Searching a Bargain: Power of Strategic Commitment*

Selçuk Özyurt†
Sabancı University

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Abstract

It is common to think of reputation as assets–things of value that require costly investments to build, that can deteriorate if not maintained attentively. This paper shows that under reputational concerns, the equilibrium outcome of competitive markets may not be Walrasian. In particular, the ability of committing to a specific share, the opportunity of building reputation about inflexibility and the anxiety to preserve their reputation can provide significant market power to the players that are in the long side of the market, even when frictions are negligible. Therefore, the role of reputation is substantial, and so ignoring its presence would be severely misleading.

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†Faculty of Arts and Social Sciences, Sabancı University, 34956, Istanbul, Turkey. (e-mail: ozyurt@sabanciuniv.edu)

1. Introduction

Due to their institutional dissimilarities, not all competitive markets are the same. However, we–the economic theorists–handle them in the same way and use the Walrasian equilibrium to describe the outcome of a competitive market. Walrasian theory suggests that equilibrium will be achieved through a process of tâtonnement; given the supply and demand, the market will clear itself. Implicit in this conventional mistreatment is the assumption that competitive markets are frictionless, and thus the institutional structure of a competitive market, including the particulars of the trading procedure, has no or little impact on the market outcome. Since the Walrasian theory does not provide any explanation for that matter, it stays an intriguing and open question whether all competitive (or frictionless) markets are indeed Walrasian.

This query has provoked many papers, initiated by Rubinstein and Wolinsky (1985), providing attractive, non-cooperative foundations for competitive equilibrium. Following the Rubinstein's seminal paper (1982), some has considered competitive homogeneous goods markets where the price is determined as the perfect equilibrium of a bargaining game between sellers and buyers.[1] Some has investigated the effects of auctions as the trading mechanism.[2] Others have studied the impacts of informational asymmetries under different trading mechanisms.[3] All these researches share a common message: regardless of the trading procedures or informational asymmetries, the market outcome is Walrasian when all frictions vanish.

However, this paper proposes that it may be the market participants' ability to build reputation, not the details of the trading procedures, that may cause non-Walrasian outcomes in competitive markets. To prove this point, I construct a very simple benchmark model resembling a competitive market where the long side–the sellers–has no market power. There are three defining features of the model. First, a single buyer negotiates with two sellers over the sale of one item. Second, the sellers make initial posted-price offers in the Bertrand fashion. The buyer can accept one of these costlessly, or else visit one of the stores and try to bargain for a lower price.[4] Third, each of three players suspects that the opponents might have some kind of irrational commitment forcing them to insist

[1]Rubinstein and Wolinsky (1985), Gale (1986a,b), Binmore and Herrero (1989), Bester (1988), Bester (1989), Shaked and Sutton (1984) and Rubinstein and Wolinsky (1990).

[2]Wolinsky (1988), De Fraja and Sakovics (2001), Serrano (2002), Satterthwaite and Shneyerov (2007), Satterthwaite and Shneyerov (2008), Atakan (2008).

[3]Moreno and Wooders (2002), Ponsati (2004), Lauermann (2008), Shneyerov and Wong (2010).

[4]Bertrand paradigm has been extensively used to study competitive markets. Bertrand (1883) assumes that each seller can supply the entire market as the sellers have constant average costs, and that buyers can freely accept one price that the sellers post simultaneously. As a result of these specifications, the presence of two price-setting firms suffices to yield the perfectly competitive outcome. Because of this result, many accepted models in the information economics and the industrial organization literatures have employed the Bertrand approach to reproduce competitive markets. See, for example, Spence (1973), Rothschild and Stiglitz (1976) and Stiglitz and Weiss (1981).

on a specific allocation.[5] That is, the players can be obstinate with small probabilities which affects their negotiating tactics and provides incentives to build reputation on their resoluteness.[6] For analytical clarity, I construct the benchmark model so that it is almost frictionless. Therefore, I assume that the initial priors of each player being obstinate is small but positive, the set of obstinate types (demands) is finite and the search cost that the rational buyer incurs at each time he switches his bargaining partner is very small but positive.

The analysis of the benchmark model shows that even in the limit where the frictions vanish, a range of prices that includes the monopoly price and zero are compatible in equilibrium.[7] Therefore, reputational concerns may give the sellers significant market power in a highly competitive market environment. This surprising result is not contradicting with the findings of the existing literature. This is true because the previous literature assumes that market participants are randomly rematched with new partners at each round if an agreement is not reached, and the equilibrium analyses are restricted only to stationary beliefs. That is, players' ability or desire to build reputation has been ignored.

In addition to this notable contribution to the literature initiated by Rubinstein and Wolinsky (1985), the formalization I propose in this article has four major benefits. First, the model facilitates the investigation of the roles of strategic commitment and reputation that are elements missing in existing formal models of search and multilateral bargaining. For example, the important finding of bargaining models in search markets is that an outside option plays a limited or no role when the continuation of negotiation is at least as valuable as that of the outside option. The current model, however, makes

[5]Shelling (1960) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996) and Ellingson and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if (revoking) it is costly. However, I adopt the approach following Kreps and Wilson (1982) and Milgrom and Roberts (1982) where commitments are modeled as behavioral types that exist in the society so that the rational players can mimic if they like to do so. Abreu and Sethi (2003) supports the existence of commitment types from evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with evolutionary stability in bargaining environments.

[6]Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991), Abreu and Gul (2000), Kambe (1999) and Atakan and Ekmekci (2009), a commitment player always demands a particular share and accepts an offer if and only if it weakly exceeds that share. An obstinate seller, for example, always offers his original posted price, and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount, and will never agree to pay more. Thus, a rational player must choose either to mimic an inflexible type, or reveal his rationality and continue negotiation with no uncertainty regarding his actual type. Therefore, reputation of a player is the posterior probability (attached to this player) of being the obstinate type.

[7]This is true regardless of the players' time preferences. By vanishing frictions I mean that initial priors and the buyer's search cost converge to zero and the set of obstinate types converge to the unit interval (indicating the area of possible bargaining agreements).

this prediction invalid by showing that the availability of an endogenous outside option substantially affects the outcome in the bargaining between a buyer and a pair of sellers, if reputational concerns are present.

Besides, equilibrium analysis shows that sellers has no bargaining power when they fail to coordinate on their initial offers or when the buyer's initial reputation is sufficiently high, i.e. the buyer is *strong*. For this reason, there is no equilibrium in which rational sellers post different prices, and the unique equilibrium price is zero if the buyer is strong. The reason behind this finding is simple. First, in equilibrium, the buyer's outside option of leaving a seller is high means that he prefers to walk away from this seller's store rather than to accept the seller's price. Clearly, this is the case when the buyer's reputation is sufficiently more elevated than the sellers' reputation or the other seller posts a lower price.[8] Second, in standard models where obstinate types are nonexistent, a seller can always offer the buyer his continuation value and prevent the buyer leaving him empty-handed. However, when commitment types are present, offering something different than his posted price reveals a seller's type (flexibility), which yields surplus smaller than what he can achieve by accepting the buyer's offer (see Myerson 1991; Compte and Jehiel 2002). As a result, if the buyer's outside option is sufficiently high, then the buyer's bargaining power becomes substantially strengthened so that the sellers accept any positive share the buyer is about to offer.

However, when the buyer is *weak*, i.e. his initial reputation is low enough, then reputation has a *lock-in* effect (see Klemperer, 1987) which provides leverage to the sellers so that non-Walrasian prices are sustainable in equilibrium. On the one hand, for the rational buyer, conceding to the first seller is at least as good as visiting the second seller when the buyer is weak and the sellers post the same price. The rational buyer can credibly threaten the first seller to terminate the negotiation only if he maintains enough reputation to make his obstinacy credible against the second seller. But, this is possible if the rational buyer is playing a strategy in which he accepts the seller's price with a positive probability. Therefore, the rational buyer cannot abandon a seller unless he guarantees a positive expected surplus to that seller. On the other hand, price undercutting is never optimal for the sellers. We reach this conclusion in two steps. First, if a seller price undercuts, then he will be perceived as obstinate. Second, as I argued previously, posting different prices will improve the buyer's bargaining power remarkably. As a result, in a competitive environment, being perceived as an obstinate seller reduces the chance that his offer is accepted because the rational buyer prefers to visit the seller, who is very

[8] If the buyer's reputation is sufficiently high, then he can sustain a long delay to convince a seller about his resoluteness. In this case, the rational buyer expects to receive some surplus from the seller, that is closer to his own terms.

likely to be flexible, first.[9] And, this restrains a rational seller from underbidding his competitor.

The second significant benefit of the formalization is that given the sellers' initial offers, the equilibrium strategies in the multilateral bargaining game is essentially unique.[10] This makes the benchmark model a fruitful ground to answer further questions regarding the impacts of reputation on market outcomes and structures. One immediate extension I examine in the paper investigates the effects of reputation in "large markets". The current model presumes that the buyer's moves throughout the haggling process are observable by the sellers. Therefore, the buyer can use his reputation that is built in one store against the other seller. This might be a strong assumption for large markets where the buyers are usually anonymous. For this reason, in Section 7, I relax this condition and suppose that the buyer's arrival time to stores, initial offers and the time he spends in each store are not publicly observable. The simple model in this section shows that anonymity increases the sellers' market power even further.

In another extension, I study the impacts of reputation when search friction is large. In the benchmark model, the rational buyer can costlessly learn and accept the sellers' posted prices. Therefore, price search is indeed costless. However, for analytical convenience, searching for a bargain price is assumed to be costly as the buyer suffers very small but positive switching cost each time he changes his bargaining partner. In Section 6, I consider the case where the cost of searching a bargain price is high. Somewhat counterintuitive result in this section shows that high search friction reduces the sellers' market power down to zero. That is, zero is the unique equilibrium price.

Therefore, in the light of our earlier findings, we can conclude that under reputational concerns, there is an inverse relationship between the players' bargaining power and their equilibrium payoffs, if the buyer is weak. That is, higher search friction decreases the buyer's bargaining power but most likely increases his equilibrium payoff. The reason of this unconventional correlation, that highlights an eminent aspect of reputation for the sustainability of non-Walrasian outcomes, is simple. Regardless of his initial reputation, the rational buyer believes that he can achieve a lower price by haggling with the sellers, and low cost for searching a deal makes haggling more attractive than accepting a seller's posted price. Indeed, the rational buyer strictly prefers to visit sellers if his initial reputation is high, and is indifferent between visiting stores and immediate acceptance of the lowest price if he is weak.

However, when the buyer is weak, then the rational buyer's desire or hope to make a

[9]This contrasts with the prediction in the two-person bargaining model of Abreu and Gul (2000). In their model, being perceived as an obstinate type causes the concession by the rational opponent.

[10]This finding differs from the standard conclusion in non-cooperative bargaining games that informational asymmetries give rise multiplicities. See, for example, Osborne and Rubinstein (1990)

better deal turns into a trap–the lock-in effect of reputation. This trap drags the rational buyer into a situation where he may get much less than what he would achieve if he would have committed himself to accept the lowest posted price. The problem is that the rational buyer cannot commit himself to immediate acceptance because searching for a bargain is equally attractive to him. For this reason, the rational sellers do not have to compete with each other over their posted prices, yielding non-Walrasian outcomes consistent with equilibrium. High search cost clearly makes this trap go away as the rational buyer knows that high cost decreases the attractiveness of searching for a deal.

Arguably, this trap–caused by the buyer's reputational weakness and low search cost–probably is the reason for significant markups in some markets, e.g. oriental bazaars, where there are many stores next to one another, selling (almost) identical products.

The third advantage of the formalization is that its predictions are robust in many aspects. For instance, in Section 5, I check if the impacts of reputation decrease in "larger" markets where the number of sellers is greater than two, and show that a range of prices including the monopoly price and zero are still consistent with equilibrium. In addition, Section 8 shows that the premises on the obstinate buyer's store selection has no significant effect. That is, even if the obstinate buyer is committed to immediately leave a seller's store once his offer is not accepted, then the lock-in effect of the reputation will still be in play and lead to non-Walrasian equilibrium prices. Finally, in Section 9, I show that reputational concerns of the players overwhelm their behaviors so that equilibrium has a war of attrition structure–each player is indifferent between accepting his opponents' initial demand and waiting for acceptance. As a result, given the sellers' posted prices, the equilibrium of the haggling process is unique and robust in the sense that it is "independent" of the exogenously assumed bargaining protocols (unlike more familiar but relatively less sophisticated models).[11]

Finally, the benchmark model has potential benefits to market microstructure literature. Although negotiating over prices is common in many markets, it is not clear how a haggling price policy can help a firm gain a strategic advantage or whether it is even sustainable in a competitive market. Riley and Zeckhauser (1983), Bester (1993), Wong (1995), Desai and Purohit (2004), and Camera and Delacroix (2004) compare negotiated prices with posted prices and show that each argument has specific merits.[12] It is a widely accepted approach in this literature that price posting requires irreversible commitment. That is, sellers either post price and act absolutely inflexible in their demands or do not

[11] Likewise, Chatterjee and Samuelson (1987), Samuelson (1992), Caruana, Eirav and Quint (2007) and Caruana and Einav (2008) show that credible commitment to certain promises, threats or actions would wash out technical specifications of the bargaining procedures.

[12] Parallel approaches are extensively used in labor market literature to investigate the wage determination in competitive labor markets. See Rogerson, Shimer and Wright (2005) and the references therein.

post a price but behave completely flexible and bargain with each buyer. However, the current model shows that dedication to such extreme strategies (absolute flexibility or inflexibility) that postulate pure commitment is not optimal in a competitive environment. Very roughly, rational players prefer to randomize (in a sense) these two strategies optimally.

2. The Competitive-Bargaining Game in Continuous-Time

Here I define the *competitive-bargaining game* in continuous-time with multiple commitment types. I then analyze two special cases in Section 3, in which each player has only one commitment type. These special cases both convey the flavor of the analysis and are furthermore the basic building blocks for the multiple type cases studied subsequently.

The Players: There are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[13] The valuation of the good is one for the buyer and zero for the sellers. Both the buyer and the sellers have some small, positive probability of being a "commitment" type. An obstinate (or commitment) type of player $n \in \{1, 2, b\}$, where b represents the buyer, 1 and 2 represents the sellers, is identified by a number $\alpha_n \in [0, 1]$. A type α_i of seller $i \in \{1, 2\}$ always demands α_i, accepts any price offer greater or equal to α_i and rejects all smaller offers. On the other hand, a type α_b of the buyer always demands α_b, accepts any price offer smaller or equal to α_b and rejects all greater offers. I use the terms rational (flexible) or obstinate (inflexible) with the identity of a player (buyer or seller) whenever I want to differentiate the types of the player. Not mentioning these terms with the identity of a player should be understood that I mean both rational and obstinate types of that player.

I denote by $C \subset [0, 1)$ with $0 \in C$ the finite set of obstinate types for all three players and by $\pi(\alpha_n)$ the conditional probability that player n is obstinate of type α_n given that he is obstinate.[14] Hence, π is a probability distribution on C and is common for all three players. I assume that $\pi(\alpha) > 0$ for all $\alpha \in C$. In case I need to emphasize different obstinate types of player n, I use α_n, α'_n and so on. The initial probability that n is obstinate (i.e. player n's initial reputation) is denoted by z_n. I assume without loss of generality that the sellers' initial reputations are the same (that is $z_i = z_s$ for $i = 1, 2$),

[13]In Section 5, I consider the case where the number of sellers is some $N > 2$. I take the short side of the market as the demand side. The unique buyer assumption is consistent with markets where the buyer has some monopsony power, or each seller has a large number of goods to sell (so no competition between the buyers) and the buyers cannot convey information to one another (no interaction between the buyers). On the other hand, In Sections 4 and 5 I show that non-Walrasian prices can be supported in equilibrium even though the buyer has monopsony power. In this respect, having more than one buyer can only strengthen the findings of these sections.

