot make a sale will be eliminated from the market. This clearly is not necessarily the case if the game \(G\) is just a “stage game” of a more complicated dynamic game. For example, one may think that there are two long-lived, competing venture capital firms in the market, constantly buying shares of successful businesses, and a flow of small, short-lived sellers (owners of emerging growth companies) entering to the market to fund their businesses. Another example would be that the buyers are two long-lived executive headhunters and short-lived sellers (candidates) enter to the job market over time. Beliefs of the potential sellers could naturally be heterogeneous, and so, some sellers believe that buyer 1 is tougher negotiator and others believe that buyer 2 is tougher. As a result, some sellers may end up visiting buyer 1 first while others visit buyer 2 first. Moreover, if the buyers cannot observe all the sellers entering the market, then a buyer may not have chance to realize that his model of the world is wrong.

Furthermore, disagreement is the rule rather than the exception in practice. Just to mention a few instances, there is typically considerable disagreement even among economists working on a certain topic. Acemoglu, Chernozhukov and Yildiz (2007) show that if Bayesian individuals have different initial priors and are uncertain about the informativeness of signals, then they will never agree, even after observing the same infinite sequence of signals. In this regard, buyers de facto have to deal with different signals all at the same time, and so their persistent
disagreement about one another’s flexibility is not a surprising defect. Morris (1995) provides a
detailed discussion why common prior assumption is indeed a very strong one. Furthermore, if
one insists on the idea that the buyers’ divergent beliefs shall eventually converge one another,
and so frictions must vanish, then we should expect that the set of equilibrium outcomes will
converge to the unique monopoly outcome as \( \epsilon \) (indicating the size of divergence) converges to
0. However, Theorems 3 and 4 show that this is not the case.

In terms of equilibrium calculations, the only difference of this model with the common-
prior version is that there are two types of rational sellers. Therefore, in equilibrium, buyer 1’s
strategy must be a best response to the other players’ strategies given buyer 1’s beliefs. Since
buyer 1 believes that the rational seller is of type 1, then buyer 1 will ignore type 2 seller’s
strategy. Similarly, rational buyer 2 will ignore type 1 seller’s strategy. Likewise, both sellers’
(type 1 and 2) strategies must be optimal given the buyers’ strategies and the sellers’ beliefs.
Finally, I assume that both types of the rational seller visit each buyer with equal probabilities
whenever they are indifferent between visiting buyer 1 and 2 first. With this further restriction,
I eliminate some unattractive and trivial equilibrium strategies where both type 1 and 2 sellers
are indifferent between the buyers, and each buyer believes that the seller will definitely visit
him first. That is, I restrict my attention to equilibria where each buyer assigns probability one
to the event that the seller will visit him first only if this buyer believes that the seller strictly
prefers to visit him first.

Furthermore, in order to eliminate unnecessary complications, I will restrict the sellers’ strate-
gies in stage 1 and assume that both type 1 and type 2 sellers have to choose their initial demands
from the set \( \{\alpha_s\} \) where \( \alpha_s \in [0,1] \). I adopt this restriction because the results in this section
are existence results. That is, for any \( \alpha_s \in [0,1] \), there exists a sequential equilibrium of the
bargaining game where both type of the sellers post \( \alpha_s \) and the buyers post some \( \alpha_1, \alpha_2 \in [0,1] \)
with \( \alpha_1, \alpha_2 < \alpha_s \). Furthermore, the last two results of this section provides some upper bounds
for \( \alpha_s \) that can be supported in equilibrium for vanishingly small initial priors, and these upper
bounds are away from the monopoly price for a large set of parameter values for \( r_b \) and \( r_s \).

For what follows, I fix the values of the primitives \( r_b, r_s, \delta \) and \( z_h, z_l \in (0,1) \) with \( z_h > z_l \). Let \( \tilde{G}(r_b, r_s, \delta, z_h, z_l) \), or \( \tilde{G} \) in short, denote the bargaining game \( G \) with the new information
structure.

**Definition.** In any equilibrium where the players’ first stage announcements satisfy \( 1 > \alpha_s > \alpha_b = \alpha_1 = \alpha_2 > 0 \), I call a buyer **powerless** if this buyer is weak and he cannot make himself
strong by deviating to a price other than \( \alpha_b \). That is, a buyer is powerless if the inequalities
\( z_h \geq (z_i/\alpha_s) ^ {\frac{(1-\alpha_i) r_b}{\alpha_s r_s}} \) and \( z_h \geq z_l ^ {\frac{(1-\alpha_i) r_b}{\alpha_s r_s}} \) hold for all \( \alpha \in (0, \alpha_s) \) when \( A = \frac{\delta \alpha_2 - \alpha_b}{\delta \alpha_2 - \alpha_1} > z_h \), and only
the inequality \( z_h \geq z_l ^ {\frac{(1-\alpha_i) r_b}{\alpha_s r_s}} \) holds for all \( \alpha \in (0, \alpha_s) \) when \( z_h \geq A \).
As the equilibrium characterizations in Section 4 indicates, if a buyer is powerless in equilibrium, then he must be weak, and his expected payoff is at most $1 - \alpha_s$. This payoff is attainable in equilibrium only if the seller plays a strategy in which he visits the buyer first with probability 1. However, if a buyer is weak, then he is not necessarily powerless. The next result indicates that the buyer who announces the lowest price must be weak in any equilibrium of the game $\tilde{G}$.

**Proposition 3.1.** In any equilibrium of the game $\tilde{G}$ where $1 > \alpha_s > \alpha_1 \geq \alpha_2 > 0$, buyer 2 must be weak and $z_h \geq z_l \frac{(1 - \alpha_s \rho_h)}{r_s}$ must hold for all $\alpha \in (0, \alpha_s)$.

The next result is important because it provides an upper bound for the value of the seller(s) posted price $\alpha_s$ in equilibrium (given the values of other primitives such as $z_h$ and $z_l$). For example, if $\alpha_s$ is very close to 1, then for some values of $z_h$ and $z_l$, there cannot exist an equilibrium (in pure strategies for the first stage) where the sellers post $\alpha_s$ and the buyers post some $\alpha_b \leq \alpha_s$. This is true because for high values of $\alpha_s$ the buyers may be weak but will not be powerless. Recall from previous arguments that buyers must be weak in any equilibrium. However, if a buyer is weak but not powerless, then he can deviate to a price very close to but smaller than $\alpha_s$ and make himself strong. By this deviation the buyer can achieve expected payoff higher than $1 - \alpha_s$ (highest expected payoff weak buyers can ever achieve in the game $\tilde{G}$).

**Theorem 2.** There exists an equilibrium strategy profile of the game $\tilde{G}$ where both buyers announce $\alpha_b \in (0, \alpha_s)$ if and only if the buyers are powerless under this strategy profile.

The last two results of this section underline two important properties of the equilibrium outcomes. First, there exists a large set of equilibrium prices that are away from the monopoly price even in the limit where $z_h$ and $z_l$ converge one another (i.e., the common prior assumption is restored in the limit). Second, in some of these equilibria, the buyers’ prices can be significantly different from each other, and this is also true when the initial priors $z_h$ and $z_l$ are very close to 0.

To state these claims more formally, we need the following definitions. Fix the values of the parameters $r_h, r_s$ and $\delta$. For any $z_h, z_l \in (0, 1)$ with $z_h > z_l$, let $\tilde{G}(z_h, z_l)$ denote the bargaining game $\tilde{G}$ where $z_h$ and $z_l$ represent the players’ initial priors. I say the bargaining game $\tilde{G}(z_h, z_l)$ converges to $\tilde{G}(K)$ when the sequences $\{z_h^m\}$ and $\{z_l^m\}$ of initial priors satisfy $\lim_{m \to \infty} z_h^m = 0$, $\lim_{m \to \infty} z_l^m = 0$ and $\log z_l^m / \log z_h^m = K < 1$ for all $m \geq 0$. Likewise, I say the bargaining game $\tilde{G}(z_h, z_l^m)$ converges to the game $G(z_h)$, where $z_h$ represents the players’ common prior of being the obstinate type, when the sequence $\{z_l^m\}$ of initial prior satisfy $\lim_{m \to \infty} z_l^m = z_h$. The next result directly follows from Theorem 2.

**Theorem 3.** If there exists an equilibrium of the game $\tilde{G}(z_h, z_l^m)$ where $\alpha_s^m$ is the seller’s price, the game $\tilde{G}(z_h, z_l^m)$ converges to $G(z_h)$, and if $\alpha_s \in (0, 1)$ is the limit point of $\alpha_s^m$, then we must have $\alpha_s \leq \frac{r_s}{r_h + r_s}$.
Proof. According to Theorem 2, in equilibrium, the sellers’ bid $\alpha^m_s$ must satisfy $z_h \geq (z^m_l)^{(1-\alpha^m_s)r_b/\alpha r_s}$ for all $m > 0$ and $\alpha \in (0, \alpha^m_s)$. Taking the log and then the limit of both sides, as $z^m_l \to z_h$ and $\alpha^m_s \to \alpha_s$, will result in $\alpha r_s \leq (1 - \alpha_s) r_b$ for all $\alpha \in (0, \alpha_s)$. The last inequality yields the required inequality. □

Theorem 3 states that if, for example, all players’ time preferences are the same (i.e., $r_b = r_s$), then the seller’s bid in equilibrium must be no more than $1/2$ in the limit where the bargaining game $\tilde{G}$ converges to the bargaining game with common prior (i.e., $G$). Recall that I restrict the seller to post a pre-specified price $\alpha_s \in (0, 1)$ at the first stage. What if the seller is free to choose any price from the set $[0, 1]$ he wants? In that case, with the light of the previous result, one may wonder if there exists an equilibrium (for the values of $z_l$ and $z_h$ very close to each other) such that the seller of either type selects the same price $\alpha_s = 1/2$. The sellers’ expected payoffs are strictly increasing functions of $\alpha_s$ for sufficiently small values of $z_h$ and $z_l$, and so the sellers will pick the highest price that can be supported in equilibrium. Therefore, for sufficiently small and close values of the initial priors, $1/2$ will be the unique equilibrium price that the sellers will announce.

Theorem 4. For any small values of $z_h, z_l \in (0, 1)$ with $z_h > z_l$, there exists a large set of equilibria of the game $\tilde{G}(z_h, z_l)$ in which the buyers’ price announcements are different. Furthermore, if there exists an equilibrium of the game $G(z^m_h, z^m_l)$ where $\alpha^m_s$ is the seller’s price, the game $\tilde{G}(z^m_h, z^m_l)$ converges to $\tilde{G}(K)$, and if $\alpha_s \in (0, 1)$ is the limit point of $\alpha^m_s$, then we must have $\alpha_s \leq \frac{Kr_b}{Kr_b + r_s}$.

4. Equilibrium Characterization of the Third Stage

In this section I will characterize the rational (flexible) players’ equilibrium strategies in the third stage of the game $G$.

Proposition 4.1. In any (sequential) equilibrium of the bargaining game $G$ following a history $h_T$, where players’ price announcements (bids) are $1 > \alpha_s > \alpha_1, \alpha_2$, the seller arrives at buyer $i$ at time $T$ and his actual type has not yet revealed, the players’ concession game strategies are $F^i_{s,T}(t) = 1 - c^i_s e^{-\lambda^i_s(t-T)}$ and $F^T_i(t) = 1 - c^i e^{-\lambda^i(t-T)}$ for all $t \geq T$, where $\lambda^i = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_i}$ and $\lambda^i = \frac{\alpha r_s}{\alpha_s - \alpha_i}$, satisfying $c^i_s, c_i \in [0, 1]$ and $F^i_{s,T}(T)F^T_i(T) = 0$.

I defer the proofs to the Appendix. For the rest of the paper, I will use $F^i_s$ and $F^i_T$ for each $i = 1, 2$ to indicate the players’ concession game strategies, although these strategies depend on the previous history of the game. For notational simplicity I omit these connections and manipulate the subsequent notation and reset the clock once the seller and a buyer begins a
concession game. Thus, I define each players concession game strategies (distribution functions) as if the concession game with each buyer starts at time 0.