[14]Having $1 \notin C$ does not affect the analyses and the results of the paper but eliminates additional cases that produce nothing new.

and that z_b and z_s take sufficiently small values.[15] Although some results will require smaller upper bounds for these priors, the assumption $z_b, z_s < \alpha$ for all $\alpha \in C \setminus \{0\}$ is sufficient for most of the results in the paper. Note that imposing such upper bounds on priors is an innocuous restriction because unlike z_b and z_s, a demand α being in the interval $[0, 1)$ is nothing but a normalization. Finally, I denote by r_b and r_s the rate of time preferences of the rational buyer and the sellers, respectively.

The Timing of the Game: The competitive-bargaining game between the sellers and the buyer is a two-stage, infinite horizon, continuous-time game. The sellers make initial posted-price offers; the buyer can accept one of these costlessly (over the phone, say), or else visit one of the stores and try to bargain for a lower price. The buyer can negotiate only with the seller whom he is currently visiting. The buyer is free to walk out of one store and try the other, but at a cost (delay) of switching which is assumed to be very small. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town, and so changing the bargaining partner is costly for the buyer because it takes time to move from one store to the other and the buyer discounts time.

More formally, stage 1 starts and ends at time zero and the timing within the first stage is as follows. Initially, each seller simultaneously announces (posts) a demand (price) from the set C and it is observable by the buyer.[16] After observing the sellers' demands, the buyer has two options. He can accept one of the posted prices and finish the game. Or, he can make a counter offer that is observable by the sellers and visit one of the sellers to start the second stage (the bargaining phase).

Note that if seller i is rational and posting the price of $\alpha_i \in C$ in stage 1, then this is his strategic choice. If he is the obstinate type, then he merely declares the demand corresponding to his type. Given the description of the obstinate players, if the buyer accepts α_i and finishes the game at time zero, then he is either rational and finishing the game strategically or obstinate of type α_b such that $\alpha_b \geq \alpha_i$. Likewise, if the buyer makes a counter offer $\alpha_b \in C$ which is incompatible with the sellers' demands, i.e., $\alpha_b < \min\{\alpha_1, \alpha_2\}$, then this may be because the buyer is rational and strategically demanding this price or because the buyer is the obstinate type α_b.[17]

Upon the beginning of the second stage (at time zero) the buyer and seller i, who

[15] This restriction is consistent with the analysis in Section 5 since I eventually analyze the limiting case where z_b and z_s approach zero.

[16] For analytical simplicity, I assume that the set of offers is common for all the players and is equal to the set of obstinate types C. This restriction is dispensable and can be removed with no impact on equilibrium outcomes.

[17] Therefore, if the buyer makes a counter offer and demands α_b that is greater than or equal to the minimum of the posted prices, then the buyer is rational and strategically demanding this price.

is visited by the buyer first, immediately begin to play the following *concession game*: At any given time, a player either accepts his opponent's initial demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. If the buyer leaves store i and goes to store $j \in \{1, 2\}$ with $j \neq i$, the buyer and seller j start playing the concession game upon the buyer's arrival at that store.[18] Assuming that the sellers' are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time, $\Delta > 0$, required to travel from one store to the other. That is, $\delta = e^{-r_b \Delta}$. Note that δ (the search friction) is the cost that the buyer incurs at each time he switches his bargaining partner.[19] I assume that the search friction is very small, and thus δ is very close to one.[20] Concession of the buyer or seller i, while the buyer is in store i, marks the completion of the game; if the agreement $\alpha \in \{\alpha_b, \alpha_i\}$ is reached at time t, then the payoffs to seller i, the buyer and seller j are $\alpha e^{-r_s t}$, $(1-\alpha)e^{-r_b t}$ and 0, respectively. In case of simultaneous concession, surplus is split equally.[21]

I denote the two stage competitive-bargaining game in continuous-time by G. The competitive-bargaining game is modeled as a modified war of attrition game. This model is justified in Section 9. There, I show that under some restrictions, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers converge to zero, and this limit is equivalent to the unique outcome of the second stage of the game G.

The Information Structure: There is no informational asymmetry regarding the players' valuations and time preferences. Moreover, all three players' initial offers, the buyer's timing and store selection are observable by the public.[22] However, players have

[18] After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again. However, the buyer will never behave that way in equilibrium.

[19] One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

[20] More specifically, I assume that for all $\alpha, \alpha' \in C$ with $\alpha > \alpha'$ we have $(1-\alpha) < \delta(1-\alpha')$. The idea behind this assumption is very simple; the search friction should not prevent the rational buyer to walk away from a store if he knows that the other seller has posted a lower price. Analysis starting with Section 3.B utilize this assumption extensively. Section 6 emphasizes its importance by investigating the case where the search friction is large. Search frictions have strong impacts on economic activities, and it is of great interest to many researcher. However, in this paper I aim to investigate the impacts of reputation in (almost) frictionless competitive markets. Hence, the motivation of the current formalization and the assumptions is to create an environment which resembles such markets. One alternative would be eliminating the search friction entirely. However, I need arbitrarily small but positive search friction to eliminate impractical indifferences that would occur during concession games when the sellers post the same prices.

[21] This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[22] When stakes are high, the negotiation becomes (to some degree) public mainly because the bargainers' incentive to scrutinize their opponents' moves throughout the negotiation process is higher.

private information about their resoluteness. That is, each player knows its own type but does not know the opponents' true types.

More Details on Obstinate Types: Strategies of an obstinate player is simple; never back down from the initial offer. Although the remaining assumptions are dispensable, I give them for the sake of completeness. I assume that the obstinate buyer of any type (or demand) $\alpha_b \in C$ understands the equilibrium and leaves his bargaining partner permanently when he is convinced that his partner will never concede. Furthermore, if the sellers' posted prices (α_1 and α_2) are the same, or the obstinate buyer's type (α_b) is incompatible with these prices, then the obstinate buyer visits each seller with equal probabilities. Section 8 elaborates on these assumptions and analyzes some possible alternatives.

Moreover, if a seller's posted price is compatible with the obstinate buyer's type α_b, that is $\min\{\alpha_1, \alpha_2\} \leq \alpha_b$, then he immediately visits the seller who posts the lowest price (without making any announcement), accepts his demand and finishes the game at time zero. Finally, the obstinate buyer with demand α_b never visits a seller who is known to be the commitment type with demand $\alpha > \alpha_b$.[23]

Strategies of the Rational Players: In the first stage of the competitive-bargaining game G, a strategy for rational seller i is a pure action $\alpha_i \in C$. Since the subsequent analysis is quite involved, I restrict sellers to play pure strategies in stage 1. However, the buyer can employ mixed strategies. A strategy for the rational buyer consists of two parts; μ and σ_i. Although the strategy μ is a function of the sellers' announcements and σ_i is a function of all three players' announcements, these connections are omitted for notational simplicity. Given that each seller posts α_i, $\mu(\alpha_b)$ is the probability that the rational buyer announces the demand $\alpha_b \in C$ with $\alpha_b \leq \alpha$ where $\alpha = \min\{\alpha_1, \alpha_2\}$. That is, μ is a probability measure over $C_\alpha = \{x \in C | x \leq \alpha\}$. I require that the game G ends in stage 1 when the rational buyer announces α. That is, immediate concession of the buyer is represented by the buyer's announcement of α. Moreover, σ_i denotes the probability of the rational buyer visiting seller i first, and so $\sigma_1 + \sigma_2 = 1$.

YouTubes flirt with Google and Yahoo before Google has acquired YouTube for \$1.65 billion and Yahoos negotiation with Microsoft and AOL Time Warner are just two examples on this account. Therefore, I consider an extreme case where the buyer's actions (demands) are perfectly observable. Clearly, in some circumstances, e.g. in large markets where traders are rather anonymous, the sellers may not be able to attain all the information nor can the buyer convey it perfectly. For this reason, in Section 7, I consider the other extreme case where the buyer's arrival to the market and moves in negotiating with a seller is unobservable by the public. The simple model in that section shows that anonymity increases sellers' market power further.

[23]This assumption is consistent with the story that the obstinate buyer can understand the equilibrium; he knows that visiting an obstinate seller with a demand higher than α_b has no point because it is impossible to reach an agreement with him.

If the competitive-bargaining game proceeds to stage 2 and the first stage strategies of the players are $(\alpha_i, \sigma_i)_i$ and μ, then the Bayes' rule implies the followings: The probability of seller i being obstinate conditional on posting price α'_i is z_s if $\alpha'_i = \alpha_i$ and 1 otherwise. Likewise, the probability that the buyer is the commitment type conditional on announcing his demand as $\alpha_b < \alpha$ and visiting seller i first is

$$\frac{\frac{1}{2} z_b \pi(\alpha_b)}{\frac{1}{2} z_b \pi(\alpha_b) + (1 - z_b)\sigma_i \mu(\alpha_b) \left[\sum_{x<\alpha} \pi(x)\right]} \tag{1}$$

Second stage strategies are relatively more complicated. A nonterminal history of length t, h_t, summarizes the initial demands chosen by the players in stage 1, the sequence of stores the buyer visits and the duration of each visit until time t (inclusive). For each $i = 1, 2$, Let $\hat{H}^i_t$ be the set of all nonterminal histories of length t such that the buyer is in store i at time t. Also, let H^i_t denote the set of all nonterminal histories of length t with which the buyer just enters store i at time t.[24] Finally, set $\hat{H}^i = \bigcup_{t\geq 0} \hat{H}^i_t$ and $H^i = \bigcup_{t\geq 0} H^i_t$.

The buyer's strategy in the second stage has three parts. The first part determines the buyer's location at any given history. For the other two parts, $\mathscr{F}^i_b$ for each i, let $\mathbb{I}$ be the set of all intervals of the form $[T, \infty]$ $(\equiv [T, \infty) \cup \{\infty\})$for $T \in \mathbb{R}_+$, and $\mathbb{F}$ be the set of all right-continuous distribution functions defined over an interval in $\mathbb{I}$. Therefore, $\mathscr{F}^i_b : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F^{i,T}_b : [T, \infty] \to [0, 1]$ representing the probability of the buyer conceding to seller i by time t (inclusive). Similarly, seller i's strategy $\mathscr{F}_i : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F^T_i : [T, \infty] \to [0, 1]$ representing the probability of seller i conceding to the buyer by time t (inclusive).

Player n's reputation $\hat{z}_n$ is a function of histories and n's strategies, representing the probability that the other players attach to the event that n is obstinate. It is updated according to the Bayes' rule. At the beginning of the game we have $\hat{z}_b(\emptyset) = z_b$ and $\hat{z}_i(\emptyset) = z_s$ for each seller i, where $\emptyset$ represents the null history. Given the rational buyer's first stage strategies and a history h_0 where the buyer announces α_b and visits seller i first, the buyer's reputation at the time he enters store i, i.e. $\hat{z}_b(h_0)$, is given by Equation (1). Following the history h_0, if the buyer plays the concession game with seller i until some time $t > 0$, and the game has not ended yet (call this history h_t), then the buyer's reputation at time t is $\frac{\hat{z}_b(h_0)}{1-F^{i,0}_b(t)}$, assuming that the buyer's strategy in the concession game is $F^{i,0}_b$.

Note from the last arguments that the buyer's reputation at time t reaches 1 when $F^{i,0}_b(t)$ reaches $1 - \hat{z}_b(h_0)$. This is the case because $F^{i,0}_b(t)$ is the sellers' belief about the buyer's play during the concession game with seller i. That is, it is the strategy of

[24] That is, there exits $\epsilon > 0$ such that for all $t' \in [t - \epsilon, t)$, $h_{t'} \notin \hat{H}^i_t$ but $h_t \in \hat{H}^i_t$.

the buyer from the point of view of the sellers. More generally, the upper limit of the distribution function $F_b^{i,T}$ is $1 - \hat{z}_b(h_T)$ where $\hat{z}_b(h_T)$ is the buyer's reputation at time $T \geq 0$, the time that the buyer (re)visits store i. Same arguments apply to the sellers' strategies.

Since I will use z_b, z_s and $\hat{z}_b^i$ extensively in the paper, it is crucial to emphasize what they refer to. I will denote the buyer's and the sellers' initial reputations by z_b and z_s, respectively. The term $\hat{z}_b^i$ represents the buyer's reputation at the beginning of the second stage conditional on him visiting store i first. Clearly, $\hat{z}_b^i$ is a function of the rational buyer's strategy and the realized history of the first stage, however I omit this connection only for notational simplicity.

Given $F_b^{i,T}$, rational seller i's expected payoff of conceding to the buyer at time t (conditional on not reaching a deal before time t where $T \leq t$,) is

$$\begin{aligned} U_i(t, F_b^{i,T}) &:= \alpha_i \int_0^{t-T} e^{-r_s y} dF_b^{i,T}(y) \\ &\quad + \frac{1}{2}(\alpha_i + \alpha_b)[F_b^{i,T}(t) - F_b^{i,T}(t^-)]e^{-r_s(t-T)} + \alpha_b[1 - F_b^{i,T}(t)]e^{-r_s(t-T)} \end{aligned} \tag{2}$$

with $F_b^{i,T}(t^-) = \lim_{y \uparrow t} F_b^{i,T}(y)$.

In a similar manner, given F_i^T, the expected payoff of the rational buyer who concedes to seller i at time t is

$$\begin{aligned} U_b^i(t, F_i^T) &:= (1 - \alpha_b) \int_0^{t-T} e^{-r_b y} dF_i^T(y) \\ &\quad + \frac{1}{2}(2 - \alpha_i - \alpha_b)[F_i^T(t) - F_i^T(t^-)]e^{-r_b(t-T)} + (1 - \alpha_i)[1 - F_i^T(t)]e^{-r_b(t-T)} \end{aligned} \tag{3}$$

where $F_i^T(t^-) = \lim_{y \uparrow t} F_i^T(y)$.[25]

3. Single Commitment Types

I now turn to the analysis of equilibrium in case each player has only one commitment type. Therefore, the set of obstinate types, C_n, is singleton and possibly different for each player n. In particular, for the rest of this section, I assume that seller i's obstinate type $\alpha_i \in (0, 1)$ is incompatible with the buyer's commitment type $\alpha_b \in (0, 1)$, i.e. $\alpha_i > \alpha_b$ for $i = 1, 2$.

[25]Expected payoffs are evaluated at time T, and they are conditional on the event that the buyer visits seller i at time $T \geq 0$.

A Benchmark Model: One Seller

To have a benchmark result, suppose for now that there is only one seller, denoted by s, with single obstinate type $\alpha_s \in (0,1)$ which is incompatible with the buyer's demand α_b. The timing of the modified version of the game G goes as follows. In stage 1, the seller and then the buyer announce their demands. Since each player has a unique obstinate type, this stage has no strategic content. In the second stage (time zero), players begin to play the concession game as described in Section 2 with one important difference; the buyer has no outside option of leaving the seller's store. This model is identical to the Abreu and Gul (2000) setup and the equilibrium strategies are characterized by the following three conditions:

$$\begin{aligned} &F_n(t) = 1 - c_n e^{-\lambda_n t} \text{ for all } t \leq T^e \\ &c_n \in [0,1], (1-c_b)(1-c_s) = 0, \text{ and} \\ &F_n(T^e) = 1 - z_n \text{ for all } n \in \{b, s\} \end{aligned} \tag{4}$$

During the concession game, the rational buyer and seller concede by choosing the timing of acceptance randomly with constant hazard (or instantaneous acceptance) rates $\lambda_b = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_b}$ and $\lambda_s = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$, respectively. They play the concession game until T^e, when both players' reputations simultaneously reach 1. Since rational player n is indifferent between conceding and waiting at all times, his expected payoff during the concession game v_n is equal to what he can achieve at time 0. Therefore, by Equations (2) and (3) we have

$$\begin{aligned} v_b &= F_s(0)(1-\alpha_b) + [1 - F_s(0)](1-\alpha_s), \text{and} \\ v_s &= F_b(0)\alpha_s + [1 - F_b(0)]\alpha_b \end{aligned} \tag{5}$$

Note that $1 - c_n$ indicates the probability of player n's initial concession (or player n's *initial probabilistic concession*), and the second condition in (4) implies that only one player can make concession at time zero. Abreu and Gul (2000) call a player ***strong*** if his opponent makes an initial probabilistic concession at time zero and ***weak*** otherwise. Therefore, if the rational buyer (or the seller) is weak, then his expected payoff is $1 - \alpha_s$ (or α_b).

A. Symmetric Obstinate types for the Sellers

Now, I resume the case with two sellers. In this subsection, I assume that the sellers' obstinate types are identical, that is $\alpha_i = \alpha_s$ for $i = 1, 2$. The equilibrium of the competitive-bargaining game is virtually unique.[26] The three conditions provided in (4)

[26] See Proposition 3.2.

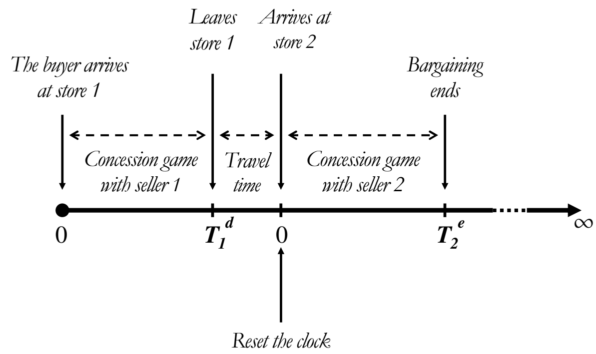

Figure 1: The time-line of the buyer's equilibrium strategy

will not characterize the equilibrium strategies of the competitive-bargaining game G, as the game gets complicated with the existence of outside option for the buyer. However, Arguments parallel to Hendricks, Weiss and Wilson (1988) and Lemma 1 of Abreu and Gul (2000) ensures that the equilibrium strategies in the concession games between the buyer and each seller i will *partially* satisfy these three conditions.

A short descriptive summary of the equilibrium strategies is as follows. In stage 1, sellers post α_s and the buyer makes counter offer of α_b and visits seller i with a positive probability σ_i. In stage 2 (see *Figure 1*), the buyer visits each store at most once. He enters, for example, store 1 at time zero. His reputation at this time is $\hat{z}_b^1 = z_b/[z_b + 2(1 - z_b)\sigma_1]$. If the buyer's reputation, $\hat{z}_b^1$ is high enough (relative to z_s), then the rational buyer makes a *take it or leave it offer* to seller 1. That is, he leaves store 1 immediately following his arrival at that store. Then, he goes directly to store 2 and plays the concession game with the second seller. However, for small values of $\hat{z}_b^1$, the rational buyer starts playing the concession game with seller 1 until time $T_1^d > 0$. At time T_1^d, the buyer leaves store 1 for sure, if the game has not yet ended, and goes directly to store 2. The value of this deterministic departure time from store 1 depends on the primitives.

When there are two sellers, building reputation on inflexibility by negotiating with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. In equilibrium, the rational buyer leaves the first store when his discounted expected payoff in the second store is at least as high as his continuation payoff in the first store. Therefore, in equilibrium, if $\hat{z}_b^1$ is low relative to z_s, the rational buyer needs to build up his reputation before leaving the first store.

During the concession game, the rational buyer and seller 1 concede by choosing the timing of acceptance randomly with constant hazard rates that are given in the previous subsection, i.e. λ_b and λ_s respectively. Seller 1's concession game strategy $F_1(t)$ satisfies all three conditions of (4), but the buyer's strategy F_b^1 satisfies only the first two.[27] That is, conditional on the game lasting until time T_1^d, seller 1's reputation reaches one, and the buyer's reputation reaches $\frac{\hat{z}_b^1}{1-F_b^1(T_1^d)}$. The last term is less than one because it provides enough incentive to the buyer to walk away from the first store and to search a deal in the second.

Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, the time that both players' reputations simultaneously reach 1. For notational simplicity, I manipulate the subsequent notation and reset the clock once the buyer arrives in store 2 (but not the players' reputations). Thus, I define each player's distribution function as if the concession game in each store starts at time zero. In the second store, the rational buyer and seller 2 concede with constant hazard rates λ_b and λ_s respectively. The concession game strategies of the buyer and seller 2, F_b^2 and F_2 respectively, satisfy all three conditions in (4) with one important adjustment; we have $F_b^2(T_2^e) = 1 - \hat{z}_b^1/[1 - F_b^1(T_1^d)]$ because the buyer's reputation at the time he arrives at store 2 is different (higher) than his reputation once he enters store 1.

Proposition 3.1. *In any (sequential) equilibrium of the competitive-bargaining game G, the rational buyer visits each store at most once. Moreover, the rational buyer leaves the first store at some finite time for sure, given that the game does not end before, and directly goes to the other store if and only if the first seller is obstinate. Finally, in an equilibrium where the rational buyer visits seller 1 first with probability σ_1, leaves store 1 at time T_1^d and finalizes the game in store 2 at time T_2^e if the game has not yet ended before, the players' concession game strategies must be*

$$F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t} \qquad F_1(t) = 1 - z_s e^{\lambda_s (T_1^d - t)}$$
$$F_b^2(t) = 1 - e^{-\lambda_b t} \qquad F_2(t) = 1 - z_s e^{\lambda_s (T_2^e - t)}$$

satisfying

$$F_b^1(0)F_1(0) = 0 \qquad \text{and} \qquad F_b^2(T_2^e) = 1 - \frac{\hat{z}_b^1}{1 - F_b^1(T_1^d)}$$

where $\hat{z}_b^1 = \frac{z_b}{z_b + 2(1-z_b)\sigma_1}$, $\lambda_s = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_b}$ *and* $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$.

I defer the proofs of all the results in this section to Appendix. In equilibrium, the rational buyer's continuation payoff is no more than $1 - \alpha_s$ if he reveals his rationality.[28]

[27] That is, $F_1(t) = 1 - c_1 e^{-\lambda_s t}$ and $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$ for all $t \le T_1^d$; $c_1, c_b^1 \in [0,1]$ and $(1-c_1)(1-c_b^1) = 0$; $F_1(T_1^d) = 1 - z_s$ but $F_b^1(T_1^d) < 1 - \hat{z}_b^1$.

[28] Arguments similar to the proof of Lemma 2 in the Online Appendix and the one-sided uncertainty result of Myerson (1991, Theorem 8.4) imply this result.

Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is also inflexible, leaving the first seller "earlier" (or "later") than this time would reveal the buyer's rationality. Moreover, since the cost of switching stores is positive, the rational buyer never leaves a seller if there is a positive probability that the seller is flexible, and he immediately leaves otherwise. Clearly the buyer does not revisit a seller once he knows that this seller is obstinate.

Next, I will characterize the buyer's departure time from the first store, T_1^d, the time that the competitive-bargaining game ends in store 2, T_2^e, the rational buyer's *initial probabilistic concession* in the first store, $F_b^1(0)$, and the rational buyer's store selection at time zero, σ_1. The rational players' equilibrium payoffs in the concession games are calculated by the equations of (5). That is, for each seller i

$$\begin{aligned} v_b^i &= F_i(0)(1-\alpha_b) + [1-F_i(0)](1-\alpha_s), \text{and} \\ v_i &= F_b^i(0)\alpha_s + [1-F_b^i(0)]\alpha_b \end{aligned} \tag{6}$$

However, the rational players' equilibrium payoffs in the game G is different as they should take into account the buyer's outside option and store selection in stage 1. I will provide the rational buyer's payoffs because they are important for the analyses in the subsequent sections.[29]

In equilibrium where the buyer first visits seller 1, the rational buyer leaves the first seller when he is convinced that this seller is obstinate. At this moment, walking out of store 1 is optimal for the rational buyer if his discounted continuation payoff in the second store, δv_b^2, is no less than $1-\alpha_s$, payoff to the rational buyer if he concedes to the obstinate seller 1. Let z_b^* denote the level of reputation required to provide the rational buyer enough incentive to leave the first store. Assuming that $\hat{z}_b^1 < z_b^*$ (i.e., the rational buyer needs to build up his reputation before walking out of store 1), the game ends in store 2 at time $T_2^e = -\log(z_b^*)/\lambda_b$.[30] Thus, given the value of $F_2(0)$ and the rational buyer's discounted continuation payoff in store 2, z_b^* must solve

$$1-\alpha_s = \delta[1-\alpha_b - z_s(\alpha_s-\alpha_b)(z_b^*)^{-\lambda_s/\lambda_b}]$$

implying that $z_b^* = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda_s}}$ and $A = \frac{\delta(1-\alpha_b)-(1-\alpha_s)}{\delta(\alpha_s-\alpha_b)}$. Note that z_b^* is well-defined, i.e. $z_b^* \in (0,1)$, as A is positive. In fact, A is very close to 1 since the cost of traveling is assumed to be very small.

On the other hand, when $\hat{z}_b^1 \geq z_b^*$, the rational buyer's discounted continuation payoff in store 2 is higher than $1-\alpha_s$ at the beginning of the second stage. Therefore, the rational buyer prefers going to store 2 and playing the concession game with this seller

[29] The sellers' expected payoff calculations are more involved, and hence presented in the appendix.

[30] According to Proposition 3.1, $F_b^2(T_2^e) = 1 - z_b^*$, which implies the value of T_2^e.

over conceding to seller 1. In equilibrium, rational seller 1 anticipates that the buyer will never concede to him but rather plans to leave his store immediately, and so he accepts the buyer's demand at time zero without any delay.

Lemma 3.1. *In equilibrium where the rational buyer visits seller 1 first with probability σ_1 and $\hat{z}_b^1 \geq (z_s/A)^{\lambda_b/\lambda_s}$ holds, the rational buyer makes a take it or leave it offer to the seller and goes directly to store 2. Rational seller 1 immediately accepts the buyer's demand and finishes the game at time zero with probability one. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is obstinate, and so he never comes back to this store again. The concession game with the second seller may continue until the time $T_2^e = -\log(\hat{z}_b^1)/\lambda_b$ with the following strategies: $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - z_s(\hat{z}_b^1)^{-\lambda_s/\lambda_b} e^{-\lambda_s t}$.*

This result shares the flavor of the arguments of Compte and Jehiel (2002) on the role of outside options for the obstinate negotiators and of one-sided uncertainty result of Myerson (1991, Theorem 8.4). In equilibrium, if the value of the buyer's outside option is high, then the seller is forced to reveal his rationality, implying immediate concession by the rational seller. Assuming that the rational buyer visits seller 1 first and $\hat{z}_b^1 \geq (z_s/A)^{\lambda_b/\lambda_s}$ holds, the rational buyer's equilibrium payoff of visiting seller 1 first is given by

$$\begin{aligned} V_b^1 &= (1 - z_s)(1 - \alpha_b) + \delta z_s \left[v_b^2 \right] \\ &= (1 - \alpha_b) \left[1 - z_s(1 - \delta) - \frac{\delta z_s^2}{[\hat{z}_b^1]^{\lambda_s/\lambda_b}} \right] + (1 - \alpha_s) \frac{\delta z_s^2}{[\hat{z}_b^1]^{\lambda_s/\lambda_b}} \end{aligned} \tag{7}$$

Lemma 3.2. *In equilibrium where the rational buyer visits seller 1 first with probability σ_1 and $\hat{z}_b(1) \leq (z_s^2/A)^{\lambda_b/\lambda_s}$ holds, the buyer leaves store 1 at time $T_1^d = -\log(z_s)/\lambda_s$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time $T_2^e = -\log(z_s/A)/\lambda_s$. The players' concession game strategies are $F_b^1(t) = 1 - \hat{z}_b^1 (A/z_s^2)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$ and $F_1(t) = 1 - e^{-\lambda_s t}$ in store 1, and $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$ in store 2.*

In equilibrium, $\hat{z}_b^1 \leq (z_s^2/A)^{\lambda_b/\lambda_s}$ implies that the rational buyer's initial reputation is very low and thus he needs to spend significant amount of time to build up his reputation before leaving the first seller. In this case, $F_1(0) = 0$, i.e. the buyer does not receive an initial probabilistic gift from seller 1, implying that the rational buyer is weak and so his expected payoff during the concession game with seller 1, v_b^1, is $1 - \alpha_s$. Therefore, the rational buyer's expected payoff of visiting seller 1 first, V_b^1, is also $1 - \alpha_s$.

Lemma 3.3. *In equilibrium where the rational buyer visits seller 1 first with probability* σ_1 *and* $(z_s^2/A)^{\lambda_b/\lambda_s} < \hat{z}_b^1 < (z_s/A)^{\lambda_b/\lambda_s}$*, the buyer leaves store 1 at time* $T_1^d = -\log(\hat{z}_b^1)/\lambda_b + \log(z_s/A)/\lambda_s$ *for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time* $T_2^e = -log(z_s/A)/\lambda_s$*. The players' concession game strategies are* $F_b^1(t) = 1 - e^{-\lambda_b t}$ *and* $F_1(t) = 1 - (z_s^2/A)(\hat{z}_b^1)^{-\lambda_s/\lambda_b} e^{-\lambda_s t}$ *in store 1, and* $F_b^2(t) = 1 - e^{-\lambda_b t}$ *and* $F_2(t) = 1 - Ae^{-\lambda_s t}$ *in store 2.*

In this particular case, the rational buyer's equilibrium payoff of visiting seller 1 first is

$$V_b^1 = (1-\alpha_b)\left[1 - \frac{z_s^2}{A[\hat{z}_b^1]^{\lambda_s/\lambda_b}}\right] + (1-\alpha_s)\frac{z_s^2}{A[\hat{z}_b^1]^{\lambda_s/\lambda_b}} \tag{8}$$

I call the buyer ***strong*** if the first seller he visits makes an initial probabilistic concession and ***weak*** otherwise.[31] Similarly, seller i is called strong if the rational buyer concedes to him with a positive probability at the time he visits store i first at time zero, and weak otherwise.

The last three Lemmas show that the rational buyer's expected payoff of visiting seller i first increases with $\hat{z}_b^i$ (See Equations (7) and (8)). As a result, if the buyer's initial reputation z_b is high, then in equilibrium the rational buyer must visit each seller with equal probabilities so that $\hat{z}_b^1 = \hat{z}_b^2$ holds. This is true because if the rational buyer's strategy is such that $\sigma_i > \sigma_j$, then we would have $\hat{z}_b^i < \hat{z}_b^j$, implying that the rational buyer benefits from deviating and visiting seller j with probability one. On the other hand, when z_b is weak, the buyer can choose each store with different probabilities as long as these probabilities are not too distinct from 1/2.

Along with Lemmas 3.1-3.3, the next result implies three things. First, the buyer is weak (or strong) if and only if the sellers are strong (or weak). Second, the buyer is weak if $z_b \le (z_s^2/A)^{\lambda_b/\lambda_s}$. And third, the rational buyer's expected payoff in the game G is $1-\alpha_s$ if he is weak and strictly higher than $1-\alpha_s$ otherwise.[32]

Proposition 3.2. *In equilibrium, the rational buyer visits seller 1 first with probability* $\sigma_1 = \frac{1}{2}$ *whenever the buyer is strong, i.e.* $z_b > (z_s^2/A)^{\lambda_b/\lambda_s}$*. Otherwise,* $\sigma_1 \in [D, 1-D]$ *where* $D = \frac{z_b\left[1-(z_s^2/A)^{\lambda_b/\lambda_s}\right]}{2(1-z_b)(z_s^2/A)^{\lambda_b/\lambda_s}}$.

The last result in this section shows that extreme greediness makes the buyer weak independent of his initial reputation z_b.

Proposition 3.3. *Suppose now that* $\alpha_b = 0 < \alpha_s$*. In equilibrium, the rational buyer visits seller* i *first with probability* $\sigma_i \in (0,1)$ *and accepts* α_s *immediately following his*

[31] Note that, the second seller (the one who is visited after the first seller) always makes an initial probabilistic concession in equilibrium.

[32] In fact, if the buyer is strong, then the rational buyer's expected payoff in the game is given by the Equations (7) or (8), depending on the value of z_b.

arrival at store i. The rational sellers never concede to the buyer. Hence, equilibrium payoff of the buyer is $1 - \alpha_s$.