Since the buyers are identical in all other aspects, I will focus, without loss of generality, on the equilibrium strategies where the buyers’ price announcements satisfy $\alpha_1 \geq \alpha_2$. In an equilibrium where the players’ bids satisfy $1 > \alpha_s > \alpha_1 \geq \alpha_2 > 0$ and $\alpha_2 > \delta \alpha_1$, there are two exhaustive cases that we need to consider. The first one is that the buyers’ prices are sufficiently close to each other (i.e., $\alpha_2 > \delta \alpha_1$). The second case is that the gap between the buyers’ bids are big, that is $\delta \alpha_1 \geq \alpha_2$. The structures of the equilibrium strategies are slightly different in these two cases. Therefore, the next two subsections separately characterize the equilibrium strategies falling into these two categories.

The Gap Between the Buyers’ Bids is Small

In this section, I characterize the equilibrium strategies where the players bids in the first stage satisfy $1 > \alpha_s > \alpha_1 \geq \alpha_2 > 0$ and $\alpha_2 > \delta \alpha_1$. The following results of this subsection characterize equilibrium strategies of the players following a history where the seller visits buyer 2 first. By interchanging the numbers 1 and 2 in all following subscript and superscript, we get the characterization of the equilibrium strategies following a history where the seller visits buyer 1 first. Finally, the next two inequalities will play crucial role in the characterization of the equilibrium strategies (following a history where the seller visits buyer 2 first):

\begin{equation}
\delta > \frac{\alpha_2}{\alpha_s}
\end{equation}

\begin{equation}
A_2 = \frac{\delta \alpha_2}{\delta \alpha_s - \alpha_1} > z_b
\end{equation}

**Proposition 4.2.** In any equilibrium where the players’ initial bids satisfy the inequalities (1), (2) and $\alpha_2 > \delta \alpha_1$, the rational seller visits each buyer at most once, and a rational buyer does not allow the seller to leave him without reaching an agreement. Moreover, in this equilibrium if the rational seller visits buyer 2 first, leaves 2 at time $T_d^2$ and finalizes the game with buyer 1 at time $T_e^1$ if the game has not yet ended before, then the players’ concession game strategies must satisfy

\[
F_s^2(t) = 1 - c_s^2 e^{-\lambda_s^2 t} \quad F_2(t) = 1 - z_b e^{\lambda_2 (T_d^2 - t)} \\
F_s^1(t) = 1 - e^{-\lambda_s^1 t} \quad F_1(t) = 1 - z_b e^{\lambda_1 (T_e^1 - t)}
\]

satisfying

\[
F_s^2(0)F_2(0) = 0 \quad \text{and} \quad F_s^1(T_e^1) = 1 - \frac{z_s}{1 - F_s^2(T_d^2)}
\]

In equilibrium, after a history where the rational seller visits buyer 2 first, second buyer’s reputation reaches 1 at time $T_d^2$ whereas the seller’s reputation increases to a level which makes
him indifferent between conceding to buyer 2 and traveling to the first buyer and playing the concession game with the first buyer. Finally, the first buyer’s and the seller’s reputation simultaneously reach 1 at time $T_1^d$.

Next, I will characterize the times that the concession games with buyer 1 and 2 ends (i.e., $T_e^1$ and $T_d^2$, respectively), and the rational seller’s initial probabilistic concession against the second buyer $F_{s}^{2}(0)$. The rational players’ equilibrium payoffs in the concession games are calculated by the equations of (4). That is, for each buyer $i$

$$v_s^i = F_s(0) \alpha_s + (1 - F_s(0)) \alpha_i, \text{ and } v_i = F_s^i(0)(1 - \alpha_i) + (1 - F_s^i(0))(1 - \alpha_s) \quad (5)$$

However, the rational players’ equilibrium payoffs in the game G is different as they should take into account the seller’s outside option and buyer selection in stage 1. In what follows, I will provide the rational seller’s equilibrium payoff, for each case, because they are important for the analyses in the subsequent sections.\(^{6}\)

In an equilibrium where the seller first visits buyer 2, the rational seller leaves the buyer when he is convinced that this buyer is obstinate. At this moment, abandoning buyer 2 is optimal for the rational seller if his discounted continuation payoff of negotiating with the first buyer, $\delta v_s^1$, is no less than $\alpha_2$, payoff to the rational seller if he concedes to the obstinate buyer 2. Let $z_s^*$ denote the level of reputation required to provide the rational seller enough incentive to leave the second buyer. Assuming that $z_s < z_s^*$ (i.e., the rational seller needs to build up his reputation before walking out of negotiation with buyer 2), the game ends with buyer 1 at time $T_1^* = -\log(z_s^*)/\lambda_s^1$.\(^{7}\) Thus, $z_s^*$ must solve $\alpha_2 = \delta v_s^1$, and given the value of $F_1(0)$ by Proposition 4.2, we must have

$$\alpha_2 = \delta \left[ \alpha_1 + (\alpha_s - \alpha_1)(1 - z_s(z_s^*)^{-\lambda_1/\lambda_s^1}) \right]$$

implying that $z_s^* = \left( \frac{z_s}{A_2} \right)^{\lambda_1/\lambda_s^1}$ and $A_2 = \frac{\delta \alpha_s - \alpha_2}{\delta (\alpha_s - \alpha_1)}$. Note that $z_s^*$ is well-defined (i.e., $z_s^* \in (0, 1)$) since $A_2$ is positive.

On the other hand, when $z_s \geq z_s^*$, the rational seller’s discounted continuation payoff of negotiating with buyer 1 is higher than $\alpha_2$ before stage three begins. Therefore, the rational seller prefers going to buyer 1 and playing the concession game with this buyer over conceding to buyer 2. In equilibrium, rational buyer 2 anticipates that the seller will never concede to him but rather plan to leave him immediately, and so accepts the seller’s demand at time 0 without any delay.

\(^{6}\)The buyers’ expected payoff calculations are more involved, and hence presented in the appendix.

\(^{7}\)According to Proposition 4.1 we have $F_s^1(T_1^1) = 1 - z_s^*$, which implies the value of $T_1^1$.\(^{20}\)
Lemma 4A–1. Consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and the players’ initial bids satisfy the inequalities (1), (2), \( \alpha_2 \geq \delta \alpha_1 \) and \( z_s \geq z_s^* = (z_b/A_2)^{\lambda_2/\lambda_1} \). Then, the continuation strategies of this equilibrium strategy will be as follows. The rational seller makes a take it or leave it offer to the buyer and goes directly to buyer 1. Rational buyer 2 immediately accepts the seller’s demand and finishes the game at time 0 with probability 1. In case buyer 2 does not concede to the seller, the seller infers that 2 is obstinate, and so he never comes back to negotiate with this buyer again. The concession game with buyer 1 may continue until the time \( T \) is not yet ended, and goes directly to buyer 1. The concession game with the first buyer may continue until the time \( T^* = -\log s/\lambda_1 \) with the following strategies: \( F^1_s(t) = 1 - e^{-\lambda_1 t} \) and \( F_1(t) = 1 - s z_s^{-\lambda_1/\lambda_1} e^{-\lambda_1 t} \).

This result shares the flavor of the arguments of Compte and Jehiel (2002) on the role of outside options for the obstinate negotiators and of one-sided uncertainty result of Myerson (1991, Theorem 8.4). In equilibrium, if the value of the seller’s outside option is high, then the buyer is forced to reveal his rationality, implying immediate concession by the rational buyer. Assuming that the rational seller visits buyer 2 first and \( z_s \geq (z_b/A_2)^{\lambda_2/\lambda_1} \) holds, the rational seller’s equilibrium payoff of visiting buyer 2 first is given by Equations (5) as follows:

\[
V_s^2 = (1 - z_b)(\alpha_s) + \delta z_b \left[ v_s^1 \right] = \alpha_s \left[ 1 - z_b (1 - \delta) - \frac{\delta z_b^2}{\lambda_1/\lambda^1} \right] + \alpha_1 \frac{\delta z_b^2}{\lambda_1/\lambda^1}.
\]

(6)

Lemma 4A–2. Consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and the players’ initial bids satisfy the inequalities (1), (2), \( \alpha_2 \geq \delta \alpha_1 \) and \( (z_b/A_2)^{\lambda_1/\lambda_1} z_b^{\lambda_2/\lambda_2} \geq z_b \). Then, the continuation strategies of this equilibrium strategy will be as follows. The seller leaves buyer 2 at time \( T^d_2 = -\log(z_b)/\lambda_2 \) for sure, if the game has not yet ended, and goes directly to buyer 1. The concession game with buyer 1 may continue until the time \( T^* = -\log(z_b/A_2)/\lambda_1 \). The players’ concession game strategies are \( F^2_s(t) = 1 - z_s(z_b/A_2)^{-\lambda_1/\lambda_1} z_b^{\lambda_2/\lambda_2} e^{-\lambda_2 t} \), \( F_2(t) = 1 - e^{-\lambda_2 t} \), \( F^1_s(t) = 1 - e^{-\lambda_1 t} \) and \( F_1(t) = 1 - A_2 e^{-\lambda_1 t} \).

In equilibrium, \( (z_b/A_2)^{\lambda_1/\lambda_1} z_b^{\lambda_2/\lambda_2} \geq z_s \) implies that the rational seller’s initial reputation is very low, and thus he needs to spend significant amount of time to build up his reputation before leaving buyer 2. In this case, \( F_2(0) = 0 \) (i.e., the seller does not receive an initial probabilistic gift from buyer 2), implying that the rational seller is weak against buyer 2 and his expected payoff during the concession game with buyer 2, \( v_s^2 \), is simply \( \alpha_i \), which follows from Equations (5). Therefore, the rational seller’s expected payoff of visiting buyer 2 first, \( V_s^1 \), is \( \alpha_i \).

Lemma 4A–3. Consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and the players’ initial bids satisfy the inequalities (1), (2), \( \alpha_2 \geq \delta \alpha_1 \) and \( (z_b/A_2)^{\lambda_1/\lambda_1} > z_s > (z_b/A_2)^{\lambda_1/\lambda_1} z_b^{\lambda_2/\lambda_2} \). Then, the continuation strategies of this equilibrium
strategy will be as follows. The seller leaves buyer 2 at time $T_2^d = -\log(z_s)/\lambda^2 + (\lambda^1/\lambda_1) \log(z_b/A_2)/\lambda^2$ for sure, if the game has not yet ended, and goes directly to buyer 1. The concession game with buyer 1 may continue until the time $T_1^e = -\log(z_b/A_2)/\lambda_1$. The players’ concession game strategies are $F^1_s(t) = 1 - e^{-\lambda^1 t}$, $F^2_1(t) = 1 - A_2 e^{-\lambda^1 t}$, and $F^2_2(t) = 1 - z_b(z_b/A_2)^{\lambda^1/\lambda_1} (z_s)^{-\lambda^2/\lambda^1} e^{-\lambda^2 t}$.

In this particular case, the rational seller’s equilibrium payoff of visiting buyer 2 first is

$$V_2^s = \alpha_2 \left[ 1 - z_b \left( \frac{(z_b/A_2)^{\lambda^1/\lambda_1}}{z_s} \right)^{\lambda^2/\lambda^1} \right] + \alpha_2 z_b \left( \frac{(z_b/A_2)^{\lambda^1/\lambda_1}}{z_s} \right)^{\lambda^2/\lambda^1}. \tag{7}$$

**Proposition 4.3.** Consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and the players’ initial bids satisfy $\alpha_2 > \delta \alpha_1$ but fail to satisfy at least one of the inequalities (1) and (2). Then, the continuation strategies of this equilibrium strategy will be as follows. The rational seller never leaves buyer 2 and the concession game ends by the time $T_2^e = \min \left\{ -\log z_b/\lambda^2, -\log z_s/\lambda^2 \right\}$ for sure, if the game has not yet ended. The players’ concession game strategies are $F^2_s(t) = 1 - z_s e^{\lambda^2 (T_2^e - t)}$ and $F_2(t) = 1 - z_b e^{\lambda^2 (T_2^e - t)}$. If the seller leaves 2 at time $T_2^e$ and goes to 1, then buyer 1 will immediately concede to the seller believing that he is the commitment type.