Proof. Clearly, these strategies form an equilibrium. Next, suppose for a contradiction that there is some other equilibrium where the sellers concede to the buyer at some time $t > 0$. First, conceding to the buyer gives instantaneous payoff of zero to the sellers and waiting is definitely optimal if the sellers believe that the buyer is rational with some positive probability. Therefore, in any equilibrium, a seller concedes to the buyer at time t if (a) he is convinced by this time that the buyer is obstinate or (b) that the buyer is leaving his store once and for all. The buyer's reputation reaches one at time t if and only if the rational buyer makes concession with probability one before or at time t. However, according to Lemma A.1(in the appendix) if the seller is not making a concession until time t, then in equilibrium, the rational buyer does not make concession until time t either. On the other hand, the second case, i.e. (b), contradicts with subgame perfection because the rational buyer's continuation payoff of leaving a seller and not returning is $(1 - \alpha_s)/\delta$. □

B. Asymmetric Obstinate Types for the Sellers

This section characterizes the unique equilibrium strategy of the competitive-bargaining game G when the sellers' commitment types are different. Without loss of generality, I assume that $\alpha_1 > \alpha_2 > \alpha_b$. In this case, the structure of the equilibrium strategy drastically changes (relating to the case where $\alpha_1 = \alpha_2$). In equilibrium, the bargaining phase never ends with the buyer's concession to the seller who has the higher demand (seller 1). If the buyer ever visits store 1, the rational seller 1 concedes to the buyer (upon the buyer's arrival at this seller) because the buyer has the tendency to opt out instantly from the concession game in store 1.

More formally, consider the case where the buyer is in store 1 and playing the concession game with this seller. This means that the rational buyer should be indifferent between, on the one hand, accepting seller 1's demand, thus receiving the instantaneous payoff of $1-\alpha_1$, and on the other hand, waiting for the concession of the seller. However, if the rational buyer leaves (immediately) seller 1 and goes directly to the second store to accept the demand of seller 2, his discounted payoff will be $\delta(1-\alpha_2)$. Thus, if the rational buyer ever visits store 1 in equilibrium, then he will never accept seller 1's demand. This is because we have $(1 - \alpha_1) < \delta(1 - \alpha_2)$ by the assumption that the search friction is very small. Therefore, in equilibrium, the rational buyer does not concede to nor spend time with seller 1 given that he ever visits store 1. As a result, it must be the case that rational seller 1 instantaneously accepts the buyer's demand with probability one upon his arrival, and the buyer immediately leaves store 1 if seller 1 does not concede to him.

Since the buyer plays an equilibrium strategy that impels rational seller 1 to reveal his type immediately, the rational buyer's expected payoff of visiting this seller is $(1 - z_s)(1 - \alpha_b) + \delta z_s v_b^2$. I denote by v_b^2 the buyer's expected payoff in store 2 when he visits this store knowing that seller 1 is the obstinate type. Thus, if the rational buyer initially chooses to visit seller 2, then he concedes to this seller and receives the instantaneous payoff of $1 - \alpha_2$, if and only if $1 - \alpha_2 \geq \delta[(1 - z_s)(1 - \alpha_b) + \delta z_s v_b^2]$.

This inequality holds when $z_s \geq \bar{z}$ holds, where $\bar{z}$ is very close to one as δ is close to one.[33] However, assuming that initial priors are small enough, we have $z_s < \bar{z}$, implying that if the buyer visits seller 2 first in equilibrium, then the rational buyer strictly prefers leaving this seller immediately upon his arrival. Hence, rational seller 2 must concede to the buyer at time 0 with probability one. The next result characterizes the second-stage equilibrium strategies of the competitive-bargaining game G.

Proposition 3.4. *Suppose that $0 < \alpha_b < \alpha_2 < \alpha_1$. In the unique sequential equilibrium of the competitive-bargaining game G*

(i) if the buyer visits seller 1 first, then the rational buyer makes a take it or leave it offer to the seller and goes directly to store 2. Rational seller 1 immediately accepts the buyer's demand and finishes the game at time zero with probability one. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is obstinate, and so he never comes back to this store again. The concession game with seller 2 may continue until the time $T_2^e = \min\{-\frac{\log \hat{z}_b^1}{\lambda_b}, -\frac{\log z_s}{\lambda_2}\}$ where $\lambda_2 = \frac{(1-\alpha_2)r_b}{\alpha_2 - \alpha_b}$ and $\lambda_b = \frac{\alpha_b r_s}{\alpha_2 - \alpha_b}$ with the following strategies: $F_2(t) = 1 - z_s e^{\lambda_2(T_2^e - t)}$ and $F_b^2(t) = 1 - \hat{z}_b^1 e^{\lambda_b(T_2^e - t)}$ for all $t \geq 0$.[34]

(ii) if the buyer visits seller 2 first, then rational seller 2 immediately accepts the buyer's demand upon his arrival. Otherwise, the buyer leaves seller 2 immediately at time zero (knowing that seller 2 is the obstinate type), and goes directly to seller 1. Rational seller 1 instantly accepts the buyer's demand with probability one upon the buyer's arrival. In case seller 1 does not concede, the rational buyer immediately leaves this seller, directly returns to seller 2, accepts the seller's demand α_2 and finalizes the game.

Therefore, in equilibrium, when the buyer visits seller 1 first, he sends a take it or leave it ultimatum to this seller. If seller 1 does not accept the buyer's demand, then the buyer will go to the second seller. In this case, an agreement might be reached with seller 2, but possibly after some delay. On the other hand, when the buyer visits seller 2 first, he sends the same ultimatum to both sellers (first to seller 2 and then to 1). If

[33] See the proof of Proposition 3.4 in Appendix

[34] Note that for notational simplicity, I reset the clock once the buyer enters store 2.

no seller accepts the buyer's demand, then the rational buyer will come back to seller 2 and accept his demand α_2.[35] Hence, the rational buyer visits seller 1 first only when he is strong relative to seller 2 (i.e., $\hat{z}_b^1$ is sufficiently higher than z_s) so that the initial probabilistic concession he will receive from seller 2 is high enough. This implies that in equilibrium, the rational buyer will visit seller 1 first with a very low probability. The following result summarizes the last argument formally.

Proposition 3.5. *In the unique equilibrium of the competitive-bargaining game G, the rational buyer visits seller 1 first with a very small probability. That is, $\sigma_1 = \frac{z_b(1-\bar{A}_2)}{2\bar{A}_2(1-z_b)}$ where $\bar{A}_2 = [(\alpha_2 - \alpha_b)/1 - \alpha_b - \delta(1-\alpha_2)]^{\lambda_b/\lambda_2}$.*

Thus if the obstinate types are such that $0 < \alpha_b < \alpha_2 < \alpha_1$, then the equilibrium payoff of the rational buyer is

$$V_b = (1 - z_s)(1 - \alpha_b) + z_s\delta[(1 - z_s)(1 - \alpha_b) + z_s\delta(1 - \alpha_2)] \tag{9}$$

The final result in this section investigates the equilibrium when the buyer is extremely greedy, i.e. $\alpha_b = 0$.

Proposition 3.6. *Suppose now that $0 = \alpha_b < \alpha_2 < \alpha_1$. In equilibrium of the competitive-bargaining game G, the payoff to the rational buyer is $1-\alpha_2$, and the payoff to the rational seller 1 is 0.*

Proof. It is clear that the buyer will never concede to seller 1 as $(1 - \alpha_1)$ is less than the discounted payoff he can achieve by accepting the second seller's demand. Hence, in any equilibrium, the payoff of the first seller must be 0. Moreover, in equilibrium, the rational buyer must choose $\sigma_i \in (0, 1)$, implying that he must be indifferent between the sellers to visit first. However, the rational seller 2 will never accept the buyer's demand in equilibrium. This is because the rational buyer will come back to second store for sure to accept his demand once he realizes that seller 1 is obstinate, implying positive expected payoff for the rational seller 2. Hence, the highest expected payoff the rational buyer can attain in the second store is $1-\alpha_2$, i.e. immediate concession to the second seller. Hence, the rational buyer's expected payoff in the game must be $1 - \alpha_2$. $\square$

[35] Rational seller 2's immediate concession to the buyer (and receiving the payoff of α_b) is optimal because otherwise the rational seller can achieve at most $\alpha_2 z_s$ (since the buyer revisits seller 2 only if seller 1 is the obstinate type) and we have $\alpha_2 z_s < \alpha_b$ by assumption.

4. Multiple Commitment Types

In this section, I resume the case where the players have multiple types $C \subset [0,1)$ with $0 \in C$. For the rest of this section, I fix the value of the search friction δ and the set of obstinate types C. For any $z_b, z_s \in (0,1)$, let $\mathrm{G}(z_b, z_s)$ denote the competitive-bargaining game G where the initial reputations of the sellers and the buyer are z_b and z_s, respectively.

Remark that $0 \in C$ is always an equilibrium demand selection of the sellers in the first stage of the competitive-bargaining game. In this section I will characterize the set of equilibrium prices for any (small) values of z_b and z_s. Also note that, in equilibrium, the rational buyer will demand 0 and visit each store first with some positive probability, that is $\mu(0) > 0$ and $\sigma_i \in (0,1)$ for each seller i. These are true, independent of the sellers' posted prices or initial reputations, because otherwise the buyer would benefit from deviating to be perceived as an obstinate type.

Proposition 4.1. *There exists no sequential equilibrium of the game $G(z_b, z_s)$ in which the sellers declare different demands in stage 1.*

Proof. Suppose for a contradiction that (α_1, α_2) are equilibrium demand selections of the sellers in the first stage of the game $\mathrm{G}(z_b, z_s)$, and without loss of generality that $\alpha_1 > \alpha_2 > 0$. By Proposition 3.5, if the buyer announces $\alpha_b = 0$, then his expected payoff in equilibrium will be $1 - \alpha_2$. However, if he announces $\alpha_b' = \min\{C \setminus \{0\}\}$ then his expected payoff will be $(1 - z_s)(1 - \alpha_b') + z_s\delta[(1 - z_s)(1 - \alpha_b') + z_s\delta(1 - \alpha_2)]$ as given by Equation (9), which is clearly higher than $1 - \alpha_2$. Hence, the rational buyer prefers posting α_b' over 0, contradicting the fact that in equilibrium the buyer must announce zero with some positive probability. $\square$

Since the sellers are *ex-ante* identical, it is natural to suspect that in equilibrium both sellers should choose the same demand. The following two results characterizes the equilibrium payoff of a rational seller who price undercuts his opponent.

Proposition 4.2. *Consider a history at which sellers post the prices α_1 and α_2 with $\alpha_1 > \alpha_2$, seller 2 is known to be obstinate whereas the true types of seller 1 and the buyer are unknown. Then the unique sequential equilibrium of the continuation game followed by this history is as follows.*

(i) *If $\alpha_2 > 0$, then the rational buyer announces his demand as 0 and visits seller 1 first (with probability one) to make the take it or leave it offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes* *directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand.*[36]

(ii) *If $\alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage.*

Proof. Consider a history and the strategies prescribed above. It is straightforward to show that they constitute an equilibrium. To show that there is no other equilibrium, recall that $1 - \alpha_1 < \delta(1 - \alpha_2)$ because the search friction is assumed to be sufficiently small. Therefore, it is optimal for the rational buyer to go to store 2 and to accept α_2 instead of accepting α_1. Moreover, regardless of the buyer's announcement α_b, postponing concession is not optimal for rational seller 1 since the buyer will never accept α_1 in equilibrium. Thus, rational seller 1 accepts the buyer's demand upon his arrival at store 1, and the rational buyer will choose $\alpha_b = 0$ in equilibrium. The remaining parts of the equilibrium strategies immediately follow for small values of z_b and z_s. $\square$

Therefore, if seller 2 deviates from his strategy and price undercuts his opponent, then the buyer infers that seller 2 is obstinate with certainty (as sellers are playing pure strategies in the first stage). Being perceived as an obstinate seller reduces the chance that his offer is accepted by the buyer. This is true because the rational buyer prefers to use the obstinate seller's low price as an "outside option" to increase his bargaining power against seller 1 whom he can negotiate and possibly get a much better deal. As a result of this, deviating from an equilibrium price leads to a very low expected payoff for a rational seller as the following result indicates.

Corollary 4.1. *Consider an equilibrium where both sellers post price $\alpha_s > 0$. Suppose that rational seller 2 deviates and posts α_2 in stage 1. Then, his expected payoff in the game will be zero if $\alpha_2 > \alpha_s$ and $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + z_s(1 - z_b)\right]$, which is strictly less than $(z_b + z_s)\alpha_2$, otherwise.*

Proof. Recall that rational sellers' price posting strategies are pure. Therefore, if rational seller 2 deviates to α_2 at time zero, then other players will conclude that seller 2 is obstinate of type α_2. Given the assumptions on obstinate types, the rational buyer's expected payoff of posting $\alpha_2 > \alpha_s$ is zero. On the other hand, Proposition 4.2 gives the equilibrium strategies of the continuation game following a history where seller 2 price undercuts his opponent. Deviation to $\alpha_2 = 0$ clearly implies expected payoff of 0. However, if $\alpha_2 > 0$, then the second seller's expected payoff will be $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + z_s(1 - z_b)\right]$ where $z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b)$ is the probability that the buyer is an obstinate type with demand higher than or equal to α_2. $\square$

[36] Therefore, in case the game does not end in store 1, the buyer infers that 1 is the obstinate type with demand α_1.

Remark that the parameters provided in Section 3.A, in particular, A, λ_b and λ_s, all depend on the sellers' and the buyer's announced demands α_s and α_b, although these notations omit this connection for simplicity. The next result characterizes the set of equilibrium prices of the game $\mathrm{G}(z_b, z_s)$. The main message of the result is simple. A demand $\alpha_s \in C \setminus \{0\}$ is an equilibrium selection of the rational sellers if and only if the buyer is weak for all demands $\alpha_b \in C$ with $\alpha_b < \alpha_s$. Hence, in an equilibrium where the sellers post the price of α_s, the rational buyer's expected payoff in the game is $1 - \alpha_s$.

Proposition 4.3 . *Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ if and only if for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$ we have $z_b \leq (z_s^2/A)^{\lambda_b/\lambda_s}$.*

I defer the proof to Appendix. Given that both sellers choose the same demand α_s that is higher than 0, the rational buyer's strategy choosing each seller with equal probabilities and declaring a demand $\alpha_b < \alpha_s$ according to $\mu^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$ (together with the second stage strategies as characterized in Section 3) are equilibrium strategies for the buyer when he is weak at all demands in the support of μ^*. Under these strategies, the equilibrium payoff of a rational seller is greater than $\frac{u}{2}\left[1 - z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b)\right]$ where $u = \sum_{\alpha_b<\alpha_s} \alpha_b \mu(\alpha_b)$. On the other hand, as Corollary 4.1 shows, a rational seller's expected payoff is much less than $z_b + z_s$ if he deviates from α_s. Hence, for sufficiently small values of z_b and z_s, posting non zero prices is an optimal strategy for the sellers as their expected payoff under these strategies is strictly greater than what they can achieve by price undercutting.

The last result in this section, that follows directly from Propositions 4.1 and 4.3, shows that all obstinate demands in C can be supported in equilibrium for some z_b and z_s small enough.