**The Gap Between the Buyers’ Bids is Sufficiently Big**

In this section, I characterize the equilibrium strategies where the players bids in the first stage satisfy $1 > \alpha_s > \alpha_1 > \alpha_2 > 0$ and $\delta \alpha_1 \geq \alpha_2$. I furthermore assume that in equilibrium, if the seller is indifferent between conceding to his current partner and abandoning his partner to visit the other buyer, the rational seller will choose to abandon his partner. This assumption is binding only when $\alpha_2 = \delta \alpha_1$. When this equality holds there are infinitely many equilibria. This restriction implies that among all possible equilibria, we will select the one that yields the highest payoff to the seller in the game G.

In equilibrium where the buyers’ bids satisfy $\delta \alpha_1 \geq \alpha_2$, the bargaining phase (stage three) never ends with the seller’s concession to buyer 2. More formally, consider the case where the seller is with buyer 2 and playing the concession game with this buyer. This means that the rational seller should be indifferent between, on the one hand, accepting buyer 2’s demand, thus receiving the instantaneous payoff of $\alpha_2$, and on the other hand, waiting for the concession of the buyer. However, if the rational seller leaves (immediately) buyer 2 and goes directly to the first buyer to accept his demand, his discounted payoff will be $\delta \alpha_1$. Thus, if the rational seller ever visits buyer 2 in equilibrium, then he will never accept buyer 2’s demand because we have $\delta \alpha_1 \geq \alpha_2$ by assumption. However, rational buyer 2 immediately concedes to the seller (upon
the beginning of the concession game) because of the seller’s tendency to opt out instantly from the concession game with buyer 2.

**Lemma 4B–1.** Consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and the players’ initial bids satisfy \(\delta \alpha_1 \geq \alpha_2\). Then, the continuation strategies of this equilibrium strategy will be as follows. The rational seller makes a take it or leave it offer to buyer 2 and, if not accepted, goes directly to buyer 1. Rational buyer 2 immediately accepts the seller’s demand and finishes the game at time 0 with probability 1. In case buyer 2 does not concede to the seller, the seller infers that buyer 2 is the commitment type, and so he never comes back to this buyer again. The concession game with buyer 1 may continue until the time \(T^c_1 = \min\{-\log z_s, -\log z_b\}\) with the following strategies: \(F_1(t) = 1 - z_b e^{\lambda_1 (T^c_1 - t)}\) and \(F_s^1(t) = 1 - z_s e^{\lambda_1 (T^c_1 - t)}\) for all \(t \geq 0\).

In this particular case, the rational seller’s equilibrium payoff of visiting buyer 2 first is

\[
V^2_s = \alpha_s (1 - z_b) + \delta z_b v^1_s
\]

where \(v^1_s = (1 - z_b e^{\lambda_1 T^c_1}) \alpha_s + z_b e^{\lambda_1 T^c_1} \alpha_1\). Hence, the seller is always strong against buyer 2 if the buyers’ bids satisfy \(\delta \alpha_1 \geq \alpha_2\).

**Lemma 4B–2.** Consider an equilibrium strategy following a history where the rational seller visits buyer 1 first and the players’ initial bids satisfy \(\delta \alpha_1 \geq \alpha_2\) and \(z_b \geq \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}\). Then, the continuation strategies of this equilibrium strategy will be as follows. The rational seller never leaves 1 and the concession game ends by the time \(T^c_1 = \min\{-\log z_s, -\log z_b\}\) for sure, if the game has not yet ended. The players’ concession game strategies are \(F_1^1(t) = 1 - z_b e^{\lambda_1 (T^c_1 - t)}\) and \(F_s^1(t) = 1 - z_s e^{\lambda_1 (T^c_1 - t)}\). If the seller leaves 1 at time \(T^c_1\) and goes to 2, then buyer 2 will immediately concede to the seller believing that he is the commitment type.

In this particular case, the rational seller’s equilibrium payoff of visiting buyer 1 first is

\[
V^1_s = (1 - z_b e^{\lambda_1 T^c_1}) \alpha_s + z_b e^{\lambda_1 T^c_1} \alpha_1
\]

Hence, the seller is strong against buyer 1 if the buyers’ bids satisfy the inequalities stated in the premises of Lemma 4B–2.

**Lemma 4B–3.** Consider an equilibrium strategy following a history where the rational seller visits buyer 1 first and the players’ initial bids satisfy \(\delta \alpha_1 \geq \alpha_2\), \(\delta \alpha_s - \alpha_1 \delta > z_b\) and \(z_s^{\lambda_1 / \lambda_1} \geq z_b\). Then, the continuation strategies of this equilibrium strategy will be as follows. Rational buyer 1 immediately accepts the seller’s demand with certainty upon his arrival. Otherwise, the seller leaves 1 immediately at time 0 (knowing that buyer 1 is obstinate), and goes directly to buyer 2.

\(\text{Note that for notational simplicity, I reset the clock once the buyer enters store 2.}\)
2. Rational buyer 2 instantly accepts the seller’s demand with probability 1 upon the seller’s arrival. In case buyer 2 does not concede, the rational seller immediately leaves this buyer, directly returns to buyer 1, accepts 1’s demand $\alpha_1$ and finalizes the game.

In this particular case, the rational seller’s equilibrium payoff of visiting buyer 1 first is

$$V_s^1 = (1 - z_b)\alpha_s[1 + \delta z_b] + \delta^2 z_b^2 \alpha_1$$

(10)

Therefore, the seller is strong against buyer 1 if the buyers’ bids satisfy the inequalities stated in the premises of Lemma 4B–3.

**Lemma 4B–4.** Consider an equilibrium strategy following a history where the rational seller visits buyer 1 first and the players’ initial bids satisfy $\delta \alpha_1 \geq \alpha_2$ and $\frac{\delta \alpha_1 - \alpha_2}{\delta (\alpha_s - \delta \alpha_1)} > z_b > z_s^{\lambda_1/\alpha_1}$. Then, the continuation strategies of this equilibrium strategy will be as follows. Rational buyer 1 does not concede to the seller’s demand at time 0. The seller leaves 1 immediately at time 0 (still believing that buyer 1 is the obstinate type with probability $z_b$), and goes directly to 2. Rational buyer 2 instantly accepts the seller’s demand with probability 1 upon the seller’s arrival. In case buyer 2 does not concede, the rational seller immediately leaves this buyer, directly returns to buyer 1. With the return of the seller, buyer 1 and the seller immediately starts playing the concession game, which may continue until the time $T_e^1 = -\frac{\log z_b}{\lambda_1}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_1 t}$ and $F_s^1(t) = 1 - (z_s/z_b^{\lambda_1/\lambda_1})e^{-\lambda_1 t}$ for all $t \geq 0$.

In this particular case, the rational seller’s equilibrium payoff of visiting buyer 1 first is

$$V_s^1 = \delta[(1 - z_b)\alpha_s + \delta z_b v_s^1]$$

(11)

where $v_s^1 = (1 - z_b e^{\lambda_1 T_e^1})\alpha_s + z_b e^{\lambda_1 T_e^1} \alpha_1$. Note that if the buyers’ bids satisfy the inequalities stated in the premises of Lemma 4B–4, then the seller is weak against buyer 1. However, the seller’s expected payoff of visiting buyer 1 is strictly higher than $\alpha_1$.

5. Closing Remarks

This paper aims to understand if a specific bargaining tactic—playing the tough bargainer, or standing firm and not backing down from an offer—would help rational negotiators in competitive environments, where searching and bargaining for a better deal are the key factors. The analyses show that even in a highly competitive environment, buyers can possess significant market power by utilizing this tactic if they have the chance to build reputation for their obstinacy. However, this conclusion is possible if the buyers’ prior beliefs about each other’s toughness is divergent (i.e., when the competing negotiators are slightly “overconfident” about
being a flexible negotiator). Surprisingly, having higher reputation for being a tough negotiator hurts a competing negotiator.

What is perhaps more interesting is that in equilibrium, the buyers can achieve positive surplus even when the size of the divergence in the buyers’ prior beliefs is very small. This result indicates that what matters to sustain outcomes other than the monopoly outcome is not the heterogeneous prior beliefs per-se. But, it is the opportunity that the buyers obtain to make the seller demand less. The analysis show that the players’ reputational concerns give rise this valuable opportunity.

If each buyer conjectures that his opponents are tougher negotiators, then in equilibrium each buyer will believe that the seller will visit him first. Therefore, divergent prior beliefs will weaken the intensity of the competition between the buyers. However, in equilibrium, each buyer knows that the seller will immediately leave him if he does not accept the seller’s price upon his arrival. That is, the competition between the buyers still has strong bite. Nevertheless, the belief that the seller will visit him first is enough for a buyer to get a positive ex-ante surplus in the game. The seller’s reputational concern will prevent him to make a greedy initial demand because high demands will make him weak. When the seller is weak, the seller cannot abandon the first buyer he visits to negotiate with the second buyer unless the seller builds his reputation for obstinacy up to a level that is enough for him to be strong against the second buyer. A player builds his reputation during the concession game by following a mixed strategy; mixing between conceding and waiting for concession. Therefore, building reputation is costly for the seller. Thus, in order to minimize the cost of reputation building (even if the seller’s reputational concerns are very small) the seller will choose to be less greedy, and thus announce a price that is possibly much less than the monopoly price.

**APPENDIX**

Proof of Theorem 1. Suppose for a contradiction that for some \(z_b, z_s, \delta, r_b\) and \(r_s\) there exists an equilibrium strategy profile, which I will denote it by \(\sigma^*\), where the players’ strategies in stage 1 satisfy \(1 > \alpha_s > \alpha_1, \alpha_2\). Next, I will show that the following four results must simultaneously hold. However, since they are incompatible, we will achieve the desired contradiction. The case where \(1 = \alpha_s > \alpha_1, \alpha_2\) will be examined separately at the end of this proof.

Proposition 1.1. In any equilibrium where \(1 > \alpha_s > \alpha_1, \alpha_2\) holds, at least one buyer must be weak.

Proof of Proposition 1.1. Suppose for a contradiction that there exists an equilibrium where \(1 > \alpha_s > \alpha_1, \alpha_2\) holds and both buyers are strong. Since both buyers are strong, the seller must
be weak against both buyers. Thus, the rational seller’s expected payoff of visiting buyer $i$ first is simply $\alpha_i$. That is, the seller’s continuation payoff, following a history where the seller visits buyer $i$ first at time 0 to play the concession game, is $\alpha_i$.

If the last argument is true, then the buyers’ prices must be the same (i.e., $\alpha_1 = \alpha_2 = \alpha_b < \alpha_s$). To prove this claim I need to consider two cases:

**CASE 1:** Suppose for a contradiction that $\alpha_1 > \alpha_2$ and $\delta\alpha_1 \geq \alpha_2$. The seller is weak against buyer 1 if and only if premises of Lemma 4B-2 and $z_b > z_s^{\lambda_1/\lambda_s}$ hold. According to Lemma 4B-2, rational seller never leaves buyer 1 in equilibrium, which implies that buyer 2’s expected payoff in the game is strictly less than $1 - \alpha_s$ (indeed it is $z_b z_s (1 - \alpha_s) e^{\epsilon (T^d + \Delta)}$.) However, rational buyer 2 can profitably deviate by posting $\alpha_s + \epsilon$ in stage 1 for some small $\epsilon \geq 0$, contradicting with the optimality of the equilibrium.

**CASE 2:** Suppose for a contradiction that $\alpha_1 > \alpha_2$ and $\delta\alpha_1 < \alpha_2$. Since $\alpha_1 > \alpha_2$, the rational seller will visit buyer 1 first (and get the expected payoff of $\alpha_1$). According to Proposition 4.2, the rational seller visits buyer 2 when he builds enough reputation for obstinacy so that he becomes strong against buyer 2. Hence, Buyer 2’s expected payoff in the game must be strictly less than $1 - \alpha_s$. Similar arguments in the previous case shows that buyer 2 can profitably deviate, contradicting with the optimality of the equilibrium.

Since both cases lead to a contradiction, we can conclude that both buyers must post the same price, say $\alpha_b < \alpha_s$. Since the seller is weak against both buyers and both buyers bid the same price in stage 1, then one of the followings must hold:

(A) $z_s < z_b^{\lambda_s/\lambda_b}$ (as given by Proposition 4.3) holds if $\delta \alpha_s \leq \alpha_b$ or $z_b \geq A$ is true, or

(B) $z_s < (z_b^{\lambda_b/A})^{\lambda_s/\lambda_b}$ (as given by Lemma 4A-2) holds if the inequalities $\delta \alpha_s > \alpha_b$ and $z_b < A$ are satisfied.

Note that, we have $\lambda_s = (1 - \alpha_s) \frac{\rho_s}{\alpha_s - \alpha_b}$, $\lambda_b = \frac{\alpha_b \rho_s}{\alpha_s - \alpha_b}$ and $A = \frac{\delta \alpha_s}{\delta (\alpha_s - \alpha_b)}$. Since both buyers are strong in the game, the inequalities that $z_b$ and $z_s$ must satisfy are strict.

**CASE A:** If the first inequality (i.e., $z_s < z_b^{\lambda_s/\lambda_b}$) holds, then I will show that the buyers can profitably deviate, which contradicts with the optimality of the equilibrium. First note that the seller must be indifferent between the buyers as both post the same price. Let $p_i$ be the probability that the seller visits buyer $i$ first according to the equilibrium strategy. If $p_i < p_j$, then I say buyer $i$ has more incentive to deviate. The buyers’ incentive to deviate is equal if $p_i = 1/2$. Therefore, pick the buyer $i$ such that $p_i \leq p_j$. Suppose, w.l.o.g, that it is buyer 1. Instead of posting $\alpha_b$, he would post $\alpha_1 = \alpha_b + \epsilon$ where $\epsilon > 0$ small enough so that $\alpha_1 > \delta \alpha_s$ and $\alpha_b > \delta \alpha_1$. With these parameter values, according to Proposition 4.3, rational seller never leaves the buyer he visits first. Moreover, since $\epsilon$ can be selected very small, we can guarantee that $z_s < z_b^{\lambda_s/\lambda_b} < z_b^{\lambda_1/\lambda_s}$ as $\lambda_s/\lambda_b > \lambda_1^{\epsilon}/\lambda_1$, where $\lambda_i = (1 - \alpha_i) \frac{\rho_i}{\alpha_s - \alpha_i}$, $\lambda_i = \frac{\alpha_i \rho_s}{\alpha_s - \alpha_i}$. That is, the seller
will still be weak against both buyers. As a result, the seller will pick buyer 1 who posts a higher price in stage 1 with probability 1.

Next, I need to show that with this deviation buyer 1 will get a higher expected payoff. The rational buyer’s expected payoff before deviation is \( V_1 = p_1 v_1 + (1 - p_1) z_s z_b e^{r s(T_2^d + \Delta)} (1 - \alpha_s) \) where \( T_2^d = -\frac{\log z_s}{\lambda_b} \) and \( v_1 = (1 - \alpha_b)(1 - z_s z_b^{-\lambda_s/\lambda_b}) + (1 - \alpha_s) z_s z_b^{-\lambda_s/\lambda_b} \), which can be calculated by Proposition 4.3. The second part of \( V_1 \) corresponds to the buyer 1’s expected payoff when the seller visits buyer 2 first. Note that according to Proposition 4.3, the rational buyer 2 will never let the seller leave his store and the rational seller will never leave buyer 2 either. On the other hand, buyer 1’s expected payoff after deviating to \( \lambda \) for sure. Similar arguments used in Case A suffice to show that rational buyer 1 can gain by deviating to \( \lambda \). Instead of posting \( \alpha_b \), he would post \( \alpha_1 = \alpha_b + \epsilon \) where \( \epsilon > 0 \) small enough so that these three inequalities simultaneously hold: (1) \( \delta \alpha_s > \alpha_1 > \alpha_b \), (2) \( z_b < A_i = \frac{\delta \alpha_s - \alpha_b}{\delta (\alpha_s - \alpha_b)} \); and (3) \( z_s < (z_b/A_i)^{\lambda_i/\lambda_b} z_b^{\lambda_i/\lambda_b} \) for each \( i = 1, 2 \) and \( j \in \{1, 2\} \) with \( j \neq i \).

We can always pick \( \epsilon > 0 \) small enough so that these three inequalities simultaneously hold. As a result of this deviation, the seller will still be weak against both buyers, and hence, optimality of the equilibrium implies that the seller will pick the buyer who posts the higher price (i.e., buyer 1) for sure. Similar arguments used in Case A suffice to show that rational buyer 1 can gain by deviating to \( \alpha_b + \epsilon \).

Since we attain contradiction in both case, A and B, we can conclude that at least one buyer must be weak in equilibrium.

Q.E.D.

**Proposition 1.2.** In any equilibrium where \( 1 > \alpha_s > \alpha_1, \alpha_2 \) holds, if a buyer is weak, then the buyers’ prices, \( \alpha_1 \) and \( \alpha_2 \), are different (i.e., \( \alpha_1 \neq \alpha_2 \)).

**Proof of Proposition 1.2.** Suppose for a contradiction that there exists an equilibrium where \( 1 > \alpha_s > \alpha_1, \alpha_2 \) holds, one of the buyers is weak and the buyers post the same price \( \alpha_b \).

Now, I will show that the seller must visit the weak buyer, say buyer 1, with probability 1. I suppose for a contradiction that the seller visits buyer 1 with probability \( p_1 < 1 \). Therefore, rational buyer 1’s expected payoff in the game is less than \( p_1 (1 - \alpha_s) + (1 - p_1) z_b \delta (1 - \alpha_s) \), where the first part is buyer 1’s expected payoff when he is visited first and the second part is strictly
greater than his expected payoff if he is visited as second (recall that rational buyer 2 never lets the seller leave him without an agreement.) That is, rational buyer 1’s expected payoff in the game is strictly less than $1 - \alpha_s$. However, by posting his price as $\alpha_s$ (if $\alpha_b < \alpha_s$) or $\alpha_s + \epsilon$ where $\epsilon > 0$ is small enough if $\alpha_b = \alpha_s$, buyer 1 can ensure strictly higher payoff.

Note that both buyers must be weak in equilibrium because the buyers are identical and they both post the same price. Thus, in equilibrium, the seller must visit both buyers with probability 1, leading to contradiction.

Q.E.D.

**Proposition 1.3.** In any equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds, buyer 1 must be strong.

**Proof of Proposition 1.3.** Therefore, suppose for a contradiction that there exists an equilibrium where $1 > \alpha_s > \alpha_1 > \alpha_2$ holds and buyer 1 is weak. Next, I will show that buyer 2 has incentive to deviate, contradicting with the optimality of equilibrium. There are two exhaustive cases we need to consider:

**Case I: $\delta \alpha_1 \leq \alpha_2$** Similar arguments used in Proposition 1.2 suffices to show that the seller must visit buyer 1 with probability 1 (or else buyer 1 can deviate and post a price $\alpha_s$ to guarantee higher expected payoff in the game). Therefore, in equilibrium the second buyer will be visited after the seller visits 1.

Moreover, since $\delta \alpha_1 \leq \alpha_2$, the rational seller leaves buyer 1 only if he is strong against buyer 2 in the concession game they play after the seller visits buyer 1. Hence, buyer 2’s expected payoff in the game is strictly less than $z_b(1 - \alpha_s)$. However, rational buyer 2 can ensure the payoff of $1 - \alpha_s$ by posting the price of $\alpha_s$.

**Case II: $\delta \alpha_1 > \alpha_2$** Then there are two subcases we should consider.

**Case II-A:** Suppose that $z_b \leq z_s^{\lambda_1/\lambda_s}$ or $z_b \geq \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$. In this case, according to Lemma 4B-2 and 4B-3, buyer 1 will concede to the seller at the time he visits him at time 0. Therefore, similar arguments used in Proposition 1.2 suffices to show that the seller must visit buyer 1 with probability 1. Similar arguments used in Case I ensures that the second buyer has incentive to deviate.

**Case II-B:** Suppose that $z_s^{\lambda_1/\lambda_s} < z_b < \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$. Then, buyer 1 is weak but buyer 1 does not concede to the seller at the time the seller visits him first at time 0. Therefore, the seller does not have to visit buyer 1 first with probability 1. However, since $z_b > 0$, we must have $\delta \alpha_s > \alpha_1$ (i.e., $\alpha_1$ and $\alpha_s$ are apart from one another). Therefore, by Lemma 4B-4, the seller’s expected payoff of visiting buyer 1 is $V_s^1 = \delta[(1 - z_b)\alpha_s + \delta z_b \alpha_1]$, and by Lemma 4B-1 the seller’s expected
payoff of visiting buyer 2 first is \( V_s^2 = (1 - z_b)\alpha_s + \delta z_b \alpha_1 \). Clearly, \( V_s^2 > V_s^1 \). That is, the seller must visit buyer 2 first in equilibrium.

Since the seller visits buyer 2 first with probability 1, Lemma 4B-4 implies that rational buyer 2’s expected payoff in the game is \( 1 - \alpha_s \). However, by deviating to a price \( \hat{\alpha}_2 = \alpha_s - \epsilon \) where \( \epsilon > 0 \) is small enough, buyer 2 can increase his payoff. Here is why: Since \( z_b > z_{s_1}/\lambda_1 > z_s^2/\lambda_2 \) as \( (\lambda_1/\lambda_2) = (\alpha_1 r_b)/(1 - \alpha_s) r_b < (\alpha_2 r_b)/(1 - \alpha_s) r_b = \lambda_2/\lambda_2 \) and \( \epsilon \) is sufficiently small so that \( \delta \alpha_s < \hat{\alpha}_2 \) (and therefore \( z_b > 0 > \delta \alpha_s - \hat{\alpha}_2 \)), then by Lemma 4B-2, rational seller never leaves buyer 2, and thus, the seller is weak against buyer 2. Since buyer 2 becomes strong, he will achieve an expected payoff that is a combination of \( 1 - \alpha_s \) and \( 1 - \hat{\alpha}_2 \) (as buyer 2’s expected payoff is \( F_s^2(0)(1 - \hat{\alpha}_2) + (1 - F_s^2)(1 - \alpha_s) \) and \( F_s^2(0) > 0 \) as buyer 2 is strong).

However, this deviation is profitable for buyer 2 when the seller visits buyer 2 first (once he deviates to \( \hat{\alpha}_2 \)) with a sufficiently high probability. Buyer 2 can ensure that the seller visits his store with probability 1 if he picks \( \epsilon > 0 \) small enough so that \( V_s > V_s^1 \), where \( V_s^1 = \alpha_s(1 - z_b) + \delta z_b \hat{\alpha}_2 \) (by Lemma 4B-1). That is, if \( \epsilon < 0 \alpha_s \frac{z_b(1 - \delta)}{1 - \delta z_b} \) holds, then buyer 2 can profitable deviate from \( \alpha_s \).

Q.E.D.

**Proposition 1.4.** In any equilibrium where \( 1 > \alpha_s > \alpha_1 > \alpha_2 \) holds, buyer 2 is strong.

**Proof of Proposition 1.4.** Suppose for a contradiction that there exists an equilibrium where \( 1 > \alpha_s > \alpha_1 > \alpha_2 \) holds and buyer 2 is weak. Similar arguments in the proof of Proposition 1.2 ensure that the seller must visit buyer 2 with probability 1 in equilibrium, and so, buyer 2’s expected payoff in the game is \( 1 - \alpha_s \). Then, there are two exhaustive cases we need to consider.

**Case I:** \( \delta \alpha_1 < \alpha_2 \) In this case, if the seller visits buyer 1 after visiting buyer 2, then the seller will be strong against buyer 1. Hence, buyer 1’s expected payoff in the game is much less than \( z_b(1 - \alpha_s) \). However, buyer 1 can deviate to a price \( \hat{\alpha}_1 = \alpha_s + \epsilon \) where \( \epsilon > 0 \) and achieve a payoff that is very close to \( 1 - \alpha_s \).