Corollary 4.2. *For all $\alpha_s \in C$, there exists some small $z_b, z_s \in (0, 1)$ such that α_s is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$.*

5. The Limiting Case of Complete Rationality

This section characterizes the set of equilibrium prices when the frictions vanish. For this purpose, first fix the parameters C, π, r_b, r_s and the search friction δ. I say the competitive-bargaining game $\mathrm{G}(z_b^m, z_s^m)$ converges to $\mathrm{G}(K)$ when the sequences $\{z_s^m\}$ and $\{z_b^m\}$ of initial priors satisfy

$$\lim z_s^m = 0, \lim z_b^m = 0 \text{ as } m \to \infty \text{ and } \log z_s^m / \log z_b^m = K \text{ for all } m \geq 0 \qquad (10)$$

Proposition 5.1. *If the game $G(z_b^m, z_s^m)$ converges to $G(K)$ and α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m)$, then $\lim \alpha_s^m = \alpha_s \in C$ satisfies $2K\alpha_b r_s \leq (1-\alpha_s)r_b$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

Proof. Recall that Proposition 4.3 implies that for any given z_b^m and z_s^m small enough the demand α_s^m is the equilibrium posted price of the sellers in the game $\mathrm{G}(z_b^m, z_s^m)$ if and only if $z_b^m \leq [(z_s^m)^2/A]^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s^m$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha_b r_s}{(1-\alpha_s^m)r_b}\left(2\log z_s^m - \log A\right)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $2K\alpha_b r_s \leq (1-\alpha_s)r_b$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s^m$. $\square$

Now fix the values of r_b and r_s. I say the competitive-bargaining game $\mathrm{G}(z_b^m, z_s^m, C^m, \delta^m)$ converges to $\mathrm{G}(K, [0,1])$ when the sequences $\{z_s^m\}$ and $\{z_b^m\}$ satisfy (10), C^m converges $[0,1]$ and $\delta^m \to 1$ in such a way that $\{z_s^m\}$, $\{z_b^m\}$ and the search friction is sufficiently small for all m. In particular, for all $\alpha, \alpha' \in C^m$ with $\alpha' < \alpha$ we have $(1-\alpha) < \delta^m(1-\alpha')$.

Corollary 5.1. *If the game $G(z_b^m, z_s^m, C^m, \delta^m)$ converges to $G(K, [0,1])$ and α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m, C^m, \delta^m)$, then $\lim \alpha_s^m = \alpha_s \in [0,1]$ satisfies $\alpha_s \leq \frac{r_b}{r_b + 2Kr_s}$*

As a special case, when the players' interest rates are common, i.e. $r_b = r_s$, and $\{z_s^m\} = \{z_b^m\}$ for all m, then the set of equilibrium prices for the sellers converge to the set $[0, \frac{1}{3}]$. Notice that higher impatience for the rational buyer (higher r_b) will increase the maximum price attainable in equilibrium. On the contrary, increasing impatience for the sellers (higher r_s) decreases the maximum price that can be supported in the limit.

Finally, note that all prices can be supported in equilibrium with carefully selected and vanishing initial priors. The monopoly price of 1, for example, can be arbitrarily approached if the priors z_b^m and z_s^m are selected so that K is sufficiently close to zero.

The final result of this section examines a straightforward extension of the model to the case with $N > 2$ identical sellers. Namely, let $\mathrm{G}^N(z_b^m, z_s^m, C^m, \delta^m)$ denote the competitive-bargaining game where the number of sellers is N; it is identical to $\mathrm{G}(z_b^m, z_s^m, C^m, \delta^m)$ except the number of players. Let the convergence of $\mathrm{G}^N(z_b^m, z_s^m, C^m, \delta^m)$ to the game $\mathrm{G}^N(K, [0,1])$ be identical to the convergence of its 2-seller counterpart. Therefore,

Proposition 5.2. *If the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$ converges to $G^N(K, [0,1])$ and α_s^m is the equilibrium posted price of the rational sellers in the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$, then $\lim \alpha_s^m = \alpha_s \in [0,1]$ satisfies $\alpha_s \leq \frac{r_b}{r_b + NKr_s}$.*

Therefore, for any large but finite number of sellers N, we can find small enough z_b^m relative to z_s^m so that $K < 1/N$, and thus prices arbitrarily close to 1 can be supported in equilibrium with vanishing frictions.

6. High Search Friction

In this section I argue that high search friction may destroy the sellers' market power which may exist when the buyer is weak. For this purpose, I make two changes in the model. First, I assume that the search friction is large enough so that for all $0 < \alpha \in C$, there exists $\alpha' \in C$ with $\alpha' < \alpha$ such that $(1-\alpha) > \delta(1-\alpha')$. That is, the search friction may prevent the rational buyer to walk away from a store even if he knows that the other seller has posted a lower price. Second, I assume that the set C is dense enough.[37]

It is important to note that if the sellers' posted prices satisfy the inequality mentioned above, then the equilibrium strategies of the rational players in the second stage will be the same as those characterized in Section 3-A, with (possibly) one difference; the rational buyer's hazard rate will be different if the sellers' posted demands are different. Therefore, in equilibrium, the buyer will play the concession game with the seller who posts the higher price. However, if the sellers' posted prices are apart from one another, so that the above inequality does not hold for the sellers' demands, then the second stage equilibrium strategies will be the same as those characterized in Section 3-B.

With the similar reasoning of Propositions 4.1 and 4.3, it is easy to show that the unique equilibrium price of the sellers is 0 if the buyer is strong, and the sellers will never post different prices in equilibrium. Along with these arguments, the following result ensures that the unique equilibrium price selection of the sellers in the game G will be 0 when the search friction is sufficiently high.

Proposition 6.1. *Take any z_b and z_s small enough and $\alpha_s \in C\backslash\{0\}$ with $z_b \leq (z_s/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b < \alpha_s$. If C is sufficiently dense, then there exists no equilibrium of the competitive-bargaining game $G(z_b, z_s)$ supporting α_s as the rational sellers' selection in the first stage.*

I defer the proof to Appendix.

7. The Buyer's Moves are Unobservable by the Public

Next, I investigate the case where the buyer's moves and demand announcements are not public. I will show that the sellers' market power will increase further in this

[37] That is, for all $\alpha \in C$ and $\alpha' := \max\{C_\alpha \setminus \{\alpha\}\}$, we have $\alpha - \alpha' \leq \epsilon$ for some $0 < \epsilon$ sufficiently small. This assumption is stronger than what I need to prove Proposition 6.1, but it simplifies the proof without affecting the main message of this result.

case. For this reason, I make three modifications on the competitive bargaining game G. First, the rational buyer announces his demand at the sellers' stores and he can offer different demands in each store.[38] Second, the buyer's moves including his arrival to the market are unknown by the public. That is, sellers can observe the buyer only when he visits their stores. Third, related to the previous one, the buyer arrives at the market according to a Poisson arrival process. Given that the rational buyer plays a strategy in which he visits both sellers with positive probabilities upon his arrival at the market, the last assumption ensures that sellers cannot learn the buyer's actual type and if they are the first or the second store visited by the buyer.[39]

The next result shows that if z_b is sufficiently small, then the following strategies support any $\alpha_s \in C \setminus \{0\}$ as an equilibrium demand selection of the sellers. Strategies are as follows: In stage 1, both sellers post α_s. In stage 2, upon his arrival at time $T \geq 0$, the rational buyer (immediately) visits the sellers with equal probabilities. Upon the buyer's entry to store i (at time T), the rational buyer immediately declares his demand $\alpha_b < \alpha_s$ according to $\mu^T_{\alpha_i}(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$ and starts concession game with seller i. The players' strategies in the concession games are $F^T_b(t) = 1 - \frac{\hat{z}^{T,i}_b}{z_s^{\lambda_b/\lambda_s}} e^{-\lambda_b t}$ and $F^T_i(t) = 1 - e^{-\lambda_s t}$ where $\hat{z}^{T,i}_b$ is the probability that the buyer is the commitment type α_b conditional on him visiting seller i at time T and demanding $\alpha_b < \alpha_i$. The rational players' hazard rates λ_b, λ_s are as characterized in Section 3. The concession game with a seller may last until time $T - \log(z_s)/\lambda_s$ at which point both the buyer's and the seller's reputations simultaneously reach one.

According to these strategies, the rational buyer will visit only one seller. Moreover, due to the Poisson arrival process and Bayes' rule, the sellers will believe very highly that the buyer is rational conditional on his arrival at their stores. In particular, $\hat{z}^{T,i}_b$ is independent of i and it equals to either z_b or a number very close to z_b. In other words, sellers will learn nothing about the buyer's actual type upon his arrival at their stores because the sellers' prior belief will stay (almost) the same for the entire arrival process.[40] Given that the buyer arrives at the market at time T, the concession game with the seller does not end by the time $-\log(z_s)/\lambda_s + T$ if both the buyer and the seller are commitment types. The obstinate buyer with demand α_b leaves the first seller at this

[38] Parallel to the assumptions made in Section 2, the obstinate buyer also announces his demand at the sellers' store if his demand is less than the posted prices. Otherwise, he immediately accepts the lowest posted price and finalize the game in stage 1.

[39] In the modified game, the rational players' strategies, that may depend on time T indicating the buyer's arrival time, are equivalent to the strategies defined in Section 2 with one exception. Now, $\mu^T_{\alpha_1}, \mu^T_{\alpha_2}$ are parts of the buyer's second stage strategies and functions of the sellers' posted prices and the arrival time $T \geq 0$. Note that, the first stage is time 0 where the sellers announce their demands and the buyer observes these prices. The second stage starts at the time that the buyer arrives at the market.

[40] I calculate $\hat{z}^{T,i}_b$ formally in the proof of Proposition 7.1

time (if the game has not yet ended) and directly goes to the second seller. However, the rational second seller will play the concession game with the (obstinate) buyer believing that his opponent is the obstinate type with probability $\hat{z}_b^{-\log(z_s)/\lambda_s+T,i}$ which is very close to z_b.

Proposition 7.1. *Take any z_b and z_s small enough. Then, $\alpha_s \in C\backslash\{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ if and only if for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$ we have $z_b \leq \frac{z_s^{\lambda_b/\lambda_s}}{1+z_s(1-z_s^{\lambda_b/\lambda_s})}$.*

I defer the proof to Appendix. Similar to the analyses in Section 5, the following result characterizes the equilibrium prices of the sellers for vanishing frictions.

Proposition 7.2. *If the game $G(z_b^m, z_s^m)$ converges to $G(K)$ and α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m)$, then $\lim \alpha_s^m = \alpha_s \in C$ satisfies $K\alpha_b r_s \leq (1-\alpha_s)r_b$ for all $\alpha_b \in C_{\alpha_s}$.*

Proof. Recall that Proposition 7.1 implies that for any given z_b^m and z_s^m small enough the demand α_s^m is the equilibrium posted price of the sellers in the game G(z_b^m, z_s^m) if and only if $z_b^m \leq \frac{(z_s^m)^{\lambda_b/\lambda_s}}{1+(z_s^m)[1-(z_s^m)^{\lambda_b/\lambda_s}]}$ for all $\alpha_b \in C_{\alpha_s^m}$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha_b r_s}{(1-\alpha_s^m)r_b}\left(\log z_s^m - \log\left[1 + z_s^m[1-(z_s^m)^{\lambda_b/\lambda_s}]\right]\right)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $K\alpha_b r_s \leq (1-\alpha_s)r_b$ for all $\alpha_b \in C_{\alpha_s}$. □

Finally, since the buyer cannot carry his improved reputation when he leaves a seller, the buyer is weak, regardless of the number of sellers in the market, if $z_b \leq \frac{z_s^{\lambda_b/\lambda_s}}{1+z_s(1-z_s^{\lambda_b/\lambda_s})}$. Therefore, the immediate counterpart of Proposition 5.2 will be as follows.

Corollary 7.1. *If the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$ converges to $G^N(K, [0,1])$ and α_s^m is the equilibrium posted price of the rational sellers in the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$, then $\lim \alpha_s^m = \alpha_s \in [0,1]$ satisfies $\alpha_s \leq \frac{r_b}{r_b+Kr_s}$.*

Note that the upper bound for the equilibrium prices of the sellers is larger than the one provided in Proposition 5.2. Thus, we can conclude that if the buyer's moves are unobservable by the public, then the sellers' market powers increase as higher prices can be supported in equilibrium.

8. Some Extensions

Obstinate Players

An obstinate player is a man of unyielding perseverance. Sellers may manifest such a steadfast attitude because they might be confined to do so. A company may be inflexible in a wage negotiation due to some regulations within the company. For example, a car dealer, a sales clerk or a realtor may be restricted by the owner regarding how flexible he can be in his demands while negotiating with a buyer. A fresh college graduate who is competing with other candidates for a specific job opening may commit to a certain salary because he wants to pay his student loan without too much financial difficulty.

Steady persistence in adhering to a course of action as assumed for an obstinate (type) buyer would be reasonable when, for example, the "buyer" is looking to advance his position. A worker (negotiating with more than one firm) may accept the new job offer if it provides a significant jump in his salary or title relative to the position he is already holding. On the other hand, a successful investor (a venture capitalist) whose portfolio have assets having high profit margins may commit to buy a small business only if it is a real bargain because otherwise it may not be worth including it in his portfolio. An entrepreneur who is running a successful small business may commit to his initial demands while negotiating with investors to sell his business or a franchise because of his overly optimistic expectations about the future of his business.

To justify the current assumptions on the obstinate buyer, one may suppose that the obstinate buyer is a player that is the "least strategic" or naive in terms of store choice and timing of departure, or a man who "plays it cool." Alternatively, one may assume that the obstinate buyer (1) does not discount time and (2) incurs a positive (but very small) switching cost ($\epsilon_b > 0$) every time he switches his bargaining partner.[41]

The assumption that the obstinate buyer visits each seller at time zero with equal probabilities is a simplification assumption. It can be generalized with no impact on the main messages of our results. For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always

[41] Therefore, according to (1), the time of an agreement is not a concern for the obstinate buyer, and thus he does not feel the need to distinguish himself from the rational buyer who wishes to reach an agreement as quickly as possible. Since the obstinate buyer does not discount time, ϵ_b is the only search friction that the obstinate buyer is subject to and it would have no impact on our analysis –the switching cost ϵ_b would work as a tie-breaking device. Moreover, the assumption "the obstinate buyer understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate" can be interpreted as an implication rather than an assumption. Since the obstinate buyer does not value time, he should be indifferent between staying with his current partner or visiting the other seller at any time (ignoring the switching cost). However, if he leaves his current partner before being convinced that he is obstinate, he will revisit this seller later if he exhausts all his hope to reach an agreement with the other seller. Therefore, since the switching cost ϵ_b is positive, the obstinate buyer will switch his partner just once and thus leaves a store when he is convinced that his opponent is also obstinate.

choose a fix seller and some visit the sellers according to their announcements while the rest are possibly a combination of these two.

The assumption on the obstinate buyer's departure habit seems a strong one since it eliminates the possibility that the rational buyer would increase his bargaining power by committing to a particular pattern of store choice. For example, consider the case where the obstinate buyer is "more strategic." That is, he commits himself to immediately switch to another seller if the first seller does not concede right away. In some situations, it will increase the rational buyers payoff. However, as the following two results show, it does not alter the main message of the paper. That is, multiple, non-Walrasian prices can be supported in equilibrium.

The case with "more strategic" obstinate buyer: Now suppose that the obstinate buyer (of any demand) leaves the first store he visits at some prespecified time T where, $0 \leq T$. Although parallel arguments are valid for any T, I focus, without loss of generality, to the case where $T = 0$.

The next result shows that any $\alpha_s \in C$ with $0 < \alpha_s$ is an equilibrium price for the sellers if the buyer is weak in equilibrium. The equilibrium strategies are as follows. In stage 1, rational sellers post the same demand α_s, the rational buyer visits each seller with equal probabilities and declares his demand as $\alpha_b < \alpha_s$ according to $\mu^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}$. At the beginning of stage 2, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time zero with probability $P_b = \frac{(z_s/A)^{\lambda_b/\lambda_s} - z_b}{(1-z_b)(z_s/A)^{\lambda_b/\lambda_s}}$ and immediately leaves store 1 with probability $1 - P_b$. Rational seller 1 never concedes to the buyer. The buyer and seller 2 play the concession game in the second store until time $T_2^e = -\frac{log(z_s/A)}{\lambda_s}$ with the following strategies $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$ where the terms λ_b, λ_s and A are as characterized in Section 3.

Proposition 8.1 . *Suppose that the obstinate buyer leaves the first store he visits immediately following his arrival. Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ when $z_b \leq \frac{(z_s/A)^{\lambda_b/\lambda_s}(\alpha_s - \alpha_b)}{\alpha_s + \alpha_b}$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

I defer all the proofs in this section to Appendix. Thus, if z_b and z_s are sufficiently small and selected carefully, then all prices in the set C can be supported in equilibrium as proved in Section 4.