**Case II:** \( \delta \alpha_1 \geq \alpha_2 \) In this case, there are two sub-cases we need to consider.

**Case II-A:** Suppose that \( \alpha_1 \geq \delta \alpha_s \). By Lemma 4B-2, if the seller visits buyer 1 first, then the seller never leaves buyer 1. If buyer 1 is weak, then \( z_s^{\lambda_1/\lambda_1} \geq z_b \) must hold. Thus, buyer 1’s expected payoff in the game will be less than \( z_b(1 - \alpha_s) \) since the seller visits 1 after visiting buyer 2 first. However, buyer 1 can profitably deviate by posting a price \( \hat{\alpha}_1 = \alpha_s + \epsilon \) where \( \epsilon \geq 0 \) small enough, which contradicts with the optimality of the equilibrium.
On the other hand, if buyer 1 is strong in the game, then we should have \( \alpha_1 < 1 - \alpha_s \). In this case, rational buyer 2’s expected payoff in the game will be \( 1 - \alpha_s \). Next, I will show that buyer 2 can achieve a higher expected payoff by deviating. Consider the case where buyer 2 posts \( \hat{\alpha}_2 = \alpha_1 + \epsilon \) in stage 1 where \( \epsilon > 0 \) is small enough so that both \( \hat{\alpha}_2 > \delta \alpha_s \) and \( \alpha_1 > \delta \hat{\alpha}_2 \) hold (so that \( \alpha_1, \hat{\alpha}_2 \) and \( \alpha_s \) are close to each other). Hence, by Proposition 4.3, the rational seller never leaves the buyers and the buyers are strong in the game as \( z_s^{\lambda_2/\lambda_1} < z_s^{1/\lambda_1} < z_b \). Hence, the seller will visit buyer 2 first as \( \hat{\alpha}_2 > \alpha_1 \). Moreover, since 2 is strong in the game once he bids \( \hat{\alpha}_2 \), his expected payoff in the game will be \([1 - F_s^2(0)](1 - \alpha_s) + F_s^2(0)(1 - \hat{\alpha}_2)\) that is strictly higher than \( 1 - \alpha_s \) as \( F_s^2(0) > 0 \). Therefore, buyer 2 can profitably deviate from \( \alpha_2 \), contradicting with the optimality of equilibrium.

**Case II-B:** Suppose that \( \alpha_1 < \delta \alpha_s \). According to Lemma 4B-2, if buyer 1 is strong in equilibrium, we must have \( \frac{\delta \alpha_s - \alpha_1}{\delta \alpha_s - \delta \alpha_1} \leq z_b \) and \( z_s^{\lambda_1/\lambda_s} < z_b \). In this case, rational buyer 2 has incentive to deviate to \( \hat{\alpha}_2 = \alpha_s - \epsilon \) where \( \epsilon > 0 \) is sufficiently small so that both \( \alpha_1 < \delta \hat{\alpha}_2 \) and \( \delta \alpha_s < \hat{\alpha}_2 \) hold. As a result of this deviation, Lemma 4B-2 ensures that the rational seller never leaves buyer 2 if he visits him first, and the seller is weak against buyer 2. Moreover, if buyer 2 picks \( \epsilon \) small enough so that \( V^2_s = \hat{\alpha}_2 \) is larger than \( V^1_s = \alpha_s(1 - z_b) + \delta z_b \hat{\alpha}_2 \) (by Lemma 4B-1 (i.e., \( \epsilon < \alpha_s z_b \frac{(1 - \delta)}{1 - \delta z_b} \)), then the rational seller visits buyer 2 with probability 1. As a result, buyer 2 guarantees expected payoff slightly bigger than \( 1 - \alpha_s \) by deviating from \( \alpha_2 \), contradicting with the optimality of \( \sigma^* \).

On the other hand, if buyer 1 is weak, then we have 2 exhaustive cases to consider.

**Case II-B-1:** If the prices and the primitives satisfy the premises of Lemma 4B-2 or Lemma 4B-3, then buyer 1 being weak means \( z_b \leq z_s^{\lambda_1/\lambda_s} \) holds. Thus, buyer 1’s expected payoff in the game will be less than \( z_b (1 - \alpha_s) \) since the seller visits buyer 1 after visiting buyer 2 first. As argued above, buyer 1 prefers to deviate to a price slightly above \( \alpha_s \), contradicting with the optimality of equilibrium.

**Case II-B-2:** If the prices and the primitives satisfy the premises of Lemma 4B-4 (i.e., \( z_s^{\lambda_1/\lambda_s} < \bar{z}_b \leq \frac{\delta \alpha_s - \alpha_1}{\delta \alpha_s - \delta \alpha_1} \)), then buyer 1’s expected payoff in the game is higher than \( 1 - \alpha_s \). However, since \( z_s^{\lambda_1/\lambda_s} < z_b \) holds, rational buyer 2 has incentive to deviate to \( \hat{\alpha}_2 = \alpha_s - \epsilon \) as I argued above (Case II-B, buyer 1 is strong in the game), where \( \epsilon > 0 \) is sufficiently small. The last argument leads to a desired contradiction and finalizes the proof. Q.E.D.

Finally, I will discuss that there does not exist an equilibrium where \( 1 = \alpha_s > \alpha_1, \alpha_2 \). Suppose for a contradiction that there exists. The first observation will be that in equilibrium both buyers’ ex-ante game payoff is positive: If, for example, buyer 1’s equilibrium payoff is 0, then he would deviate to a price \( \alpha'_1 = 1 - \epsilon \) where \( \epsilon \) is small enough so that the seller prefers to make agreement with buyer 1 and buyer 1 makes a positive payoff. However, in any equilibrium
where the seller accepts a buyer’s demand (at some time) with a positive probability, the buyer’s
best response would be never conceding to the seller. This is true simply because if a buyer
concedes to the seller, then his payoff will be 0.

Therefore, in equilibrium, the seller concedes to both buyers with positive probabilities and
the buyers never concede to the seller. That is, the seller is weak against both buyers. Hence,
the optimality of equilibrium implies that the seller should concede to the buyer with the highest
price, or the buyers’ prices are the same (i.e., $\alpha_1 = \alpha_2$). In either case, at least one of the buyers
would have incentive to overbid his opponent unless $\alpha_i = 1$ for some $i \in \{1, 2\}$, contradicting
with the optimality of equilibrium. Q.E.D. for the proof of Theorem 1

Proof of Proposition 4.1. First, I will study the properties of the equilibrium strategies
(distribution functions) in concession games. For this purpose, take any $i \in \{1, 2\}$ and history
$h_{Ti} \in H_i$, and consider a pair of equilibrium distribution functions $(F_{s,Ti}, F_{Ti})$ defined over
the domain $[T_i, T'_i]$ where $T'_i \leq \infty$ depends on the seller’s equilibrium strategy. Proofs of the
following results directly follow from Ozyurt (2014), from the arguments in Hendricks, Weiss and
Wilson (1988), and are analogous to the proof of Lemma 1 in Abreu and Gul (2000). Therefore,
I skip the details.

Lemma A.1. If a player’s strategy is constant on some interval $[t_1, t_2] \subseteq [T_i, T'_i]$, then his
opponent’s strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.

Lemma A.2. $F_{s,Ti}$ and $F_{Ti}$ do not have a mass point over $[T_i, T'_i]$.

Lemma A.3. $F_{Ti}(T_i)F_{s,Ti}(T_i) = 0$

Therefore, according to Lemma A.1 and A.2, both $F_{Ti}$ and $F_{s,Ti}$ are strictly increasing and
continuous over $[T_i, T'_i]$. Recall that

$$U_s^i(t, F_s^T) = \alpha_s \int_0^{t-T} e^{-rs} dF_s^T(y) + \alpha_i [1 - F_s^T(t)] e^{-rs(t-T)}$$

denotes the expected payoff of rational seller who concedes at time $t \geq T_i$ and

$$U_i(t, F_s^T) = (1 - \alpha_i) \int_0^{t-T} e^{-rs} dF_s^T(y) + (1 - \alpha_s) [1 - F_s^T(t)] e^{-rs(t-T)}$$

denotes the expected payoff of the rational buyer $i$ who concedes to the seller at time $t \geq T_i$.
Therefore, the utility functions are also continuous on $[T_i, T'_i]$.

Then, it follows that $D_{i,Ti} := \{t \mid U_i(t, F_s^T) = \max_{y \in [T_i, T'_i]} U_i(y, F_s^T(t))\}$ is dense in $[T_i, T'_i]$.
Hence, $U_i(t, F_s^T)$ is constant for all $t \in [T_i, T'_i]$. Consequently, $D_{i,Ti} = [T_i, T'_i]$. Therefore,$U_i(t, F_s^T)$ is differentiable as a function of $t$. The same arguments also hold for $F_{s,Ti}$. The

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differentiability of $F_{i}^{T_i}$ and $F_s^{T_i}$ follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz’s rule, we get $F_{i}^{T_i}(t) = 1 - c_i e^{-\lambda_it}$ and $F_s^{T_i}(t) = 1 - c_s' e^{-\lambda_st}$ where $c_i = 1 - F_{i}^{T_i}(T_i)$ and $c_s' = 1 - F_s^{T_i}(T_i)$ such that $\lambda_i = \frac{\alpha_s r_i}{\alpha_s - \alpha_i}$ and $\lambda_s' = \frac{(1-\alpha_s)r_s}{\alpha_s - \alpha_i}$.

Q.E.D.

**Proof of Proposition 4.2.** Given the strategies in Proposition 4.1, the rational seller’s expected payoff of playing the concession game with buyer $i$ during $[T_i, T_i']$ is $[\alpha_s F_{i}^{T_i}(T_i)) + \alpha_i(1 - F_s^{T_i}(T_i))]$. Moreover, by Lemma A.3, we know that if the seller is strong in a concession game with buyer $i$ (starting at time $T_i$), then buyer $i$ is weak. Hence, there is no sequential equilibrium of the game $G$ such that the seller visits a buyer multiple times. Suppose on the contrary that there is an equilibrium strategy in which the rational seller visits, without loss of generality, buyer 1 twice. Then, the seller must be strong in his second visit to buyer 1. Otherwise the seller would prefer to concede to buyer 2 and finish the game before making the second visit to buyer 1 (because $\delta \alpha_1 < \alpha_2$). Thus, since buyer 1 is weak, his expected payoff is $1 - \alpha_s$ when the seller visits his store for the second time. However, this continuation payoff contradicts the optimality of the equilibrium strategy because buyer 1 strictly prefers accepting the seller’s offer (for sure) when the seller first attempts to leave him to eliminate a further delay. Therefore, in equilibrium, a rational buyer will not allow the seller to leave his store without an agreement.

For notational simplicity, I reset the clock each time the seller arrives at a buyer, and denote the seller’s concession game strategy against buyer $i$ by $F_i^s$ and $i$’s strategy by $F_i$. Now, consider an equilibrium strategy following a history where the rational seller visits buyer 2 first and leaves buyer 2 at time $T_2^d$. Note that, rational seller visits buyer 1 only if $F_1(0) > 0$ is true. Suppose $F_1(0) = 0$. Then, the rational seller’s discounted continuation payoff with buyer 1, $\delta[\alpha_s F_1(0) + \alpha_i(1 - F_1(0))]$, will be $\delta \alpha_1$. Since $\delta \alpha_1 < \alpha_2$, the rational seller prefers to concede to buyer 2 instead of visiting buyer 1, yielding the required contradiction. By lemma A.3., as $F_1(0) > 0$, we must have $F_1^s(0) = 0$, implying that $c_1^s = 1$. That is, $F_s^1(t) = 1 - e^{-\lambda_it}$. Furthermore, assuming that the rational seller leaves buyer 2 at time $T_2^d$ and the concession game with buyer 1 ends at time $T_1^e$, we must have $F_2(T_2^d) = 1 - z_b$ and $F_1(T_1^e) = 1 - z_b$. Thus we have $c_2 = z_b e^{\lambda_2 T_2^d}$ and $c_1 = z_b e^{\lambda_1 T_1^e}$ as required.

Finally, Lemma A.3 implies that $F_s^2(0) F_2(0) = 0$. Since buyer 1’s reputation reaches 1 at time $T_1^e$, then the rational seller will not continue the game $G$ after this time. Thus, his reputation must also reach 1 at that time, implying that $F_s^1(T_1^e) = 1 - z_s^*$ where $z_s^* = \frac{z_s}{1 - F_2^s(T_2^d)}$ is the seller’s reputation at the time he arrives at buyer 1.

Q.E.D.

**Proof of Lemma 4A–1.** Given the history described in the premises of the Lemma, in equilibrium, the rational seller (weakly) prefers to go to buyer 1 over conceding to buyer 2. In
equilibrium, rational buyer 2 anticipates that the seller will never concede to him, and hence
accepts \( \alpha_s \) at time 0 without any delay. Therefore, if buyer 2 is rational, then the game should
finish at time 0. Otherwise, the seller leaves the second buyer at time 0 and directly goes to buyer 1. Therefore, the concession game with buyer 1 ends at time \( T \).

\[ T_1 = \tau_1 = \min\{\tau_1^s, \tau_1^b\} \]

for sure, where \( \tau_1^s = \inf\{t \geq 0 | F_1^s(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_1} \) and \( \tau_1^b = \inf\{t \geq 0 | F_1^b(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_2} \) denote the times that the seller’s and buyer 1’s reputations reach 1, respectively. Given the equilibrium strategies by Proposition 4.1, the rest follows.

Q.E.D.

**Proof of Lemma 4A–2.** Given the history described in the premises of the Lemma, in equilibrium, the rational seller prefers to play the concession game with buyer 2 over going to buyer 1 at time 0. Note that the rational seller leaves buyer 2 if and only if buyer 2 is the commitment type. The reason for this is the following:

Since the players’ concession game strategies are increasing and continuous, the buyers’ reputation will eventually converge to 1 at some finite time. Similarly, the seller’s reputation will increase to a level that is sufficiently high (but strictly less than 1) so that it will be optimal for the seller to visit the other buyer. Hence, in equilibrium, the seller will leave the second buyer when the seller is indifferent between conceding to buyer 2 and visiting buyer 1. Call this time as \( T_2^d \). Moreover, the buyer 2’s reputation must reach 1 at time \( T_2^d \). The rational seller will break his indifference at this time by leaving the buyer because according to Lemma A.2 concession game strategies must be continuous in their domain, eliminating the possibility of mass acceptance at time \( T_2^d \). Hence, buyer 2’s reputation reaches 1 at time \( T_2^d = \tau_2 = \min\{\tau_2^s, \tau_2^b\} \)

where \( \tau_1^s = \inf\{t \geq 0 | F_2^s(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_2} \) and \( \tau_1^b = \inf\{t \geq 0 | F_2^b(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_2} \) denote the times that the seller’s and buyer 2’s reputations reach 1, respectively.

However, leaving 2 is optimal for the rational seller if and only if the seller’s reputation at time \( T_2^d \) reaches \( z_s^* \), implying that

\[ c_s^2 e^{-\lambda_2 T_2^d} = \frac{z_s}{z_s^*} \tag{12} \]

Given the value of \( T_2^d \), solving the last equality yields the seller’s equilibrium strategy with buyer 2. Finally, the game ends with buyer 1 at time \( T_1^e = \tau_1 = \min\{\tau_1^s, \tau_1^b\} \) for sure where \( \tau_1^s = -\frac{\log z_s}{\lambda_1} \) and \( \tau_1^b = -\frac{\log z_b}{\lambda_2} \), at which points both players’ reputation simultaneously reach 1. Given the value of \( T_1^e \), Proposition 4.1 implies the concession game strategies in the second store.

Q.E.D.

**Proof of Lemma 4A–3.** In equilibrium, the rational seller leaves buyer 2 when his reputation reaches \( z_s^* \), implying that Equation (12) holds. If \( c_s^2 = 1 \), then \( T_2^d = -\frac{\log z_s}{\lambda_2} + \frac{\lambda_1 \log(z_b/A_2)}{\lambda_1 \lambda_2} \), and it is smaller than \( -\frac{\log z_s}{\lambda_2} \) as \((z_b/A_2)^{\lambda_1/\lambda_2} < z_s \). Similar to Lemma 4A–2, the game ends with
buyer 1 at time $T^e_1 = \frac{\log z^s}{\lambda^s}$. Given the values of $T^d_2$ and $T^e_1$, Proposition 4.1 and 4.3 imply the concession game strategies.

Q.E.D.

**Proof of Proposition 4.3.** If condition (1) does not hold, then it is clear that the rational seller will never leave buyer 2. However, if (2) fails to hold, then the rational seller will not be able to build enough reputation to make visiting the other buyer optimal. Also, rational seller will not benefit by deviating and not conceding to buyer 2, leaving 2 at time $T^e_2$ and visiting buyer 1. This is because with this deviation, rational seller’s expected payoff will be at most $(1-z_b)\delta \alpha_s + z_b\delta \alpha_1$ which is less than conceding to buyer 2 (i.e., $\alpha_2$) as condition (2) does not hold. Propositions 4.1 and 4.2 give the functional forms of the players’ strategies.

Q.E.D.

**Proof of Lemma 4B–1.** Since we have $\alpha_2 \leq \delta \alpha_1$, the rational seller prefers going to buyer 1 over conceding to 2 at any given time. That is, in equilibrium, the rational seller never concedes to buyer 2. Since rational buyer 2 anticipates that the rational seller will never accept his demand in equilibrium, he concedes to the seller with probability 1 upon his arrival without any delay. Thus, the seller leaves buyer 2 immediately if rational buyer 2 does not accept the seller’s demand and finish the game at time 0.

When the seller arrives at buyer 1 (after visiting 2), the rational seller and buyer 1 play the concession game until some finite time $T^e_1$ as the seller has no outside option worth leaving buyer 1. As characterized in the proof of Proposition 4.1, the equilibrium strategies are $F^1_s(t) = 1 - c^1_s e^{-\lambda^1_s t}$ and $F^1(t) = 1 - c^1 e^{-\lambda^1 t}$. Therefore, the concession game with buyer 1 ends at time $T^e_1 = \min\{\tau^1_s, \tau_1\}$ for sure if it does not end before, where $\tau^1_s = \inf\{t \geq 0 \mid F^1_s(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda^s}$ and $\tau_1 = \inf\{t \geq 0 \mid F_1(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda}$, denote the times that the seller’s and buyer 1’s reputations reach 1, respectively.

Q.E.D.

**Proof of Lemma 4B–2.** If the rational seller concedes to buyer 1, his instantaneous payoff is $\alpha_1$. However, if the rational seller leaves buyer 1 at time 0 and goes to buyer 2, then we know from Lemma 4B–1 that rational buyer 2 will immediately accept the seller’s demand. Therefore, the rational seller’s continuation payoff of leaving buyer 1 at time 0 is $\bar{V}_s = \delta[(1-z_b)\alpha_s + \delta z_b v^1_s]$, where $v^1_s = (1 - F_1(0))\alpha_1 + F_1(0)\alpha_s$ denotes the seller’s expected payoff in his second visit to buyer 1. In equilibrium $v^1_s$ must be equal to $\alpha_1$. Suppose for a contradiction that $v^1_s > \alpha_1$. It requires that buyer 1 offers positive probabilistic gift to the seller on his second visit. In this case, buyer 1’s expected payoff must be $1 - \alpha_s$ (as $F^1_s(0)F_1(0) = 0$ by Lemma A.3). However, optimality of the equilibrium implies that rational buyer 1 should have accepted the seller’s offer.
with probability 1 when the seller attempts to leave him for the first time. Hence, it must be that in equilibrium $v^1_s = \alpha_1$. As a result, the rational seller’s expected payoff if he leaves buyer 1 at time 0 is $\tilde{V}_s = \delta [(1 - z_b)\alpha_s + \delta z_b \alpha_1]$.

Finally, if $\tilde{V}_s$ is strictly greater than $\alpha_1$, then the rational seller prefers leaving buyer 1 immediately at time 0 over conceding to buyer 1. $\tilde{V}_s > \alpha_1$ implies that $z_b < \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$. Since the opposite of this inequality is assumed to hold, the rational seller will never leave buyer 1.

Equilibrium strategies follow from Proposition 4.1 and 4.3. Moreover, rational seller will not benefit by deviating and not conceding to buyer 1, leaving 1 at time $T_e^1$ and visiting buyer 2. This is because with this deviation, rational seller’s expected payoff will be at most $(1 - z_b)\delta \alpha_s + z_b \delta^2 \alpha_1$, and this payoff is less than the payoff of conceding to buyer 1 (i.e., $\alpha_1$) due to the condition on $z_b$.

Proof of Lemma 4B–3. Since $z_b < \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$, the rational seller will prefer to visit buyer 2 instead of conceding to buyer 1. Note that rational seller will revisit buyer 1 if he learns that buyer 2 is the commitment type. On the other hand, rational buyer 1’s payoff of conceding to the seller is $1 - \alpha_s$, but buyer 1’s payoff of letting the seller leave him and waiting for his second visit is $\delta^2 z_b \hat{v}_1$, where $\delta_b = e^{-r_b \Delta}$ and $\hat{v}_1$ denotes buyer 1’s expected payoff of playing the concession game with the seller when he visits buyer 1 for the second time. Notice that $\hat{v}_1 = F_1(0)(1 - \alpha_1) + (1 - F_1(0))(1 - \alpha_s)$ with $F_1(0) = 1 - z_s e^{\lambda_1 T^e_1}$, where $T^e_1 = \frac{-\log z_s}{\lambda_1}$ (which is strictly less than $\frac{-\log z_s}{\lambda_s}$), $z_b^* = \frac{z_b}{z_b + (1 - z_b)\mu}$, and $\mu$ denotes the probability that rational buyer 1 accepts the seller’s demand at time 0 (i.e., the probability of buyer’s concession at the seller’s first visit to buyer 1).

The equality $1 - \alpha_s \geq \delta^2 z_b \hat{v}_1$ implies that we must have $z_b^* \leq \left[\frac{(\alpha_s - \alpha_1)z_s}{1 - \alpha_1 - \frac{\delta z_b}{\delta^2 \mu}}\right]^{\lambda_1 / \lambda_s}$ which is clearly less than $z_s^{\lambda_1 / \lambda_s}$. Therefore, buyer 1 will be weak when the seller visits his store for the second time, implying that there will be no equilibrium where buyer 1 is indifferent between conceding to seller at time 0 and letting the seller leave him. Thus, rational buyer 1 will concede to the seller at time 0 with probability 1 whenever $z_b \leq z_s^{\lambda_1 / \lambda_s}$ holds. Then, the rest of the claim immediately follows.

Q.E.D.

Proof of Lemma 4B–4. Similar arguments to the proof of Lemma 4B–3 suffices to show that rational buyer 1 will not concede to the seller at time 0 whenever $z_b > z_s^{\lambda_1 / \alpha_1}$ holds. Since $z_b < \frac{\delta \alpha_s - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$ holds, the rational seller will prefer to visit buyer 2 instead of conceding to buyer 1. Propositions 4.1 and 4.3 will give the equilibrium strategies of the concession game played between buyer 1 and the seller conditional on the seller visits buyer 1 for the second time.
Proof of Proposition 3.1. I will start with an important observation. If buyer 2 is weak in equilibrium and his equilibrium payoff is less than or equal to $1 - \alpha_s$, then $z_h \geq z_l^{(1-\alpha_s)/\alpha}$ must hold for all $\alpha \in (0, \alpha_s)$. Suppose for a contradiction that there exists some $0 < \alpha^* < \alpha_s$ such that $z_h < z_l^{(1-\alpha_s)/\alpha}$. If this is the case, then $z_h < z_l^{(1-\alpha_s)/\alpha}$ must be true for all $\alpha^* \leq \alpha < \alpha_s$. Let $V_i$ indicates type 2 seller’s expected payoff of visiting buyer $i$ first. In equilibrium, we must have $V_1 < V_2$ since the buyers will not offer the seller more than $\alpha_s$, and buyer 2’s expected payoff in the game is at most $1 - \alpha_s$. Therefore, buyer 2 can increase his payoff by deviating to $\hat{\alpha}_2 = \alpha_s - \epsilon$, where $\epsilon > 0$ is very small so that $\alpha^* \leq \hat{\alpha}_2$ (i.e. buyer 2 is strong according to Proposition 4.3) and $\alpha_s > V_2 > V_1$ (type 2 seller visits buyer 2 first at time 0 with probability 1). However, this contradicts with the optimality of the equilibrium.