The case with the "most strategic" obstinate buyer: Now suppose that the obstinate buyer (of any demand) leaves all stores immediately following his arrival. The following strategies ensure that all demands in the set C can be supported as equilibrium for small values of z_b and z_s. Rational sellers post the price of $0 < \alpha_s \in C$ and the rational buyer visits each seller with equal probabilities and declares his demand as $\alpha_b < \alpha_s$ according to μ^* that is given above. At the beginning of stage 2, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time zero with probability $P_b = \frac{\alpha_s(1-z_b)-\alpha_b}{(1-z_b)(\alpha_s-\alpha_b)}$ and immediately leaves store 1 with probability $1-P_b$. Rational seller 1 never concedes to the buyer. In store 2, rational seller 2 accepts the buyer's demand upon his arrival with probability $P_s = \frac{(1-\alpha_s)(1-\delta)}{\delta(1-z_s)(\alpha_s-\alpha_b)}$ and never concedes to the buyer with probability $1-P_s$.[42] The rational buyer does not leave store 2 immediately. Instead he waits for the seller's concession. However, if the game does not end at time zero by seller 2's concession, the rational buyer concedes to the buyer immediately.

Proposition 8.2 . *Suppose that the obstinate buyer leaves both stores immediately following his arrival. Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ when $z_b \leq \frac{(\alpha_s-\alpha_b)^2}{\alpha_s(\alpha_s+\alpha_b)}$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

Different Initial Reputations for the Sellers

Suppose for now that the sellers' initial reputations are different, i.e. $z_1 \neq z_2$. This assumption would not change the essence of our results as long as z_1 and z_2 are small enough. In equilibrium, rational sellers will not post different prices because the intuition of Proposition 4.1 will still survive. Similar to Proposition 4.3, in equilibrium, rational sellers post the same price α_s if and only if the buyer is weak, which would mean $z_b \leq \left(\frac{z_1 z_2}{A}\right)^{\lambda_b/\lambda_s}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$. As the rational buyer is weak, his expected payoff is independent of the seller's initial reputations, and so this particular heterogeneity does not change the fundamentals of the competition between the sellers.

Sequential Price Quoting

Suppose now that the price announcement in the game G is sequential. Seller 1 announces its demand first. Then, the second seller posts its price after observing the first seller's announcement. Finally, the buyer declares his demand after observing the sellers' prices and the rest of the game follows as it was before. Note that, this change in the first stage does not alter the equilibrium strategies of the players in the concession game, and so they are the same as those characterized in Section 3.

Similar to the previous arguments, in equilibrium, the sellers will not post different prices. Moreover, if the buyer is strong, then the unique equilibrium price will be 0. These conclusions hold because the arguments in the proof of Proposition 4.1 and 4.3 do not depend on the sellers' timing in stage 1. On the other hand, when the buyer is weak,

[42] Note that P_s is in $(0, 1)$ as $z_s < \frac{(1-\alpha_s)(1-\delta)}{\delta(\alpha_s-\alpha_b)} < 1$.

that is $z_b \leq (z_s/A)^{\lambda_b/\lambda_s}$, then the rational sellers' expected payoff in the game increases with the price they post if z_b and z_s are sufficiently small.[43] Hence, in equilibrium, both sellers will post the same price which will be the highest price available in the set C. As a result, when all the frictions vanish, the unique equilibrium price will converge to $\frac{r_b}{r_b+NKr_s}$ (the upper bound we found in Proposition 5.2) if the buyer is weak and 0 otherwise.

9. The Discrete-Time Model and Convergence

In this section, I consider the competitive-bargaining game in discrete time and investigate the structure of its equilibria as players can make their offers increasingly frequent. I show that given the symmetric obstinate types, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers approach to zero, and this limit is equivalent to the unique outcome of the continuous-time game investigated in Section 3.

To be more specific, I suppose that each player has a single commitment type. In stage 1, first the sellers and then the buyer announces their types $\alpha_s \in (0,1)$ and $\alpha_b \in (0,1)$ respectively where $\alpha_b < \alpha_s$. Then the buyer chooses a store to visit first. Upon the buyer's arrival at store i, beginning of stage 2, the buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller i is defined as $g^i : [0,\infty) \to \{0,1,2,3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[44] Moreover, $g^i(t) = 3$ implies a simultaneous offer whereas $g^i(t) = 0$ means no offer is made at time t. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller i and for all $\bar{t} \geq 0$, the set $I^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1,2,3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non-stationary, non-alternating protocols.

In stage 2, the rational players are free to choose any offer from the set $(0,1)$. An offer $x \in (0,1)$ denotes the share the seller is to receive. If the proposer's opponent accepts his offer, the game ends with agreement x where xe^{-tr_s} denotes the payoff to seller i, 0 is the payoff to seller j and finally $(1-x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the rational buyer decides whether to stay or leave the store. If the rational buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage competitive-bargaining game in discrete-time is denoted by $\mathrm{G}\langle g, (z_n, r_n)_{n\in\{b,s\}}\rangle$ (or $\mathrm{G}(g)$ in short). The competitive-bargaining game $\mathrm{G}(g)$ ends if

[43] See the rational sellers' expected payoff, for example, in the proof of Proposition 4.3.
[44] Time 0 denotes the beginning of the bargaining phase.

the offers are compatible. In the event of strict compatibility the surplus is split equally. Throughout the game, both sellers can perfectly observe the buyer's moves. Thus, the players' actual types remain to be the only source of uncertainty.

I am particularly interested in equilibrium outcome(s) of the competitive-bargaining game $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time competitive-bargaining game where the buyer and the sellers bargain, in stage two, according to the protocol $g_\epsilon = (g_\epsilon^1, g_\epsilon^2)$ such that for all $t \geq 0$ and i, both seller i and the buyer have the chance to make an offer, at least once, within the interval $[t, t+\epsilon]$ in the bargaining protocol g_ϵ^i.[45] In this sense, the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \to 0$.[46]

Now, let σ_ϵ denote a sequential equilibrium of the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ and σ_i be the rational buyer's equilibrium strategy for store selection at time zero. Given σ_i, the random outcome corresponding to σ_ϵ is a random object $\theta_\epsilon(\sigma_i)$ which denotes any realization of an agreed division as well as a time and store at which agreement is reached.

The next result shows that in the limit as ϵ converges to zero $\theta_\epsilon(\sigma_i) \to \theta(\sigma_i)$ in distribution, where $\theta(\sigma_i)$ is the unique equilibrium distribution of the continuous-time game G. Therefore, the outcome of the discrete-time competitive-bargaining game, independent of the bargaining protocol g_ϵ, converge in distribution to the unique (given the buyers initial choice of store) equilibrium outcome of the competitive-bargaining game analyzed in Section 3-A.

Proposition 9.1. *As ϵ converges to 0, $\theta_\epsilon(\sigma_i)$ converges in distribution to $\theta(\sigma_i)$.*

I defer the proof to the Online Appendix.

Appendix

Proof of Proposition 3.1. First, I will study the properties of equilibrium strategies (distribution functions) in concession games. For this purpose, take any $i \in \{1, 2\}$ and history $h_{T_i} \in H^i$, and consider a pair of equilibrium distribution functions $(F_b^{i,T_i}, F_i^{T_i})$ defined over the domain $[T_i, T_i']$ where $T_i' \leq \infty$ depends on the buyers' equilibrium strategy. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

[45] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t+\epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t+\epsilon]$.

[46] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

Lemma A.1. *If a player's strategy is constant on some interval* $[t_1, t_2] \subseteq [T_i, T_i')$*, then his opponent's strategy is constant over the interval* $[t_1, t_2 + \eta]$ *for some* $\eta > 0$.

Lemma A.2. F_b^{i,T_i} *and* $F_i^{T_i}$ *do not have a mass point over* $(T_i, T_i']$.

Lemma A.3. $F_i^{T_i}(T_i)F_b^{i,T_i}(T_i) = 0$

Therefore, according to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_b^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Recall that

$$U_i(t, F_b^{i,T_i}) = \int_{T_i}^{t} \alpha_s e^{-r_s y} dF_b^{i,T_i}(y) + \alpha_b e^{-r_s t}(1 - F_b^{i,T_i}(t))$$

denote the expected payoff of rational seller i who concedes at time $t \geq T_i$ and

$$U_b(t, F_i^{T_i}) = \int_{T_i}^{t} (1 - \alpha_b) e^{-r_b y} dF_i^{T_i}(y) + (1 - \alpha_s) e^{-r_b t}(1 - F_i^{T_i}(t))$$

denote the expected payoff of the rational buyer who concedes to seller i at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$.

Then, it follows that $D^{i,T_i} := \{t | U_i(t, F_b^{i,T_i}) = \max_{s \in [T_i, T_i']} U_i(s, F_b^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_b^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $D^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_b^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_b^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda_s t}$ and $F_b^{i,T_i}(t) = 1 - c_b^i e^{-\lambda_b t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_b^i = 1 - F_b^{i,T_i}(T_i)$ such that $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$ and $\lambda_s = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_b}$.

Therefore, the rational buyer's expected payoff of playing the concession game with seller i during $[T_i, T_i']$ is $[F_i^{T_i}(T_i))(1 - \alpha_b) + (1 - F_i^{T_i}(T_i))(1 - \alpha_s)]$. Moreover, by Lemma A.3, we know that if the buyer is strong in a concession game with seller i (starting at time T_i), then seller i is weak. Hence, there is no sequential equilibrium of the game G such that the buyer visits a store multiple times. Suppose on the contrary that there is a strategy in which, without loss of generality, the buyer visits store 1 twice. Then, the buyer must be strong in his second visit to seller 1. Otherwise the buyer would prefer to concede to seller 2 and finish the game before making the second visit to store 1 (because $\delta < 1$). Thus, since seller 1 is weak, his expected payoff is α_b when the buyer visits his store for the second time. However, in equilibrium, this continuation payoff contradicts the optimality of seller 1's strategy because seller 1 would prefer to accept the buyer's offer (for sure) when the buyer first attempts to leave his store to eliminate a further delay.

As a result, in equilibrium, rational sellers will not allow the buyer to leave their stores. On the other hand, the rational buyer will eventually leave the first store he visits if that seller is obstinate. The reason for this is clear. Since the players' concession game strategies are increasing and continuous, the seller's reputation will eventually converge to one at some finite time. The rational buyer has no incentive to continue the concession game with an obstinate seller, and so he must either concede to the seller at that time or leave the store. However, Lemma A.2 implies that concession game strategies must be continuous in their domain, eliminating the possibility of mass acceptance at the time that the seller's reputation reaches one.

Next, for notational simplicity, I reset the clock each time the buyer arrives at a store, and denote the buyer's concession game strategy against seller i by F_b^i and i's strategy by F_i. Now, consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1, leaves store 1 at time T_1^d and finalizes the game in store 2 at time T_2^e if the game has not yet ended before. Then, rational buyer visits seller 2 only if $F_2(0) > 0$ is true. Suppose $F_2(0) = 0$. Then, the rational buyer's discounted continuation payoff in store 2, $\delta[F_2(0)(1-\alpha_b)+(1-F_2(0))(1-\alpha)]$, will be $\delta(1-\alpha)$. In this case, the rational buyer prefers to concede to seller 1 instead of traveling store 2, yielding the required contradiction. By lemma A.3., as $F_2(0) > 0$, we must have $F_b^2(0) = 0$, implying that $c_b^2 = 1$. That is, $F_b^2(t) = 1 - e^{-\lambda_b t}$. Furthermore, assuming that the rational buyer leaves store 1 at time T_1^d and the concession game in store 2 ends at time T_2^e, we must have $F_1(T_1^d) = 1 - z_s$ and $F_1(T_2^e) = 1 - z_s$. Thus we have $c_1 = z_s e^{\lambda T_1^d}$ and $c_2 = z_s e^{\lambda T_2^e}$ as required.

Finally, Lemma A.3 implies that $F_b^1(0)F_1(0) = 0$. Since seller 2's reputation reaches 1 at time T_2^e, then the rational buyer will not continue the game G after this time. Thus, his reputation must also reach 1 at that time, implying that $F_b^2(T_2^e) = 1 - z_b^*$ where $z_b^* = \frac{\hat{z}_b^1}{1-F_b^1(T_1^d)}$ is the buyer's reputation at the time he arrives at store 2 and $\hat{z}_b^1 = \frac{z_b}{z_b+2(1-z_b)\sigma_1}$ is the buyer's reputation at the time he arrives at store 1.

Proof of Lemma 3.1. Consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1 and $\hat{z}_b^1 \geq z_b^* = (z_s/A)^{\lambda_b/\lambda_s}$. Therefore, the rational buyer (weakly) prefers to go to store 2 over conceding seller 1. In equilibrium, rational seller 1 anticipates that the buyer will never concede to him, and hence accepts α_b at time zero without any delay. Therefore, if 1 is rational the game is over at time zero. Otherwise, the buyer leaves the first store at time zero and directly goes to 2. Therefore, the concession game in store 2 ends at time $T_2^e = \tau_b^2 = \min\{\tau_b^2, \tau_2\}$ for sure where $\tau_b^2 = \inf\{t \geq 0 | \ F_b^2(t) = 1 - \hat{z}_b^1\} = -\frac{\log \hat{z}_b^1}{\lambda_b}$ and $\tau_2 = \inf\{t \geq 0 | \ F_2(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_s}$ denote the times that the buyer's and seller 2's reputations reach 1, respectively. Given the equilibrium strategies by Proposition 3.1, the rest follows.

Proof of Lemma 3.2. Consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1 and $\hat{z}_b^1 \leq (z_s^2/A)^{\lambda_b/\lambda_s} < z_b^*$. Then, the rational buyer prefers to play the concession game with seller 1 over going to store 2 at time zero. Since the buyer leaves store 1 if and only if seller 1 is obstinate, seller 1's reputation reaches one at time $T_1^d = \tau_1 = \min\{\tau_b^1, \tau_1\}$ where $\tau_b^1 = \inf\{t \geq 0 | \ F_b^1(t) = 1 - \hat{z}_b^1\} = -\frac{\log \hat{z}_b^1}{\lambda_b}$ and $\tau_1 = \inf\{t \geq 0 | \ F_2(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_s}$ denote the times that the buyer's and seller 1's reputations reach 1, respectively.

However, leaving 1 is optimal for the rational buyer if and only if the buyer's reputation at time T_1^d reaches z_b^*, implying that

$$c_b^1 e^{-\lambda_b T_1^d} = \frac{\hat{z}_b^1}{z_b^*} \tag{11}$$

given the value of T_1^d, solving the last equality yields the buyer's equilibrium strategy in store 1. Finally, the game ends in store 2 at time $T_2^e = \tau_b^2 = \min\{\tau_b^2, \tau_2\}$ for sure where $\tau_b^2 = -\frac{\log z_b^*}{\lambda_b}$ and $\tau_2 = -\frac{\log z_s}{\lambda_s}$, at which points both players' reputation simultaneously reach one. Given the value of T_2^e, Proposition 3.1 implies the concession game strategies in the second store.

Proof of Lemma 3.3. Consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1 and $(z_s^2/A)^{\lambda_b/\lambda_s} < \hat{z}_b^1 < z_b^* = (z_s/A)^{\lambda_b/\lambda_s}$. Again, the rational buyer leaves 1 when his reputation reaches z_b^*, implying Equation (11) must hold. If $c_b^1 = 1$, then $T_1^d = -\frac{\log \hat{z}_b^1}{\lambda_b} + \frac{\log z_s/A}{\lambda_s}$, and it is smaller than $-\frac{\log z_s}{\lambda_s}$ as $(z_s^2/A)^{\lambda_b/\lambda_s} < \hat{z}_b^1$. Similar to Lemma 3.2, the game ends in store 2 at time $T_2^e = \frac{\log z_b^*}{\lambda_b}$. Given the values of T_1^d and T_2^e, Proposition 3.1 implies the concession game strategies.