Next, by using this observation, I will prove the main claim of the proposition under three exhaustive cases:

**Case I:** The buyers’ bids satisfy $\delta \alpha_1 \geq \alpha_2$: Then according to the equilibrium strategies characterized in Section 4.B, buyer 2 will be weak and his expected payoff following a history where the seller (of type 2) visits buyer 2 first at time 0 will be $1 - \alpha_s$. Together with the above observation, we achieve the desired result.

**Case II:** The buyers’ bids satisfy $\alpha_1 > \alpha_2$ and $\delta \alpha_1 < \alpha_2$: Suppose for a contradiction that buyer 2 is strong. By the results in Section 4.A, buyer 2 is strong if and only if $z_h < z_l^{\lambda_2/\lambda_1 (z_l/A_2)^{\lambda_1/\lambda_2}}$. However, since $(1/A_1)^{\lambda_2/\lambda_1} > (1/A_2)^{\lambda_1/\lambda_1}$ holds, we have $z_h^{\lambda_1/\lambda_1 (z_l/A_1)} > z_l^{\lambda_2/\lambda_1 (z_l/A_2)^{\lambda_1/\lambda_2}} > z_l$. The last inequality implies that type 2 seller believes that buyer 1 is strong in the game as well. Thus, in equilibrium, type 2 seller’s expected payoff of visiting buyer $i$ is simply $\alpha_i$ (since he is weak against both buyers), implying that type 2 seller will visit buyer 1 first with probability 1. However, this contradicts with the optimality of the equilibrium because if buyer 2 believes that the seller will visit buyer 1 first for sure, his expected payoff in the game would be strictly less than $1 - \alpha_s$ (since he will be visited after the seller visits buyer 1), implying that 2 would profitably deviate and post $\alpha_s$ in the first stage. Thus, buyer 2 is weak in equilibrium and according to our results in Section 4.B his expected payoff in the game is no more than $1 - \alpha_s$. Hence, together with the above observation, we reach the desired conclusion.

**Case III:** The buyers’ bids satisfy $\alpha_1 = \alpha_2 = \alpha_b$: Suppose for a contradiction that buyer 2 is strong in the game. There are four exhaustive cases we need to consider.

**Subcase 1:** The buyers’ bids satisfy $\delta \alpha_s > \alpha_b$ and $z_h, z_l < A$: Then, buyer 2 is strong in the game if and only if $z_h < z_l^{\lambda_2/\lambda_1 (z_l/A_2)^{\lambda_1/\lambda_2}}$ (Lemma 4A–2). Thus, buyer 1 is also strong in the game. Similar arguments we did in Case II will lead to the desired contradiction.
**Subcase 2:** The buyers’ bids satisfy $\delta \alpha_s > \alpha_b$ and $z_l < A \leq z_h$: According to Proposition 4.3, type 2 seller will not leave buyer 2 if he visits 2 first. Since, buyer 2 is strong in equilibrium, type 2 seller is weak and his expected payoff of visiting buyer 2, $V_s^2$, is $\alpha_b$. However, $V_s^1 \geq \alpha_b$, implying that in equilibrium, the rational seller of type 2 either visits buyer 2 with probability $1/2$ (if $V_s^1 = \alpha_b$) or with probability 0 (in case $V_s^1 > \alpha_b$). In either case, similar arguments in the proof of Proposition 1.1 (Case A) imply that buyer 2 would profitably deviate. Very briefly, the idea is that if $V_s^1 = \alpha_b$ (i.e., buyer 1 is strong in the game as well), then type 2 seller visits each buyer with probability $1/2$. But, if buyer 2 overbids slightly, then he can get the seller with certainty. If $V_s^1 > \alpha_b$, then 2’s expected payoff is strictly less than $1 - \alpha_s$, and so 2 deviates and posts $\alpha_s$ to get the seller with probability 1.

**Subcase 3:** The buyers’ bids satisfy $\delta \alpha_s > \alpha_b$ and $A < z_l < z_h$: Then similar to the second case, type 2 seller never leaves the store he visits first. Since, 2 is strong in the game, by Proposition 4.3 we must have $z_h < z_l^{\lambda_s/\lambda_b}$. Since $z_l < z_h$, then type 2 seller is also weak against the first buyer (i.e., $z_l < z_l^{\lambda_s/\lambda_b}$). Hence, type 2 seller’s equilibrium payoff of visiting either seller is just $\alpha_b$, and thus the seller visits the buyers with equal probabilities (according to our equilibrium restriction assumption). However, similar arguments in the previous case imply that buyer 2 would profitably deviate, contradicting with the optimality of equilibrium.

**Subcase 4:** The buyers’ bids satisfy $\delta \alpha_s \leq \alpha_b$: Similar arguments in the previous case will lead to a contradiction in this case as well.

Hence, we can conclude that when $\alpha_1 = \alpha_2$ buyer 2 must be weak, and according to our results in Section 4.A, 2’s expected payoff in the game is less than or equal to $1 - \alpha_s$. The last point and the initial observation we made yields the desired result and completes the proof.

Q.E.D.

**REMARK:** Consider an equilibrium strategy where the players’ bids satisfy $1 > \alpha_s > \alpha_1 \geq \alpha_2 > 0$ and $\alpha_2 > \delta \alpha_1$. Let $V_s^i$ indicates the seller’s (either type 1 or 2) expected payoff in the game following a history where the seller visits buyer $i$ first. Next, I will calculate $V_s^2$. The symmetric arguments can directly be used to calculate $V_s^1$. The following expected payoff calculations depend on the type of the seller. This is why I left the buyers’ initial reputations $z_1, z_2$ unspecified. Type 2 seller, for example, assigns $z_1 = z_h$ and $z_2 = z_l$ whereas type 1 seller assigns $z_1 = z_l$ and $z_2 = z_h$. Finally, note that $z_s$ indicates the prior belief that the buyers attach to the event that the seller of either type is obstinate. By our assumption, it is in fact equal to $z_h$. 

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1. Suppose that $\delta \alpha_s > \alpha_2$ and $A_2 = \frac{\delta \alpha_s - \alpha_2}{\delta (\alpha_s - \alpha_1)} > z_1$. Then, the seller’s expected payoff of visiting buyer 2 first is as follows.

(a) If $z_s > (z_1/A_2)^{\lambda_i/\lambda_1}$, then $V_s^2$ is given by Lemma 4A–1 as

$$V_s^2 = \alpha_s \left[ 1 - z_2(1 - \delta) - \frac{\delta z_1 z_2}{z_s^{\lambda_1/\lambda_i}} \right] + \alpha_1 \frac{\delta z_1 z_2}{z_s^{\lambda_1/\lambda_i}}$$  \hspace{1cm} (13)

(b) If $(z_1/A_2)^{\lambda_i/\lambda_1} z_s^2 < z_s < (z_1/A_2)^{\lambda_i/\lambda_1}$, then $V_s^2$ is given by Lemma 4A–3 as

$$V_s^2 = \alpha_s \left[ 1 - z_2 \left( \frac{(z_1/A_2)^{\lambda_i/\lambda_1}}{z_s} \right)^{\frac{\lambda_i}{\lambda_2}} \right] + \alpha_2 z_2 \left( \frac{(z_1/A_2)^{\lambda_i/\lambda_1}}{z_s} \right)^{\frac{\lambda_i}{\lambda_2}}$$ \hspace{1cm} (14)

(c) If $z_s \leq (z_1/A_2)^{\lambda_i/\lambda_1} z_s^{2/\lambda_2}$, then $V_s^2$ is given by Lemma 4A–2 as $V_s^2 = \alpha_2$

2. Suppose now that $\alpha_2 \geq \delta \alpha_s$ or $z_1 \geq A_2$. Then, then $V_s^2$ is given by Proposition 4.3 as

$$V_s^2 = \alpha_s [1 - z_2 z_s^{-\lambda_2/\lambda_i}] + \alpha_2 z_2 z_s^{-\lambda_2/\lambda_i}$$ \hspace{1cm} (15)

**Proof of Theorem 2.** First, I will prove the if-part. For this purpose, consider a strategy profile $\sigma^*$ where players’ bids satisfy $0 < \alpha_1 = \alpha_2 = \alpha_b < \alpha_s < 1$, seller of type $i$ visits buyer $i$ first with probability 1, and the continuation strategies in stage 3 are identical to the ones characterized in Section 4 (Lemma 4A–1, in particular). Also, suppose that the buyers are powerless in this strategy profile. Next, I want to show that $\sigma^*$ is an equilibrium of the game $\tilde{G}$.

Since buyer $i$ believes that the seller is certainly type-$i$, the seller’s deviation from $\sigma^*$ never changes the buyers’ belief that the seller is of type-$i$. Furthermore, since the buyers believe that there is one and only one type of the seller, the equilibrium characterisation that is provided in Section 4 proves that the continuation strategies of $\sigma^*$ constitute an equilibrium of the subgame of $\tilde{G}$, following stage 2.

Next, I will show that the first stage strategies of $\sigma^*$ are also optimal. Consider first the case that $A > z_h$. Since the buyers are powerless, $z_h \geq (z_l/A) \frac{(1-\alpha_j)_{\alpha_j}}{\alpha_{\alpha_j} \alpha_{\alpha_j}}$ must hold. Therefore, type-$i$ seller’s expected payoff of visiting buyer $j$ first is given by equation (13), and it is $V_j^i = \alpha_s \left[ 1 - z_l(1 - \delta) - \frac{\delta z_l z_j}{z_h^{\lambda_j/\lambda_i}} \right] + \alpha_b \frac{\delta z_l z_j}{z_h^{\lambda_j/\lambda_i}}$. Type-$i$ seller’s expected payoff of visiting buyer $i$ first is also given by equation (13) if $z_h \geq (z_l/A) \frac{(1-\alpha_j)_{\alpha_j}}{\alpha_{\alpha_j} \alpha_{\alpha_j}}$ holds, in which case $V_i^i = \alpha_s \left[ 1 - z_l(1 - \delta) - \frac{\delta z_l z_h}{z_h^{\lambda_j/\lambda_i}} \right] + \alpha_b \frac{\delta z_l z_h}{z_h^{\lambda_j/\lambda_i}}$. However, if $(z_l/A) \frac{(1-\alpha_j)_{\alpha_j}}{\alpha_{\alpha_j} \alpha_{\alpha_j}} < z_h < (z_l/A) \frac{(1-\alpha_j)_{\alpha_j}}{\alpha_{\alpha_j} \alpha_{\alpha_j}}$, then $V_i^i$ is given by equation (14). In either case, it is easy to check that $V_i^i > V_j^i$. That is, type-$i$ seller
will (strictly) prefer to visit buyer \( i \) first. Hence, buyer \( i \)’s expected payoff in the game is \( 1 - \alpha_s \).

Moreover, since \( z_h \geq z_l \frac{1 - \alpha_s}{\alpha_s} \) for all \( \alpha \in (0, \alpha_s) \), buyer \( i \) cannot increase his payoff in the game by deviating to a price other than \( \alpha_b \). Since players play equilibrium strategies in stage 3 and do not have any incentive to deviate from their first stage bids, \( \sigma^* \) is an equilibrium of the game \( G \).

Now, consider the second case where \( A \leq 0 \). Then, by Proposition 4.3 we know that the seller will not leave the first buyer he visits. Thus, since the buyers are powerless and \( A \leq 0 \), we must have \( z_h \geq z_l \frac{1 - \alpha_s}{\alpha_s} \) for all \( \alpha \in (0, \alpha_s) \). This condition implies that the buyers are weak and type-\( i \) seller’s expected payoffs are given by equation (15). Once again, since \( z_l < z_h \) we have \( V^i_s > V^j_s \). That is, type-\( i \) seller strictly prefers to visit buyer \( i \) first. Identical arguments at the end of the previous paragraph ensures that \( \sigma^* \) is an equilibrium of the game \( G \).

Next, suppose the third case where \( z_h \geq A > 0 \). There are two exhaustive cases we need to consider.

**Case 1:** Suppose that \( z_h > z_l \geq A \): Then, type-\( i \) seller does not leave the first buyer he visits. Since buyers are powerless and \( A \leq z_h \), we have \( z_h \geq z_l \frac{1 - \alpha_s}{\alpha_s} \) for all \( \alpha \in (0, \alpha_s) \). That is, type-\( i \) seller is strong against both buyers and hence the seller’s expected payoffs are given by the equation (15), implying that \( V^i_s > V^j_s \). Thus, type-\( i \) seller strictly prefer to visit buyer \( i \) first. Identical arguments at the end of the second paragraph ensures that \( \sigma^* \) is an equilibrium of the game \( G \).

**Case 2:** Suppose that \( z_h \geq A > z_l \): Then, type-\( i \) seller never leaves buyer \( i \). There are three exhaustive subcases we need to consider. In all three cases I will show that type-\( i \) seller strictly prefers to visit buyer \( i \) first. Thus, identical arguments at the end of the second paragraph ensures that \( \sigma^* \) is an equilibrium of the game \( G \).

**SubCase 1:** Suppose that \( z_h \geq (z_l/A)^{\lambda_i/\lambda_b} \) holds. Then, \( V^i_s = \alpha_s [1 - z_l (1/z_h)^{\lambda_b/\lambda_s}] + \alpha_b z_l (1/z_h)^{\lambda_b/\lambda_s} \) and \( V^j_s = \alpha_s [1 - z_h (1 - \delta) - \delta z_l z_h/z_h^{\lambda_b/\lambda_s}] + \alpha_b \delta z_l z_h/z_h^{\lambda_b/\lambda_s} \). Note that \( V^i_s > V^j_s \) because \( V^i_s - V^j_s = z_h \alpha_s (1 - \delta) - (\alpha_s - \alpha_b) [z_l/z_h^{\lambda_b/\lambda_s}] (1 - \delta z_h) > 0 \) if and only if \( 1 > \frac{z_l (1 - \delta)}{\delta z_h} = \frac{1}{\frac{\alpha_s}{\lambda_s}} \frac{\lambda_s}{\lambda_b} \alpha_s \left( \frac{A (\alpha_s - \alpha_b) + \alpha_b z_l z_h/z_h^{\lambda_b/\lambda_s}}{1 - \delta z_h} \right) \), which is equivalent to \( z_h > (z_l/A)^{\lambda_i/\lambda_b} \) and \( \frac{A (\alpha_s - \alpha_b)}{\lambda_s} < 1 \) by assumption. Thus, type-\( i \) seller strictly prefers to visit buyer \( i \) first.

**SubCase 2:** Suppose that \( z_h \leq (z_l/A)^{\lambda_i/\lambda_b} \) but \( z_h > (z_l z_h/A)^{\lambda_i/\lambda_b} \). Then, \( V^i_s \) is as given in the previous subcase but \( V^j_s = \alpha_s \left[ 1 - \frac{z_l z_h}{A z_h^{\lambda_i/\lambda_b}} + \alpha_b \frac{z_l z_h^{\lambda_b/\lambda_s}}{A z_h^{\lambda_i/\lambda_b}} \right] \). Since \( z_h \geq A \), then it is the case that \( V^i_s > V^j_s \) as required.
**SubCase 3:** Suppose that $z_h \leq (z_l/A)^{\lambda_s/\lambda_b}$ but $z_h \leq (z_lz_h/A)^{\lambda_s/\lambda_b}$. Then, $V^j_s = \alpha_b$ and it is strictly less than $V^i_s$ as required.

Second, I will prove the only-if-part. For this purpose suppose that $\sigma^*$ is an equilibrium strategy profile of the game $\hat{G}$ where both buyers bid $\alpha_b \in (0,1)$ and the seller bids $\alpha_s \in (0,1)$ satisfying $\alpha_b < \alpha_s$. Then I will show that the buyers are powerless in $\sigma^*$. Since the buyers are identical, it is enough to show that one buyer is powerless.

To start with, Proposition 3.1 ensures that the buyers must be weak in $\sigma^*$ and
$$z_h \geq z_l (1 - \alpha_s) r_b \frac{(1-\alpha_b) r_h}{\alpha_s r_s}$$

must hold for all $\alpha \in (0, \alpha_s)$. Next, I will show that we must have $z_h \geq (z_l/A) \frac{(1-\alpha_b) r_h}{\alpha_s r_s}$ in equilibrium whenever $A > z_h$.

For this purpose, consider the case where $A > z_h$ and suppose for a contradiction that $z_h < (z_l/A)^{\lambda_s/\lambda_b}$. Therefore, type-i seller’s expected payoff of visiting buyer 1 and 2 are given by the equation (14). However, one can easily check that $V^1_s = V^2_s$. That is, in equilibrium, type-i seller must visit both buyers with equal probabilities (by the equilibrium restriction we imposed). However, if this is the case in equilibrium, buyer $i$ would deviate and bid $\alpha_s$ and get the seller with certainty to increase his expected payoff. This contradicts with the optimality of $\sigma^*$ (note that we reach the same contradiction even if we do not restrict the seller—who is indifferent between the buyers—to visit each seller with 1/2 probability). Therefore, in equilibrium, if $A > z_h$, then $z_h \geq (z_l/A) \frac{(1-\alpha_b) r_h}{\alpha_s r_s}$ must hold. Together with the previous arguments, the last statement ensures that the buyers are powerless in equilibrium.

Q.E.D.

**Proof of Theorem 4.** Since we just need to show that there exists some equilibria where the buyers post different prices, I will focus my attention on a sub-class of equilibria. For this purpose suppose that the buyers’ bids are pure strategies and satisfy the followings: $\alpha_2 < \alpha_1 < \alpha_s$, $\delta \alpha_s > \alpha_1$ and $\delta \alpha_1 \geq \alpha_2$ (the last two inequality imply that the difference between the players’ bids are large enough), $(1 - \alpha_s) r_b \geq \alpha_1 r_s$, i.e. $\lambda^1_s \geq \lambda_1$, and $z_h < \frac{\delta \alpha_2 - \alpha_1}{\delta (\alpha_s - \delta \alpha_1)}$.

Next, I will prove that along with the third stage strategies as characterized in Lemma 4B–1 and 4B–3 these bids constitute an equilibrium if and only if the following three conditions hold.

1. $z_h \geq z_l (1 - \alpha_b) \frac{(1-\alpha_s) r_h}{\alpha_s r_s}$ holds for all $\alpha \in (0, \alpha_s)$,

2. $z_h - z_l > \frac{\delta \alpha_2 z_h}{\alpha_s (1-\delta)} \left[ \alpha_s - \delta \alpha_1 - \frac{\alpha_s - \alpha_1}{z_h^{1/\lambda_1}} \right]$, and

3. $z_h - z_l > \frac{\delta \alpha_2 z_h}{\alpha_s (1-\delta)} \left[ \frac{\alpha_s - \alpha_1}{z_h^{1/\lambda_1}} - (\alpha_s - \delta \alpha_1) \right]$.

To prove the “only-if” part, assume that $\alpha_1, \alpha_2$ and $\alpha_s$, that satisfy the inequalities in the previous paragraph, constitute an equilibrium. Then, according to Proposition 3.1 buyer 2 must
be weak and \( z_h \geq z_l^{(1-\alpha_s)r_h} \). According to the equilibrium strategies of the players in the third stage, as characterized in Lemma 4B–1 and 4B–3, both buyers are weak in the game and the highest payoff they can achieve is \( 1 - \alpha_s \). However, in an equilibrium, the buyers must achieve the expected payoff of \( 1 - \alpha_s \) because otherwise either one of them would deviate and post the price of \( \alpha_s \) in the first stage. Hence, in equilibrium, the seller of type-\( i \) must visit buyer \( i \) first with probability 1.

According to Lemma 4B–3, if type-\( i \) seller visits buyer 1 first, his expected payoff will be \( V^1_s(i) = (1 - z^1_i)\alpha_s + z^1_i\delta[(1 - z^2_i)\alpha_s + z^2_i\delta\alpha_1] \) where for \( i, j \in \{1, 2\} \), \( z^j_i = z_i \) whenever \( i = j \) and \( z^j_i = z_h \) otherwise. Likewise, expected payoff of type-\( i \) seller if he visits buyer 2 first is given by Lemma 4B–1; \( V^2_s(i) = \alpha_s(1 - z^2_i) + \delta z^2_i v^1_s(i) \) where \( v^1_s(i) = (1 - z^1_i)e^{\lambda T^1_i(i)}\alpha_s + z^1_i e^{\lambda T^1_i(i)}\alpha_1 \) and \( T^1_s(i) = \min\{-\log z^1_s, -\log z^2_s\} \).

First, consider **type-1** seller (or equivalently what buyer 1 believes). This seller visits buyer 1 first with certainty if and only if \( V^1_s(1) > V^1_s(2) \). Given the above payoff functions one can easily show that the last inequality implies the second inequality above. Second, consider **type-2** seller (or equivalently what buyer 2 believes). This seller visits buyer 2 first with certainty if and only if \( V^2_s(2) > V^2_s(1) \). Given the above payoff functions one can easily show that the last inequality implies the third inequality above.

It is easy to verify that the second inequality continues to hold for vanishingly small values of \( z_h \) and \( z_l \) where \( z_h > z_l \). However, we need a bit of work to show that the same is true for the third inequality. Note that the third inequality can be rewritten as \( z_l(z^K-1 - 1) > \delta z_l^{K(1-\alpha_s)} \left[ \alpha_s - \alpha_1 - z^1_l / \lambda_s(1-\gamma) \right] \). I dropped the superscript \( m \) for notational simplicity. Taking the natural log of both sides and dividing by \( \ln z_l \) we achieve \( 1 + \frac{\ln(z^K-1) - 1}{\ln z_l} < 1 + K(1 - \lambda_s / \lambda_1) + \frac{\ln X}{\ln z_l} \) where \( X = \alpha_s - \alpha_1 - z^1_l / \lambda_s(1-\gamma) \). Taking the limit of both sides and using the L’Hospital whenever necessary, we get \( \frac{\ln X}{\ln z_l} < 1 \), which is true by the assumption that the initial bids satisfy \( (1 - \alpha_s)r_h > \alpha_1 r_s \).

The “if” part is simple. Let the bids satisfy the inequalities in the first paragraph and (1)-(3). To prove that these prices can be supported as an equilibrium outcome, one shall suppose that the players’ strategies in the third stage are given in Lemma 4B–1 and 4B–3. Then, all we need to prove that (I) seller of type-\( i \) will visit buyer \( i \) first at time 0, (II) both buyers’ expected payoff in the game is exactly \( 1 - \alpha_s \), and (III) the buyers have no incentive to deviate from their initial bids. The first item (I) is guaranteed by the inequalities (1) and (2). The second and the third items directly follows from the inequality (1). Hence, the buyers have no incentive to deviate from their initial bids.

Finally, suppose that there exists an equilibrium of the game \( G(z^m_h, z^m_l) \) where \( \alpha^m_s \) is the sellers’ price, the game \( \tilde{G}(z^m_h, z^m_l) \) converges to \( G(K) \), and \( \alpha_s \in (0, 1) \) is the limit point of \( \alpha^m_s \). Then the inequality (1) must hold (i.e., \( z^m_h \geq (z^m_l)^{(1-\alpha^m_s)r_h} / \alpha^m_s \) for all \( \alpha \in (0, \alpha^m_s) \)).
Taking the log of both sides, dividing by \( \log z_t \) and taking the limit as \( m \to \infty \) yield
\[
\alpha r_s \leq K(1 - \alpha_s) r_b
\]
for all \( \alpha \in (0, \alpha_s) \). The last inequality implies the desired inequality.

Q.E.D

References