Proof of Proposition 3.2. First, let $z_b > (z_s^2/A)^{\lambda_b/\lambda_s}$. Suppose for a contradiction that $\sigma_i > 1/2$ holds. Similar arguments also works if one supposes $\sigma_i < 1/2$. By Bayes' rule, we have $\hat{z}_b^i = \frac{z_b}{z_b + 2(1-z_b)\sigma_i}$. Therefore, this would imply that $\hat{z}_b^j > \hat{z}_b^i$ and $\hat{z}_b^j > z_b$. Since the rational buyer's expected payoff of visiting store i, V_b^i, is increasing with $\hat{z}_b^i$ (see Equations 7 and 8), the buyer would strictly prefer to visit seller j first as we will have $V_b^j > V_b^i$, contradicting with $\sigma_i > 1/2$.

Now, let $z_b \leq (z_s^2/A)^{\lambda_b/\lambda_s}$. Then in any equilibrium, we must have $\hat{z}_b^i \leq (z_s^2/A)^{\lambda_b/\lambda_s}$ that yields the desired result by the Bayes' rule. On the other hand, suppose for a contradiction that there is an equilibrium where $\hat{z}_b^i > (z_s^2/A)^{\lambda_b/\lambda_s}$ holds for some i. In this case, we must also have that $\hat{z}_b^j > (z_s^2/A)^{\lambda_b/\lambda_s}$. Since the rational buyer must choose $\sigma_i \in (0,1)$ in equilibrium, implying $V_b^1 = V_b^1$, we must have $\hat{z}_b^1 = \hat{z}_b^2$. However, the last equality is possible only if $\sigma_1 = \frac{1}{2}$, implying $\hat{z}_b^1 = \hat{z}_b^2 = z_b \leq (z_s^2/A)^{\lambda_b/\lambda_s}$. The last inequality yields the desired contradiction.

Proof of Proposition 3.4. First note that there is no equilibrium in which the buyer visits store 1 multiple times and store 2 more than twice. Second, since we have $1-\alpha_1 < \delta(1-\alpha_2)$, the rational buyer prefers going to store 2 over conceding to seller 1 at any given time. That is, in equilibrium, the rational buyer never concedes to seller 1. Since rational seller 1 anticipates that the rational buyer will never accept his demand in equilibrium, he concedes to the buyer with probability one upon his arrival without any delay. Thus, the buyer leaves seller 1 immediately if rational seller 1 does not accept the buyer's demand and finish the game at time zero.

If the buyer arrives at store 2 (after visiting seller 1), then the rational buyer and seller 2 play the concession game until some finite time T_2^e as the buyer has no outside option worth leaving store 2. As characterized in the proof of Proposition 3.1, the equilibrium strategies are $F_b^2(t) = 1 - c_b^2 e^{-\lambda_b t}$ and $F_2(t) = 1 - c_2 e^{-\lambda_2 t}$ where $\lambda_b = \frac{\alpha_b r_s}{\alpha_2 - \alpha_b}$ and $\lambda_2 = \frac{(1-\alpha_2) r_b}{\alpha_2 - \alpha_b}$. Therefore, the game in store 2 ends at time $T_2^e = \min\{\tau_b^2, \tau_2\}$ for sure if it does not end before, where $\tau_b^2 = \inf\{t \geq 0 | \ F_b^2(t) = 1 - \hat{z}_b^1\} = -\frac{\log \hat{z}_b^1}{\lambda_b}$ and $\tau_2 = \inf\{t \geq 0 | \ F_2(t) = 1 - z_s\} = -\frac{\log z_s}{\lambda_2}$, denoting the times that the buyer's and seller 2's reputations reach 1 respectively.

To prove the second part of the proposition, suppose that the buyer visits seller 2 first at time 0. If the rational buyer concedes to seller 2, his instantaneous payoff is $1 - \alpha_2$. However, if the rational buyer leaves store 2 at time zero and goes to store 1, then we know from previous arguments that rational seller 1 will immediately accept the buyer's demand. Therefore, the rational buyer's continuation payoff of leaving store 2 at time 0 is $\bar{V}_b = \delta[(1-z_s)(1-\alpha_b)+\delta z_s v_b^2]$ where $v_b^2 = (1-F_2(0))(1-\alpha_2)+F_2(0)(1-\alpha_b)$ denoting the buyer's expected payoff in his second visit to store 2. In equilibrium v_b^2 must be equal to $1-\alpha_2$. Suppose for a contradiction that $v_b^2 > 1-\alpha_2$. It requires that seller 2 offers positive probabilistic gift to the buyer on his second visit. In this case, seller 2's expected payoff must be α_b (as $F_b^2(0)F_2(0) = 0$ by Lemma A.3). However, optimality of the equilibrium strategy implies that rational seller 2 should have accepted the buyer's offer with probability 1 when the buyer attempts to leave his store for the first time. Hence, it must be that in equilibrium $v_b^2 = 1-\alpha_2$. As a result, the rational buyer's expected payoff if he leaves store 2 at time 0 is $\bar{V}_b = \delta\left[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)\right]$.

Finally, if $\bar{V}_b$ is larger than $1-\alpha_2$, then the rational buyer prefers leaving store 2 immediately at time 0 over conceding to seller 2 at time zero. $\bar{V}_b > 1-\alpha_2$ implies that $z_s < \frac{1-\alpha_b-\frac{1-\alpha_2}{\delta}}{1-\alpha_b-\delta(1-\alpha_2)} := \bar{z}$. Note that $\bar{z} > 0$ since by assumption we have $\delta(1-\alpha_b) > (1-\alpha_2)$. Indeed, $\bar{z}$ takes values very close to 1 as the search friction is sufficiently small. Thus, for small values of z_s, that is $z_s < \bar{z}$, the rational buyer finds it optimal to leave store 2 immediately at time 0. On the other hand, rational seller 2 prefers immediate concession at time zero over letting the buyer leave his store because immediate concession ensures payoff of α_b which is much higher than what he can achieve if he lets the buyer leave his store at time zero, i.e. $z_s\alpha_2$.

Proof of Proposition 3.5. The rational buyer's expected payoff of visiting store 2 first, V_b^2, is $(1-z_s)(1-\alpha_b)+z_s\delta[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)]$. However, V_b^1 is either $(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)$ if the buyer is weak in store 2, i.e. $\hat{z}_b^1 \leq z_s^{\lambda_b/\lambda_2}$, or $(1-z_s)(1-\alpha_b)+\delta z_s u_b^2$ otherwise, where $u_b^2 = z_s[\hat{z}_b^1]^{-\lambda_2/\lambda_b}(1-\alpha_2)+(1-z_s[\hat{z}_b^1]^{-\lambda_2/\lambda_b})(1-\alpha_b)$ and $\hat{z}_b^1 = \frac{1/2z_b}{1/2z_b+(1-z_b)\sigma_1}$. In equilibrium, it must be true that $V_b^1 = V_b^2$.

If σ_1 is such that the buyer is weak in store 2, then the last equality implies that $z_s = \frac{1-\alpha_b-(1-z_s)(1-\alpha_b)-\delta z_s(1-\alpha_2)}{1-\alpha_b-\delta[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)]}$. However, this equality cannot be true when z_s is small and δ is close enough to 1. Hence, in equilibrium, the rational buyer must pick σ_1 in such a way that he becomes strong relative to seller 2 when he visits store 1 first. Thus, $V_b^2 = (1-z_s)(1-\alpha_b)+\delta z_s u_b^2$ implies the desired result.

Proof of Proposition 4.3. Given that both sellers choose α_s, the equilibrium strategies of the rational buyer in the first stage, σ_i and μ must satisfy the followings.

1. σ_i is the probability of visiting seller i first with $\sigma_1+\sigma_2 = 1$ and μ is a probability distribution over the set $D \subset C_{\alpha_s} = \{\alpha_b \in C | \alpha_b \leq \alpha_s\}$ with $\sum_{x\in D}\mu(x) = 1$.

2. For all $i \in \{1,2\}$ and $\alpha_b \in D$ we must have $V_b^i(\alpha_b) = V$ where

$$V_b^i(\alpha_b) = \begin{cases} 1-\alpha_s & \text{if } z_{\alpha_b} \geq 1 \\ (1-\alpha_b)\left[1-z_{\alpha_b}\right]+(1-\alpha_s)z_{\alpha_b} & \text{if } z_s < z_{\alpha_b} < 1 \\ (1-\alpha_b)[1-z_s(1-\delta)-\delta A z_{\alpha_b}]+(1-\alpha_s)\delta A z_{\alpha_b} & \text{otherwise} \end{cases}$$

where $z_{\alpha_b} = \frac{z_s^2}{A(\hat{z}_b^i)^{\lambda_s/\lambda_b}}$ and $\hat{z}_b^i = \frac{\frac{1}{2}z_b\pi(\alpha_b)}{\frac{1}{2}z_b\pi(\alpha_b)+(1-z_b)\mu(\alpha_b)\sigma_i(\sum_{x\in D}\pi(x))}$. This payoff function is implied by Lemmas 3.1-3.3.

3. $V \geq 1 - \min\{C \setminus D\}$. That is, the rational buyer should have no incentive to deviate and declare some other demand α_b' which is not in the support of μ.

Therefore, in equilibrium μ and σ_i are solutions of $\#D+1$ (nonlinear) equations for $\#D+1$ unknowns. For small values of z_b (relative to z_s), existence of these strategies is easy to show. Consider the following strategies for all $\alpha_b \in C_{\alpha_s} \setminus \{\alpha_s\}$, $\mu^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ and $\sigma_i = \frac{1}{2}$. Then, we have $\hat{z}_b^i = z_b$ for all $\alpha_b < \alpha_s$. Moreover, since we have $z_b \leq (z_s^2/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b \leq \alpha_s$, then we have $z_{\alpha_b} \geq 1$ for all $\alpha_b \leq \alpha_s$, implying that these strategies satisfy the requirements 1-3. Hence, together with the second stage strategies given in Section 3, μ^* and σ_i are the rational buyer's equilibrium strategies.

Finally, given that the rational players' second stage strategies are as characterized in section 3, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff under the strategies μ^* and σ_i that are given above. Let V_i denote seller i's expected payoff in the game. Since a deviating seller's equilibrium payoff is less than $(z_b + z_s)$ (by Corollary 3.1), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. Moreover, following the assumptions on obstinate types, if a seller deviates and posts a price above α_s, then his expected payoff in the game will be simply zero. Thus, $V_i = p\alpha_s + (\frac{1}{2} - p)(a + b)$ and we calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b\sum_{\alpha_b\geq\alpha_s}\pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer picks store i first and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1 - z_b) + z_b\frac{1}{2} - p]$, and rational seller i's expected payoff in this case is $\sum_{\alpha_b<\alpha_s}[\frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}][\alpha_b + F_b^i(0)(\alpha_s - \alpha_b)] := a$ where $F_b^i(0) = 1 - z_b(A/z_s^2)^{\frac{\alpha_b r_s}{(1-\alpha_s)r_b}}$.

Case 4. The buyer picks store i second and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and rational seller i's expected payoff in this case is $\frac{e^{-\Delta r_s}z_s}{\sum_{x<\alpha_s}\pi(x)}\sum_{\alpha_b<\alpha_s} z_s^{\frac{r_s(\alpha_s-\alpha_b)}{(1-\alpha_s)r_b}}\alpha_b\pi(\alpha_b) := b$. Note that the buyer will visit the second store only if the first seller is obstinate and the rational buyer announces $\alpha_b < \alpha_s$. Therefore, seller i's expected payoff in this case is discounted by the travel time $e^{-\Delta r_s}$ and $z_s^{\frac{r_s(\alpha_s-\alpha_b)}{(1-\alpha_s)r_b}}$ - the discount due to the delay in the first store j, i.e. T_j^d.

Note that V_i is strictly greater than $(\frac{1}{2} - p)u$ where u is the convex combination of the demands in $C_{\alpha_s} \setminus \{\alpha_s\}$, i.e., $u = \sum_{\alpha_b<\alpha_s}\alpha_b\mu(\alpha_b)$, and it is much higher than $(z_b + z_s)$ if z_b and

z_s are sufficiently small. Hence, posting α_s is optimal for each seller. This completes the proof of the only if part. That is, α_s is an equilibrium price selection of the rational sellers in stage 1.

Next, I will prove the if part. For this purpose, I assume that $0 < \alpha_s \in C$ is an equilibrium demand of the game $G(z_b, z_s)$. Suppose for a contradiction that there exists some $\alpha_b^* < \alpha_s$ such that $z_b > (z_s^2/A)^{\lambda_b^*/\lambda^*}$. Consider any first stage equilibrium strategies of the rational buyer μ and σ_i. I will reach a contradiction in two steps. First, I want to show that if there is such α_b^*, then the rational buyer's expected payoff in the game must be strictly higher than $1 - \alpha_s$ (which is the least the rational buyer can achieve in an equilibrium where both sellers post the price of α_s).

To prove the last claim, assume for a contradiction that the buyer's expected payoff under the strategies μ and σ_i is $1 - \alpha_s$. That is, for each i, $\hat{z}_b^i \leq (z_s^2/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b < \alpha_s$. However, the strategies $\mu'(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ for all $\alpha_b < \alpha_s$ and $\sigma_i' = 1/2$ ensures that $\hat{z}_b^i = z_b$ when the buyer announces α_b^*, implying that the buyer is strong if he announces α_b^* in the first stage. This means that the rational buyer's expected payoff in the game is strictly higher than $1 - \alpha_s$ under μ' and σ_i' ; the rational buyer's expected payoff is strictly higher than $1 - \alpha_s$ if he announces α_b^* and is equal to $1 - \alpha_s$ for all other demands other than α_b^*. This contradicts to the earlier assumption that μ and σ_i are equilibrium strategies for the rational buyer.

Hence, the buyer's expected payoff under equilibrium must be strictly higher than $1 - \alpha_s$ conditional on there is such a demand α_b^*. However, since μ is a mixed strategy, then the buyer's expected payoff for all realizations of demands $\alpha_b < \alpha_s$ must be strictly higher than $1 - \alpha_s$. But, this is impossible when $\alpha_b = 0$ as Proposition 3.3 shows, leading to the desired contradiction.

Proof of Proposition 5.2. The same arguments used in the proofs of Propositions 4.1 suffice to show that in equilibrium, all the sellers will choose the same demand in stage 1. Moreover, recall that the proof of Proposition 5.1 relies solely on the fact that the buyer must be weak for each α_b in the support of μ^*. Same arguments in the proof of Proposition 4.3 shows that if there are N identical sellers, the buyer must be weak in equilibrium as well. Next, I will show that being weak in equilibrium with N sellers means $z_b \leq \left(z_s^N/A^{N-1}\right)^{\lambda_b/\lambda_s}$.

For the ease of exposition, I will derive this condition for the 3-sellers case, which can be extended to N-sellers case by iterating the same process. For this reason, suppose now that there are three sellers all of which choose the same demand α_s in stage 1 and the buyer declares his demand as $\alpha_b < \alpha_s$. Without loss of generality, I assume that the buyer visits seller 1 first and seller 3 last (if no agreement have been reached with the sellers 1 and 2). The following arguments are straightforward extensions of the approach that I use in the proof of Proposition 3.1. Therefore, let T_i^d denote the time that the buyer leaves seller $i \in \{1, 2\}$ and $\hat{z}_b(T_i^d)$ denote the buyer's reputation at the time he leaves store i.

The rational buyer leaves seller 2 when his discounted continuation payoff in store 3, i.e. $\delta[1 - \alpha_b - z_s[\hat{z}_b(T_2^d)]^{-\lambda_s/\lambda_b}(\alpha_s - \alpha_b)]$, equals to $1 - \alpha_s$. This equality implies that $\hat{z}_b(T_2^d) = (z_s/A)^{\lambda_b/\lambda_s}$. As a result, the buyer's expected payoff in store 2 at the time he enters this store is $v_b^2 = 1 - \alpha_b - z_s \left[\frac{(z_s/A)^{\lambda_b/\lambda_s}}{\hat{z}_b(T_1^d)}\right]^{\lambda_s/\lambda_b} (\alpha_s - \alpha_b)$. Similarly, the buyer leaves seller 1 when his discounted continuation payoff in store 2, i.e. δv_b^2, equals to $1-\alpha_s$. Then we have $\hat{z}_b(T_1^d) = \left(z_s^2/A^2\right)^{\lambda_b/\lambda_s}$.

Also, note that we have $\hat{z}_b(T_1^d) = \frac{\hat{z}_b^1}{1-F_b^1(T_1^d)}$, $F_b^1(T_1^d) = 1 - c_b^1 e^{-\lambda_b T_1^d}$ and $c_b^1 = 1$ because the buyer is weak. Thus, it must be true that $T_1^d = -\frac{\log(\hat{z}_b^1/(z_s^2/A^2)^{\lambda_b/\lambda_s})}{\lambda_b} \geq \frac{-\log z_s}{\lambda_s}$ again because the buyer is weak. The last inequality implies $\hat{z}_b^1 \leq \left(z_s^3/A^2\right)^{\lambda_b/\lambda_s}$. In equilibrium, the last inequality must hold for all $\hat{z}_b^i$ with $i = 1, 2, 3$, implying that it must hold for z_b as well. The rest directly follows from the parallel arguments of the proof of Propositon 5.1. Iterating the above arguments suffice to prove the claim for any finite N.

Proof of Proposition 6.1. Let z_b, z_s and $\alpha_s \in C \setminus \{0\}$ satisfy the assumptions of the proposition. Suppose for a contradiction that there exists an equilibrium of the competitive-bargaining game $G(z_b, z_s)$ supporting α_s. In this equilibrium, the buyer must randomize over the set C_{α_s} according to μ and visit seller i with probability σ_i. Consider the seller whose expected payoff in the game (according to this equilibrium strategy) is smaller. Call this seller, without loss of generality seller 1. Note that the payoff of seller 1 must be much smaller than $\frac{1}{2}\alpha_s$. Consider now the continuation play following the history h where seller 1 deviates to $\alpha_1 < \alpha_s$ such that $(1-\alpha_s) > \delta(1-\alpha_1)$. Next, I want to show that in equilibrium following h, the rational buyer will visit seller 1 with probability 1 to accept α_1. As a result, the expected payoff to seller 1 will be α_1. As C is dense enough and $\alpha_1 > \alpha_s/2$, the price undercutting will be optimal, contradicting with the assumption that α_s is an equilibrium price. Therefore, we need to show that $\sigma_2 = 0$ in equilibrium following the history h. Suppose for a contradiction that $0 < \sigma_2$.

First, in any equilibrium following the history h, a commitment type buyer with demand more than or equal to α_1 directly goes to store 1 to accept seller 1's demand. An obstinate buyer with demand strictly less than α_1 goes to store 2 with probability one. Thus, the rational buyer immediately accepts α_1 if he visits seller 1 first because otherwise leaving store 1 and going to store 2 reveals his rationality, implying a lower payoff $(1-\alpha_s)$ to the buyer. Second, the rational buyer never leaves seller 2 (once he visits him) and goes to 1 because the rational buyer's payoff of accepting seller 2's demand $(1-\alpha_s)$ is higher than his payoff of traveling to store 1 and accepting seller 1's demand as we assumed $1-\alpha_s > \delta(1-\alpha_1)$. Therefore, in any equilibrium following the history h, the buyer and seller 2 play the concession game until time $T = \left\{-\frac{\ln z_s}{\lambda_s}, -\frac{\ln \hat{z}_b^2}{\lambda_b}\right\}$ if the buyer visits seller 2 first, and this implies that the rational buyer's expected payoff of visiting store 2 is $F_2(0)(1-\alpha_b) + (1-F_2(0))(1-\alpha_s)$ given that the buyer demands $\alpha_b \in C$ in stage 1. As a result, if $\sigma_2 = 1$, then the buyer must choose μ in such a way that the buyer is strong in store 2. This is possible only if $\mu(\alpha_b) < \frac{\pi(\alpha_b)}{\sum_{x \in supp(\mu)} \pi(x)}$ for all α_b in the support of μ (that is, $supp(\mu)$) since by assumption we have $z_b \leq (z_s/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b < \alpha_s$. However, if μ satisfies it, then we will have $\sum_{x \in supp(\mu)} \mu(x) < 1$ leading to a contradiction. Hence, we must have $\sigma_2 \in (0, 1)$.

Playing a mixed strategy implies that the rational buyer's expected payoff of visiting either store is $1-\alpha_1$. Hence, in any equilibrium of the continuation game following the history h, the rational buyer must choose $\sigma_2 \in (0, 1)$ and μ in such a way that $\hat{z}_b^2$ is high enough for any α_b

in the support of μ so that the buyer is strong in the second store. However, the buyer being strong in the second store implies that 0 is also in the support of μ. However, if the buyer demands 0 in stage 1, his expected payoff in the continuation game will be $1-\alpha_s$ regardless of $\hat{z}_b^2$ and z_s, contradicting that σ_2 and μ are equilibrium strategies. Hence, we must have $\sigma_2 = 0$ in equilibrium following the history h.

Proof of Proposition 7.1. The "if" part follows from similar arguments of the proof of Proposition 4.3, and so it is omitted. To prove the "only if" part, suppose that the Poisson arrival rate of the buyer is κ. First, if the players play the strategies described in the main text, then the Bayes' rule implies that the probability of the buyer being the commitment type α_b conditional on him visiting seller i during the period of $[T, T+dt]$ and demanding $\alpha_b < \alpha_i$ is

$$\hat{z}_b^{(T+dt),i} = \frac{\frac{1}{2}z_b\pi(\alpha_b)\kappa dt + \frac{1}{2}z_b z_s\pi(\alpha_b)\kappa dt}{\frac{1}{2}z_b\pi(\alpha_b)\kappa dt + \frac{1}{2}z_b z_s\pi(\alpha_b)\kappa dt + (1-z_b)\mu_{\alpha_i}^T(\alpha_b)\sigma_i\left(\sum_{x<\alpha_i}\pi(x)\right)\kappa dt}$$

The first term in the numerator corresponds to the probability that the obstinate buyer with demand α_b is visiting seller i first and arriving at the market in a short period dt. Likewise, the second term denotes the probability that the obstinate buyer visits seller i second, implying that the buyer should have arrived at the market $-\log(z_s)/\lambda_s + \Delta$ units of time ago during the short period dt.[47]

Given the strategies of the players, if the buyer arrives at the market at the period $0+dt$, then the obstinate buyer's arrival time at the second store is $\bar{T} = -\log(z_s)/\lambda_s + \Delta + dt$. Therefore, the second term in the numerator does not exists if $T < \bar{T}$. Moreover, the limiting case where dt approaches zero implies that $\hat{z}_b^{T,i}$ equals to z_b for all $T < \log(z_s)/\lambda_s + \Delta$ and to $\frac{z_b(1+z_s)}{1+z_b z_s}$ otherwise.

Second, for any $0 < \alpha_b < \alpha_s$, we have $\hat{z}_b^{T,i} < z_s^{\lambda_b/\lambda_s}$ because $z_b < \frac{z_s^{\lambda_b/\lambda_s}}{1+z_s(1-z_s^{\lambda_b/\lambda_s})}$. Moreover, according to the strategies, the rational buyer never leaves the sellers' stores. This implies that the buyer and the seller will play the concession game according to the strategies F_b and F_i's until the time $-\frac{\log(z_s)}{\lambda_s} = \min\{-\frac{\log z_s}{\lambda_s}, -\frac{\log \hat{z}_b^{T,i}}{\lambda_b}\}$ (this directly follows from Abreu and Gul (2000), Proposition 1.) As a result, the buyer's expected payoff in each store is $1-\alpha_s$ because independent of the buyer's arrival time at either store, the buyer will be weak in both. Hence, visiting each seller with equal probabilities is an optimal strategy for the rational buyer. Furthermore, if the rational buyer leaves his current bargaining partner at any point of time and goes to the other seller, then his continuation payoff will be $\delta(1-\alpha_s)$. Hence, not leaving a seller's store and playing the concession game until the time $-\log(z_s)/\lambda_s$ are also optimal strategies.

Third, independent of α_b ($\leq \alpha_s$), the rational buyer's expected payoff is $1-\alpha_s$ in each store. Thus, the mixed strategy $\mu_{\alpha_s}^T(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ is an optimal strategy for the rational buyer.

Finally, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this person, I will first calculate each seller's expected

[47] Recall that $-\log(z_s)/\lambda_s$ is the length of the concession game in the stores where $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_b}$, and Δ is the time required to travel between the stores.

payoff under the strategies given in the main text. Let $V_i(T)$ denote seller i's expected payoff in the game (evaluated in time T) given that the buyer arrives at the market at time $T \geq 0$. Then, I calculate a deviating seller's equilibrium payoff (again evaluated in time T assuming that the buyer arrives at the market at T) and argue that it is smaller than $V_i(T)$ if we choose z_b and z_s sufficiently small. Thus, $V_i(T) = [p\alpha_s + (\frac{1}{2} - p)(a+b)]$ where

Case 1. The buyer picks store i first and he is the obstinate type with demand $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2} z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b) := p$ and seller i's expected payoff is α_s.

Case 2. The buyer picks the other store j first and he is the obstinate type with demand $\alpha_b \geq \alpha_s$. Probability to this event is p and i's expected payoff is 0.

Case 3. The buyer picks store i first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1-z_b) + z_b \frac{1}{2} - p]$, and seller i's expected payoff is $\sum_{\alpha_b < \alpha_s} [\frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}][\alpha_b + F_b^T(T)(\alpha_s - \alpha_b)] := a$ where $F_b^T(T) = 1 - \hat{z}_b^{T,i} z_s^{-\frac{\alpha_b r_s}{(1-\alpha_s) r_b}}$.

Case 4. The remaining case is that the buyer picks store j first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and i's expected payoff is $\frac{e^{-r_s \Delta} z_b z_s}{\sum_{x<\alpha_s} \pi(x)} \sum_{\alpha_b < \alpha_s} \alpha_b z_s^{r_s/\lambda_s} \pi(\alpha_b) \int_0^{\frac{-\log(z_s)}{\lambda_s}} e^{-r_s t} \frac{dF_s(t)}{1-z_s} := b$ where $F_s(t) = 1 - e^{-\lambda_s t}$.

On the other hand, if seller i price undercuts j and posts α_i such that $0 < \alpha_i < \alpha_s$, then rational seller i's expected payoff is $\left(\left[z_b \sum_{\alpha_b \geq \alpha_i} \pi(\alpha_b)\right] + z_s \left[1 - z_b \sum_{\alpha_b \geq \alpha_i} \pi(\alpha_b)\right]\right) \alpha_i$, and it is less than $(z_b + z_s)\alpha_i$ (see Corollary 4.1). This is true because in any equilibrium following the history where seller i price undercuts j, the rational buyer visits seller j first with certainty, makes a take-it-or-leave-it offer 0, which will be accepted by the rational seller j, and immediately leaves if seller j does not accept 0. Then, the rational buyer immediately visits seller i to accept α_i. It is clear that $(z_b + z_s)\alpha_i < V_i(T)$ for sufficiently small values of z_b and z_s.

Proof of Proposition 8.1. I will show that the strategies given in the main text constitute and equilibrium. Suppose that the rational buyer announces $\alpha_b < \alpha_s$ in stage 1 and consider the second stage. First, at time zero, the rational buyer and seller 1 has two options; accept and reject. Rejection for the buyer means leaving the store. I assume that if the buyer chooses to leave but the seller accepts, then the game will end with the seller's acceptance. If the rational buyer does not leave the first store at time zero, he reveals his rationality, in which case the buyer's expected payoff will be no more than $1-\alpha_s$ (since the buyer is discounting time). Hence, in equilibrium, the rational buyer will either concede or leave the store at time zero.

Second, if the rational buyer finishes the game in store 1 with probability P_b, then the buyer's reputation conditional on him arriving store 2 after visiting 1 is $(z_s/A)^{\lambda_b/\lambda_s}$ as calculated by $\frac{z_b}{z_b+(1-z_b)(1-P_b)}$. Therefore, the buyer and seller 2 will play the concession game until time $T_2^e = \min\{-\frac{log(z_s/A)}{\lambda_s}, -\frac{log z_s}{\lambda_s}\}$ which is equal to $-\frac{log(z_s/A)}{\lambda_s}$ as $A < 1$. Thus, the equilibrium concession game strategies in store 2 must be as given in the main text. As a result, the rational buyer's expected payoff in the second store is $\frac{1-\alpha_s}{\delta}$.

Third, the rational buyer's expected payoff of accepting α_s in store 1 is

$$V_b(accept) = z_s(1-\alpha_s) + (1-z_s)\left[\frac{1}{2}P_s(2-\alpha_s-\alpha_b) + (1-P_s)(1-\alpha_s)\right]$$

whereas

$$V_b(reject) = z_s\delta V + (1-z_s)[P_s(1-\alpha_b) + (1-P_s)\delta V]$$

where $V = \frac{1-\alpha_s}{\delta}$ is the buyer's continuation payoff when he leaves the first seller at time zero. Note that if $P_s = 0$, then $V_b(accept) = V_b(reject) = 1-\alpha_s$, implying that the buyer's strategy P_b is a best response. Moreover, since the rational buyer's expected payoff in each store and in the game, regardless of his announcement $\alpha_b < \alpha_s$, is $1-\alpha_s$, visiting each seller with probability $1/2$ and announcing α_b according to μ^* are also best response strategies.

Similarly, rational seller i's expected payoff is

$$V_i(accept) = z_b\alpha_b + (1-z_b)\left[\frac{1}{2}P_b(\alpha_s+\alpha_b) + (1-P_b)\alpha_b\right]$$

whereas

$$V_i(reject) = z_b 0 + (1-z_b)\left[P_b\alpha_s + (1-P_b)0\right]$$

Therefore, given the value of P_b and $z_b \leq \frac{(z_s/A)^{\lambda_b/\lambda_s}(\alpha_s-\alpha_b)}{\alpha_s+\alpha_b}$, we have $V_i(accept) < V_i(reject)$. Hence, $P_s = 0$ is a best response as well.

Finally, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff in the game for the second stage strategies given in the main text. Let V^i denote seller i's expected payoff in the game. Since a deviating seller's equilibrium payoff is less than $(z_b + z_s)$ (by Corollary 3.1), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. We have $V^i = \alpha_s\left[p + \frac{(1-z_s)}{2}[P_b + e^{-r_s\Delta}(1-P_b)]\right]$ and calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b\sum_{\alpha_b\geq\alpha_s}\pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer is obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $z_b - 2p$ and rational seller i's expected payoff in this case is 0.

Case 4. The buyer picks store i first and he is rational. Probability to this event is $(1-z_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $P_b\alpha_s$.

Case 5. The buyer picks store i second and he is rational. Probability to this event is $(1-z_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $(1-P_b)e^{-r_s\Delta}\alpha_s$.

Note that for small values of z_b and z_s, the value of V^i is greater than $(z_b + z_s)$ which concludes the proof.

Proof of Proposition 8.2. Similar arguments in the proof of Proposition 8.1 will prove our claim. Note that given the value of P_b, as in the main text, the buyer's reputation conditional on him announcing α_b and arriving store 2 after visiting store 1 is $z_b^* = 1 - \frac{\alpha_b}{\alpha_s}$. The value of z_b^* makes rational seller 2 indifferent between immediate concession, with payoff of α_b, and rejection with payoff of $(1 - z_b^*)\alpha_s$. Since rational seller 2 is indifferent, immediate concession with probability P_s (as given in the main text) is optimal. Moreover, P_s ensures the expected payoff of $\frac{(1-\alpha_s)}{\delta}$ to the rational buyer, and it makes the buyer indifferent between conceding to seller 1 and leaving for seller 2. Finally, with the value of P_b and $z_b \leq \frac{(\alpha_s-\alpha_b)^2}{\alpha_s(\alpha_s+\alpha_b)}$, rational seller 1's expected payoff of rejecting the buyer's demand is higher than conceding to him as $V_1(accept) = z_b\alpha_b + (1 - z_b)[\frac{1}{2}P_b(\alpha_s + \alpha_b) - (1 - P_b\alpha_b)]$ and $V_1(reject) = (1 - z_b)P_b\alpha_s$.

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